Midterm exam II

answers (preliminary)

Instructions: Throughout, points will be deducted for insufficiently supported answers. You may use books, notes, and calculators, but no other electronic devices. You may not discuss the exam with anyone other than me until all students have turned in their exams.

Problem 1 (15 points) Find all perfect Bayesian equilibria in the extensive form game in figure **??**.

Label 3's left node x and his right node y. Then, 3 plays L if $\mu_x \geq \frac{2}{3}$. Consider 3 supports for player 3:

- L: If 3 plays L, 2 prefers a. Given this, 1 prefers A. As 3's information set is unreached, any belief is consistent with Bayes' rule for 3, including $\mu_x \geq \frac{2}{3}$. PBE.
- R: If 3 plays R, 2 plays d. Given this, 1 prefers D to A. Bayes rule then dictates that $\mu_x = 1$ in any PBE, but if $\mu_x = 1$, 3 prefers L to R. No PBE.
- mix: Implies $\mu_x = \frac{2}{3}$, which implies that either 1 plays A and 2 plays a, or that $\frac{\sigma_1(D)}{\sigma_1(D)+(1-\sigma_1(D))\sigma_2(d)} = \frac{2}{3}$. The former is possible if $\sigma_3(L) \geq \frac{1}{3}$. The latter is possible if $\sigma_1(D) = \sigma_2(a) = \sigma_3(L) = \frac{1}{3}$.

To summarize, the PBE of this game are located at: $(A, a, L, \mu_x \geq \frac{2}{3})$, $(A, a, \sigma_3(L) \geq \frac{1}{3}, \mu_x = \frac{2}{3})$, $(\sigma_1(D) = \frac{1}{3}, \sigma_2(a) = \frac{1}{3}, \sigma_3(L) = \frac{1}{3}, \mu_x = \frac{2}{3})$.

Problem 2 (15 points) Consider the extensive form game in figure **??**. Identify a SPNE that does NOT correspond to a PBE, or prove that no such SPNE exists. Support your answer.

It is a NE, and therefore a SPNE, for 1 to play L and 2 to play m. This is not a PBE, however, as m is a strictly dominated strategy for player 2, and so there are not beliefs for which m is a best response.

Problem 3 (15 points) In period t, Firm 1 and Firm 2 simultaneously select prices $p_1 \geq 0$ and $p_2 \geq 0$. If $p_1 < p_2$, Firm 1 sells quantity $q = 20 - 2p_1$ and firm 2 sells quantity 0. If $p_2 < p_1$, Firm 2 sells quantity $q = 20 - 2p_2$ and firm 1 sells quantity 0. If $p_2 = p_1$, both firms sell quantity $q = 10 - p_1$. Both firms have a constant marginal cost of \$1.

a. Determine the period t stage game Nash equilibrium (p_1^*, p_2^*).

Both firms set price equal to marginal cost, or $p_1 = p_2 = \$1$. Quantity of 18 is sold in the market.

b. Compare the Nash equilibrium outcome from part a. with the monopoly outcome that would obtain were Firm 1 and Firm 2 to merge.

A monopolist sets a price of \$5.50 and sells a quantity of 9.

c. Now suppose the same 2 firms play a repeated game, playing the stage game described above in periods $t = 0, 1, \ldots$, with discount factor $\beta \in (0, 1)$. For what values of β is there an equilibrium in which both firms set price equal to the monopoly price on the equilibrium path, using Nash reversion as the punishment path?

If firms both set a price of \$5.50, they each earn a profit of \$40.50 each period. If one of the duopolists deviates to a slightly lower price (for convenience, say he deviates to $\$5.50 - \epsilon$), he can earn a profit of \$81. On the punishment path, both firms earn a profit of \$0. Therefore, each duopolist has an incentive constraint given by:

$$40.5 \geq (1-\beta)81 + \beta 0$$
$$\beta \geq \frac{1}{2} \quad (1)$$

conclude that so long as $\beta \geq \frac{1}{2}$, neither duopolist will deviate from the equilibrium path.

Problem 4 (15 points) Consider the 2 player, simultaneous move game in figure 1:

		2: L	2: R
1	T	$-2, 2$	$-4, 4$
	M	$3, -3$	$6, -6$
	B	$0, 0$	$-3, 3$

Figure 1: *Game for problem 4*

a. Find all Nash equilibria of this game (pure as well as mixed). Support your answer.

As M is a strictly dominant strategy for player 1, the answer is trivially M, L.

b. What strategy does player 1 use to minmax player 2? What strategy does player 2 use to minmax player 1?

Since this is a zero-sum game, 1 plays M to minmax 2, and 2 plays L to minmax 1.

Problem 5 (15 points) Consider the two period stage game G in figure 2:

		2: a	2: b
1	C	$1, 12$	$5, 0$
	D	$12, 1$	$0, 2$

Figure 2: *Game for problem 5*

a. Draw a picture describing the set of payoffs that can be supported in a SPNE in the infinitely repeated version of G, $G^{\infty}(\delta)$, *using Nash reversion.*

The stage game has one Nash equilibrium, with payoffs $(\pi_1, \pi_2) = (\frac{65}{17}, \frac{24}{13})$. See the picture at the end of this answer key.

b. Draw a picture describing the set of payoffs that can be supported in a SPNE in the infinitely repeated version of G, $G^{\infty}(\delta)$, *using carrot and stick strategies.*

Player 1's minmax payoff is $\frac{65}{17}$. Player 2's minmax payoff is $\frac{24}{13}$. As both payoffs are also the Nash equilibrium payoffs, the picture is the same as in part a.

Problem 6 (15 points) Firms A and B are duopolists selling identical products in a market that is closed to entry. Market demand is $P = 20 - \frac{1}{4}q$. Both firms have identical cost functions, given by $c(q) = 2q$.

a. Suppose that A and B are Cournot oligopolists. What is the maximum firm A would be willing to pay to purchase firm B? (Assume for simplicity that the merged firm has the same cost function).

In the Cournot equilibrium, each firm earns a profit of \$144. A monopolist in this market would earn a profit of \$324. Therefore, a Cournot duopolist would be willing to pay up to \$180 to acquire his rival.

b. Now suppose that firm A is a Stackelberg leader, and firm B the Stackelberg follower. Now how much would firm A be willing to pay to purchase firm B? (Again, assume the merged firm would have the same cost function).

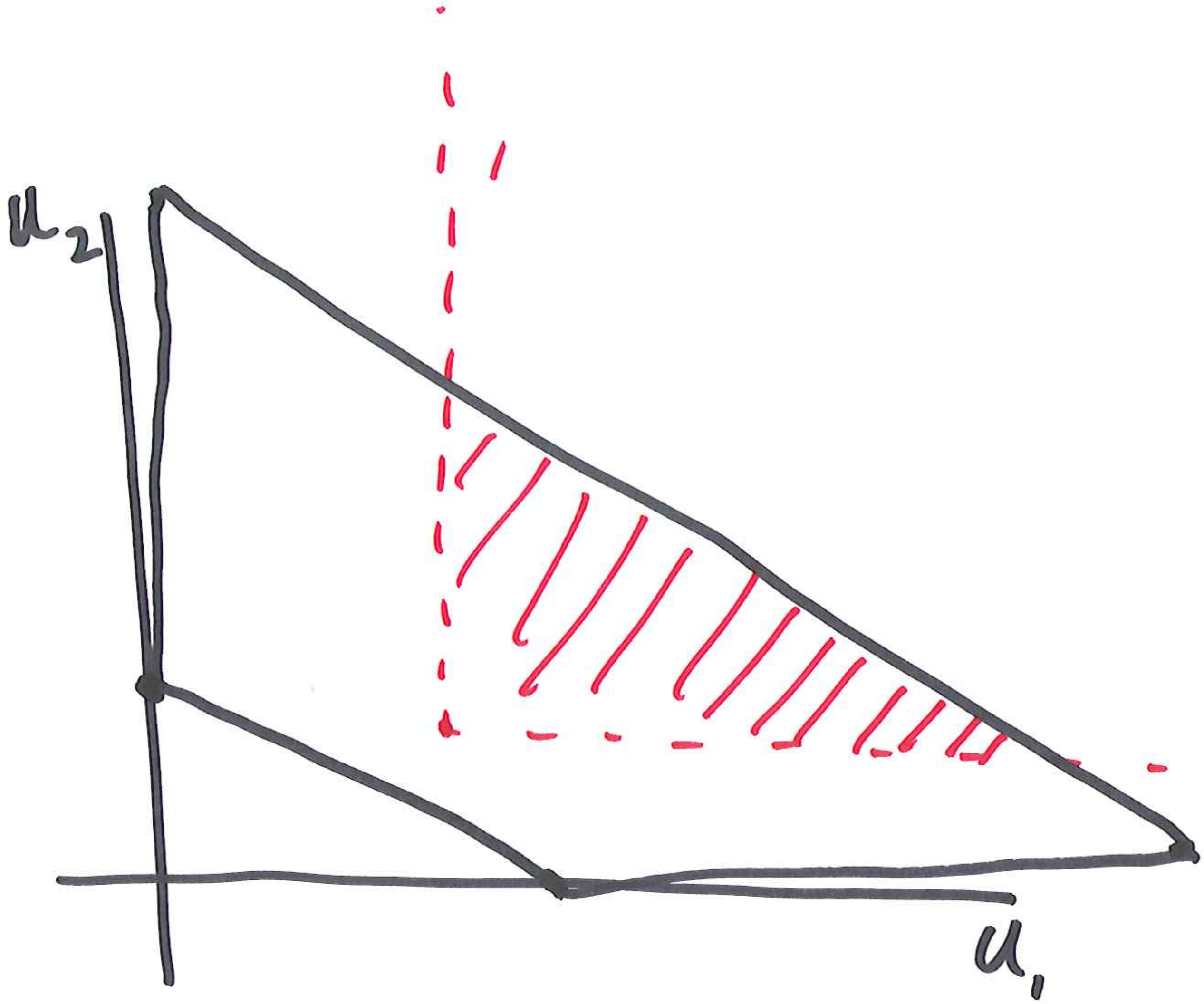
In the Stackelberg equilibrium, Firm A earns a profit of \$162, and Firm B a profit of \$81. Given that a monopolist would earn a profit of \$324, Firm A would be willing to pay up to \$162 to acquire its rival.

c. Suppose that A and B are Bertrand oligopolists. How much would firm A be willing to pay to purchase firm B? (Again, assume the merged firm would have the same cost function).

In a Bertrand duopoly, both firms earn a profit of 0. Therefore, a Bertrand duopolist would be willing to pay up to \$324 to acquire his rival.

d. What is the correlation between a firm's sales price and its profitability, in your answers above?

The Bertrand firm, who is the least profitable, has the highest potential price. The Stackelberg firm is the second-most profitable, and commands the lowest price. The Cournot firm is the most profitable, and commands an intermediate price. So there is no clear correlation between profitability and sales price, when the sales price is derived from the possibility of increased market share.



/// = payoffs supportable in an SPNE, using either Nash reversion or carrot and stick strategies.