

Midterm exam II

3/30/12

Instructions: Throughout, points will be deducted for insufficiently supported answers. You may use books, notes, and calculators, but no other electronic devices. You may not discuss the exam with anyone other than me until all students have turned in their exams.

Problem 1 (15 points) Find all perfect Bayesian equilibria in the extensive form game in figure 1.

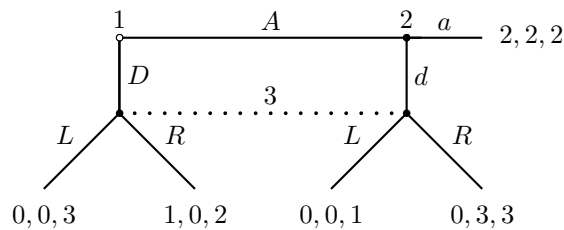


Figure 1: *Game for problem 1*

Problem 2 (15 points) Consider the extensive form game in figure 2. Identify a SPNE that does NOT correspond to a PBE, or prove that no such SPNE exists. Support your answer.

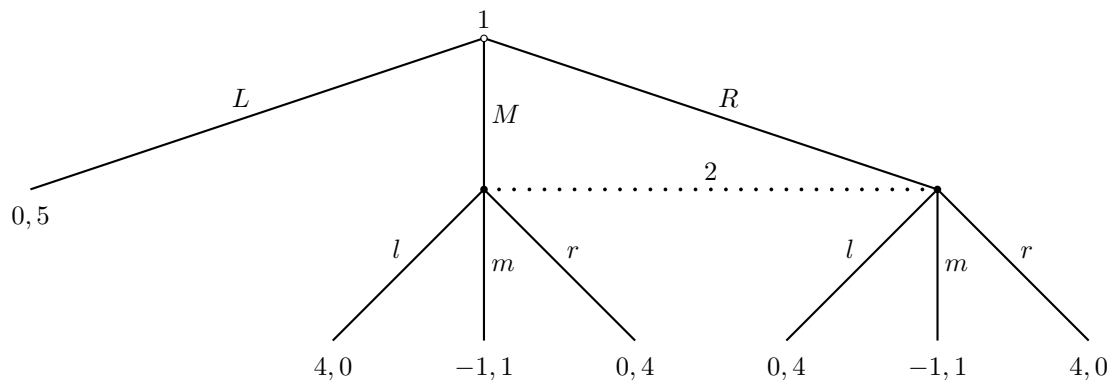


Figure 2: *Game for problem 2*

Problem 3 (15 points) In period t , Firm 1 and Firm 2 simultaneously select prices $p_1 \geq 0$ and $p_2 \geq 0$. If $p_1 < p_2$, Firm 1 sells quantity $q = 20 - 2p_1$ and firm 2 sells quantity 0. If $p_2 < p_1$, Firm 2 sells quantity $q = 20 - 2p_2$ and firm 1 sells quantity 0. If $p_2 = p_1$, both firms sell quantity $q = 10 - p_1$. Both firms have a constant marginal cost of \$1.

- a. Determine the period t stage game Nash equilibrium (p_1^*, p_2^*) .
- b. Compare the Nash equilibrium outcome from part a. with the monopoly outcome that would obtain were Firm 1 and Firm 2 to merge.
- c. Now suppose the same 2 firms play a repeated game, playing the stage game described above in periods $t = 0, 1, \dots$, with discount factor $\beta \in (0, 1)$. For what values of β is there an equilibrium in which both firms set price equal to the monopoly price on the equilibrium path, using Nash reversion as the punishment path?

Problem 4 (15 points) Consider the 2 player, simultaneous move game in figure 3:

		2	
		<i>L</i>	<i>R</i>
1	<i>T</i>	-2, 2	-4, 4
	<i>M</i>	3, -3	6, -6
	<i>B</i>	0, 0	-3, 3

Figure 3: *Game for problem 4*

- a. Find all Nash equilibria of this game (pure as well as mixed). Support your answer.
- b. What strategy does player 1 use to minmax player 2? What strategy does player 2 use to minmax player 1?

Problem 5 (15 points) Consider the two period stage game G in figure 4:

		2	
		<i>a</i>	<i>b</i>
1	<i>C</i>	1, 12	5, 0
	<i>D</i>	12, 1	0, 2

Figure 4: *Game for problem 5*

- a. Draw a picture describing the set of payoffs that can be supported in a SPNE in the infinitely repeated version of G , $G^\infty(\delta)$, using *Nash reversion*.
- b. Draw a picture describing the set of payoffs that can be supported in a SPNE in the infinitely repeated version of G , $G^\infty(\delta)$, using *carrot and stick strategies*.

Problem 6 (15 points) Firms A and B are duopolists selling identical products in a market that is closed to entry. Market demand is $P = 20 - \frac{1}{4}q$. Both firms have identical cost functions, given by $c(q) = 2q$.

- a. Suppose that A and B are Cournot oligopolists. What is the maximum firm A would be willing to pay to purchase firm B? (Assume for simplicity that the merged firm has the same cost function).
- b. Now suppose that firm A is a Stackelberg leader, and firm B the Stackelberg follower. Now how much would firm A be willing to pay to purchase firm B? (Again, assume the merged firm would have the same cost function).
- c. Suppose that A and B are Bertrand oligopolists. How much would firm A be willing to pay to purchase firm B? (Again, assume the merged firm would have the same cost function).
- d. What is the correlation between a firm's sales price and its profitability, in your answers above?