# Homework 7

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**Problem 1 (January 2012 prelim)** A firm has two types of jobs, good jobs and bad jobs. When a *qualified* worker is assigned to a good job, the firm earns a net profit of \$20,000. When an *unqualified* worker is assigned to a good job, the firm incurs a net loss of \$20,000. When a worker of either type is assigned to a bad job, the firm breaks even. Workers prefer good jobs, and get an extra \$32,000 payoff from a good job relative to a bad job.

To become qualified, a worker pays an investment cost $c$. This cost is higher for some workers than for others; the distribution of $c$ across all workers is uniform between \$0 and \$9,000. The firm cannot observe which workers are qualified and which are not.

**a.** If the firm has no additional information about new workers, how many workers become qualified in the equilibrium of the model?

Now suppose that while the firm cannot directly observe workers' investment decisions, it administers a test to new employees, with scores ranging from 0 to 1. The probability a qualified worker scores less than $t \in [0, 1]$ is $t^2$. The probability an unqualified worker scores less than $t$ is $t$.

**b.** Suppose that the firm puts all workers with a test score of $s \in [0, 1]$ or higher into a good job. Describe the incentive constraint for a worker's decision to become qualified or not. What fraction $\pi$ of workers will become qualified, as a function of $s$?

**c.** Now consider the firm's problem. Suppose that fraction $\pi$ of all workers become qualified. Show that the firm optimally puts workers scoring above some cutoff test score $s$ into good jobs, and puts low-scoring workers into bad jobs, and solve for $s$ as a function of $\pi$.

**d.** An equilibrium is $(s, \pi)$ pair such that $s$ maximizes firm profit given $\pi$ and $\pi$ is consistent with workers maximizing expected wages net of the investment cost given $s$. Characterize the equilibrium values of $\pi$ and $s$ as follows. One, show that $s = \frac{1}{4}$ and $\pi = \frac{2}{3}$ is an equilibrium. Two, show (don't try to solve for it explicitly!) that there is another equilibrium with $s > \frac{1}{4}$.

**e.** What economic interpretation does the Coate and Loury paper studied in class assign to the multiplicity of equilibria in its model?

**Problem 2 (spring 2011 final)** Consider a market with two or more firms and a continuum of workers. Each firm has two types of jobs, "old" jobs and "new" jobs. The profit to the firm and the payoff to the worker, when the worker is assigned to an old job, is 0. The payoff to a worker assigned to a new job is 1. The payoff to a firm when assigning the worker to the new job is 1 if the worker is skilled, and $-1$ if the worker is not skilled (all payoffs already include wages). A worker must pay a cost of $c$ to acquire skills. The value of $c$ differs across different workers, with $c$ being uniformly distributed on $[0, 1]$.

**a.** Suppose that workers first decide whether to acquire skills and then are matched to firms, who assign them to jobs. Suppose that the firms *can* observe whether each worker has acquired skills. Find the pure-strategy equilibrium job-assignment and skill-acquisition decisions.

**b.** Now suppose that workers first decide whether to acquire skills and then are matched to firms, who assign them to jobs. Suppose that firms *cannot* observe whether a worker has acquired skills. Find the pure-strategy equilibrium and skill-acquisition decisions.

**c.** Now suppose that workers first decide whether to acquire skills, then take a test, and then are matched to firms, who assign them to jobs. Firms cannot observe whether a worker has acquired skills, but can observe the outcome of the test, which is either a pass (p) or fail (f).[1] A worker who has acquired skills passes the test with probability $\alpha > \frac{3}{4}$ and fails with probability $1 - \alpha$, while a worker who has not acquired skills passes with probability $1 - \alpha$ and fails with probability $\alpha$. Find the equilibrium job-assignment and skill-acquisition decisions. (There are multiple such equilibria. Find all the pure strategy equilibria first. Consider mixed strategy equilibria if time permits.)

**d.** Now suppose that workers come in two varieties, red and green. The colors have no effect on the cost of acquiring skills, test outcomes, the value of acquiring skills, or anything else, but are observed by firms. Is there an equilibrium in which different colored workers behave differently?

[1]This is a similar setup to a model studied in class, but note that here the test has only two possible outcomes, whereas in class, the test score was continuously measured.

**Problem 3** Consider an economy in which there are equal numbers of men and women, and two kinds of jobs, good and bad. Some workers are qualified for the good job, and some are not. Employers believe that the proportion of men who are qualified is $\frac{2}{3}$ and the proportion of women who are qualified is $\frac{1}{3}$. If a qualified worker is assigned to the good job, the employer gains \$1,000, while if an unqualified worker is assigned to the good job, the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than $t$ is $t$. The probability that an unqualified worker will have a test score less than $t$ is $t(2-t)$. Employers are subject to a rule that requires the proportion of men assigned to the good job to be the same as the proportion of women. Otherwise, employers maximize expected profits.

**a.** Find the profit-maximizing policy for an employer. Note that in this problem we take as given employer attitudes towards men and women; they do not need to be determined endogenously.

**b.** Test your policy as follows. If you are told that a worker has just barely passed the test (and you are not told whether the worker is male or female), what is the probability that the worker is qualified? Is it the case that such a worker is a fair bet from the employer's point of view? If not, should the policy be adjusted?

**Problem 4** Suppose that business travelers have marginal willingness to pay $40 - q$ for a seat of quality $q \in [0, 40]$, meaning that their total willingness to pay for a seat of quality $\hat{q} \in [0, 40]$ is $\int_0^{\hat{q}}(40 - q)dq$ (assume that marginal willingness to pay is 0 for $q > 40$). Tourists have marginal willingness to pay of $30 - q$ for $q \in [0, 30]$, meaning their total willingness to pay for a seat of quality $\hat{q} \in [0, 30]$ is $\int_0^{\hat{q}}(30 - q)dq$ (assume tourists have marginal willingness to pay of 0 for $q > 30$). Assume that 80 tourists and 20 business travelers typically fly a given route, and the the plane used on this route is more than big enough to hold all 100 travelers, so the airline never has to worry about a capacity constraint. However, the airline cannot tell which type a given traveler is, and so cannot condition price on group membership.

Suppose the airline is able to put two sections on the plane (i.e. 1st class and coach), each with its own quality level. Assume that the cost of setting quality level $q$ in coach is $K_c * q$ and that the cost of setting quality $q$ in 1st class is $K_{fc} * q$, for $K_{fc} \geq K_c$.

**a.** For parts a-d, set $K_{fc} = K_c = 0$. Suppose the airline sets $q = 30$ in coach and $q = 40$ in 1st class. Solve for the profit maximizing prices, taking these quality levels as given.

**b.** You are hired as a consultant to advise the airline on how it can increase profits. Explain why decreasing the quality in coach — and in turn decreasing the price — can increase the airline's profit, even if the number of passengers flying the route remains 100, with 80 tourists and 20 business travelers.

**c.** Solve for the profit-maximizing price and quality levels in both coach and business class.

**d.** Now suppose that the composition of travelers changes, so that fraction $t$ of all travelers are business travelers, and fraction $1-t$ are tourists (the plane is still plenty big enough to hold all travelers, so constraints like there needing to be more seats in coach than there are passengers are not binding). Solve for the optimal price and quantity levels in coach and 1st class, as a function of $t$

**e.** Finally, suppose that $K_c = 1$ and $K_{fc} = \$K$. Suppose again that there are 80 tourists and 20 business travelers. Solve for the relationship between the price of coach and $K$, and give an intuitive explanation for why these two variables are related in this way.