Signaling games

- Player 1, **the sender**, has private information about his type, and chooses an action (message), which is observed by player 2.
- Player 2, **the receiver**, observes only player 1's message, and then chooses an action herself.

To describe a signaling game, we need to know the following:

- The set of possible types for player 1, T
- A prior distribution over 1's types, $\pi(t)$, where $\int_T \pi(t) = 1$
- The set of messages available to type t, $M(t)$
- The set of types capable of sending message m, $T(m)$
- The set of actions available to player 2 after receiving message m, $R(m)$
- Utility functions $u_i(t, m, r)$ for $i = 1, 2$.

To describe a strategy profile (for example, to be able to describe an equilibrium), we have to specify the usual assortment of strategies and beliefs:

- $\sigma_1(m|t)$: probability type t plays message m
- $\sigma_2(r, m)$: probability player 2 plays action $r \in R(m)$ after receiving message m
- $\mu(t|m)$: player 2's posterior beliefs that 1 is of type t after receiving message m.

Then, strategy profile σ is a Nash equilibrium if there is some belief μ for 2 such that:

1. For each type t which plays message m with positive probability, message m is a best response to 2 playing σ_2 ($\sigma_1(m|t) > 0 \Rightarrow m \in B_1(\sigma_2|t)$)
2. For each m **sent with positive probability** by some type t of player 1, $\sigma_2(r|m) > 0 \Rightarrow r \in B_2(\mu, m)$
3. μ comes from Bayes rule give σ

Strategy profile σ and belief μ comprise a perfect Bayesian equilibrium = sequential equilibrium if:

1. For each t, $\sigma_1(m|t) > 0 \Rightarrow m \in B_1(\sigma_2|t)$)
2. For **all** m, $\sigma_2(r|m) > 0 \Rightarrow r \in B_2(\mu, m)$
3. μ comes from Bayes rule give σ