Homework 1

answers

Problem 1 Suppose a player in an extensive form game has 4 information sets and that he chooses between 3 strategies at his first information set, 2 at his second, 7 at his third, and 4 at his fourth.

i. How many pure strategies does this player have?

He has $3 * 2 * 7 * 4 = 168$ pure strategies.

ii. What is the dimension of his set of mixed strategies? What is the dimension of his set of behavior strategies? (Hint: "dimension" means how many pieces of information you would need to completely understand his mixed or behavior strategy.)

His pure strategies have dimension 167 (you need to specify 167 pieces of information to specifiy a probability distribution over a discrete support of 168 points). For the behavior strategies, note that you must specify 2 pieces of information at information set 1, 1 piece at info set 2, etc, for a total of 12 pieces of information.

In general, if a player has m information sets with b_k actions at his k^{th} set, the set of his mixed strategies has dimension $\prod_{k=1}^{m} b_k$. the set of his behavior strategies has dimension $\sum_{k=1}^{m}(b_k - 1) = (\sum_{k=1}^{m} b_k) - m$.

Problem 2 In the game Γ, player 1 moves first, choosing between actions A and B. If he chooses B, then player 2 chooses between actions C and D. If she chooses D, then player 1 moves again, choosing between actions E, F, and G. A choice of A or C ends the game. Payoffs are irrelevant for this question.

i. Find a behavior strategy which is equivalent to the following mixed strategy:

$$\sigma_1 = (\sigma_1(AE), \sigma_1(AF), \sigma_1(AG), \sigma_1(BE), \sigma_1(BF), \sigma_1(BG)) = (\frac{1}{2}, \frac{1}{3}, 0, 0, \frac{1}{12}, \frac{1}{12})$$

$b_1(A) = \sigma_1(AE) + \sigma_1(AF) + \sigma_1(AG) = \frac{5}{6}$. $b_1(E) = \frac{\sigma_1(BE)}{\sigma_1(BE)+\sigma_1(BF)+\sigma_1(BG)} = 0$. $b_1(F) = \frac{1}{2}$.

ii. Describe *all* mixed strategies which are equivalent to the following behavior strategy:

$$b_1 = ((b_1(A), b_1(B)), (b_1(E), b_1(F), b_1(G))) = ((\frac{1}{3}, \frac{2}{3}), (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}))$$

Any σ_1 satisfying the following is equivalent to the given behavior strategy: $\sigma_1(AE)+\sigma_1(AF)+\sigma_1(AG) = \frac{1}{3}$ $\sigma_1(BF) = \frac{1}{6}$, $\sigma_1(BG) = \frac{1}{6}$, $\sigma_1(BE) = \frac{1}{3}$.

Problem 3 Two players play an extensive form game. Player 1 is either *wimpy* or *surly*, but player 2 is not sure which. He knows only that the prior probability player 1 is wimpy is .1, and the prior probability he is surly is .9.

The game is played as follows. After learning his type, player 1 chooses to have either quiche or beer for breakfast. Player 2 observes 1's choice of breakfast (but still not his type!), and then decides whether to fight player 1 or to walk away from such an encounter. 2's payoff from dueling a wimpy type is 1, from dueling a surly type is -1, and from walking away is 0. Player 1, regardless of type, prefers to walk away; his utility is lowered by 2 if a duel takes place. Player 1 gets an additional payoff of 1 from having his preferred breakfast (surly types like beer, wimpy types like quiche).

Draw a picture of the extensive form game. Note that you need to include both types of player 1 on the same picture, since 2 doesn't know which he is facing. Think carefully about how to construct information sets.

See Cho and Kreps, 1987. "Signaling Games and Stable Equilibria", QJE 102, page 183" for a drawing of the game. Not that I only gave you enough information to determine player 1's utility function up to addition of a constant. This is intentional, as VNM utility functions are only unique up to affine transformations, so in any game you can always add or subtract a constant to all payoffs or multiply all payoffs by a constant.

Problem 4 Find the reduced normal forms of the games in Figures 1 and 2. (Hint: for the 3 player game in figure 1, you will need to draw two payoff matrices, one for each of player 3's actions.)

For Figure 1:

1 \ 2	a	d
A	$2,2,2$	$0,0,1$
D	$0,0,3$	$0,0,3$

3 plays L

1 \ 2	a	d
A	$2,2,2$	$0,3,3$
D	$1,0,2$	$1,0,2$

3 plays R

For figure 2:

1 \ 2	Ll	Lr	Rl	Rr
AD	$4,3$	$4,3$	$4,4$	$4,4$
AE	$3,9$	$3,9$	$4,4$	$4,4$
B	$3,3$	$3,3$	$8,2$	$8,2$
C	$0,2$	$6,1$	$0,2$	$6,1$

Note that I have combined the redundant strategies BD and BE, and CD and CE for player 1.