Final exam

4/30/12

Instructions: Throughout, points will be deducted for insufficiently supported answers. You may use books, notes, and calculators, but no other electronic devices. You may not discuss the exam with anyone other than me until all students have turned in their exams.

Problem 1 (30 points) Consider a market with two or more firms and a continuum of workers. Each firm has two types of jobs, "good" jobs and "bad" jobs. Workers enjoy good jobs more than bad jobs; if a worker works a good job, his utility is $X \in [0, 2]$, while if he works a bad job, his utility is 0. Prior to entering the labor force, a worker chooses to either become skilled or to remain unskilled. A worker must pay a cost of c to acquire skills. The value of c differs across different workers, with c being uniformly distributed on $[0, 1]$. A firm gets a profit of 0 from any worker placed into a bad job, and a profit of 1 from a skilled worker placed into a good job and -1 from an unskilled worker placed into a good job. All payoffs include wages.

Firms cannot observe whether a worker is skilled or not, but can observe the outcome of a test that each worker takes. Specifically, a skilled worker *passes* the test with probability $\frac{3}{4}$ and *fails* with probability $\frac{1}{4}$, while an unskilled worker passes with probability $\frac{1}{4}$ and fails with probability $\frac{3}{4}$ (Important: Note that there are only two possible test scores, pass and fail, and so this setup is different than the setup we studied in class where firms chose what constituted a passing score).

a. Show that for *any* $X \in [0, 2]$ there is an equilibrium in which firms put all workers into the bad job, regardless of test score. Support your answer.

b. Show that for *no* $X \in [0, 2]$ is there an equilibrium in which firms put all workers into good jobs, regardless of test score. Support your answer.

c. For what values of X is there an equilibrium in which firms put all workers who pass the test into good jobs and put all workers who fail the test into bad jobs? Support your answer. (Hint: a key step to answering this part is calculating what the firm's prior and posterior beliefs are, where posterior beliefs depend on whether or not a worker passed. In equilibrium, these posteriors need to be consistent with firm profit maximization).

Problem 2 (15 points) True/false/uncertain: In the Spence education signaling model studied in class, the separating equilibrium satisfying the intuitive criterion is Pareto superior to the pooling equilibrium (in which all workers get zero education and are all paid the same wage). Support your answer.

Problem 3 (25 points) Little Airlines' Lexington-Mumbai route is flown by both tourists and business travelers. Tourists (80% of all travelers) have demand $p = 30 - q$ for quality level q, while business travelers (20% of all travelers) have demand $p = 40 - q$. For the sake of simplicity, assume that it does not cost the airline anything to change the quality levels in its plane, and that capacity is not a concern; the plane used on this route is big enough to hold all travelers.

a. Currently, Little Airlines has 2 sections on its plane: coach, with quality 30, and business class, with quality 40. What prices should it set for a coach ticket and a business class ticket so that tourists buy coach tickets and business travelers buy business class tickets, and so that Little Airlines maximizes profits?

b. Explain intuitively why the quality levels identified in part a. are not profit-maximizing for Little Airlines.

c. Suppose the fraction of travelers flying this route on business were to increase. Explain intuitively what effect, if any, this would have on the optimal price of a coach ticket.

d. Determine the profit-maximizing prices and quality levels for both coach and business class, both when 80% of all travelers are tourists, and when 90% of all travelers are business travelers. How does the fraction of business travelers affect the quality in coach?

Problem 4 (30 points) Consider a market in which there are a continuum of used cars for sale. Each car is worth x to its current owner, where x is uniformly distributed on the unit interval across all cars. A car worth x to its seller is worth kx to a buyer, where $k > 1$. A buyer cannot observe x until after he purchases a car.

a. Suppose that sellers first decide whether or not to offer their cars for sale, and then those cars for sale are sold at a price halway between the expected value to a seller of a car offered for sale and the expected value to a buyer of a car offered for sale. Find the equilibrium outcome of this market, as a function of k. Be sure to say what the price is and which values of x correspond to cars sold in equilibrium.

Now suppose $k = 4$. Suppose that each seller has three options: (i) not offering his car for sale, (ii) having the car subjected to a test at cost $c = \frac{5}{8}$ that perfectly and publicly reveals x and then offering the car for sale, or (iii) offering the car for sale without the test. Buyers are then randomly matched to sellers, and cars are sold at a price halfway between the actual value of the car to the seller and the actual value to the buyer (if the car has been tested), or halfway between the expected value of a randomly selected car to a seller and the expected value of an untested car to a buyer, if it has not been tested.

b. What is the net payoff to a seller of having his car tested, as a function of x?

c. What is the net payoff to a seller of selling his car without testing, as a function of μ_x, the average value of x across all untested cars?

d. Using parts b. and c., show that in any equilibrium, at least *some* sellers have their cars tested. Do this as follows: suppose all sellers sell their cars without testing, so that $\mu_x = \frac{1}{2}$. Show that, in this case, a seller with $x > \frac{3}{4}$ strictly prefers to have his car tested.

e. Show that in equilibrium sellers with $x \geq \frac{1}{2}$ prefer to have their cars tested, while those with $< \frac{1}{2}$ prefer to sell their cars without the test.

f. Going back to part a, who gains and who loses from allowing quality testing in this market? Are buyers *ex ante* better off or worse off? What about sellers? Does the answer for sellers depend on x?