Problem set 1

due 9/16/2009

Problem 1 (Existence of Nash equilibrium) Consider any two-player game of the following form (where letters represent arbitrary payoffs):

Player 2

		X	Y
Player 1	A	a,b	e,f
	B	c,d	g,h

Show that a (possibly mixed) Nash equilibrium always exists in this game. [Hint: define player 1's strategy to be his probability of choosing action A and player 2's to be her probability of choosing X. Then, examine the best response correspondences of the two players.]

Problem 2 (behavior strategies) Give an example of an extensive form game in which a player has a mixed strategy which does not admit an equivalent behavior strategy.

Problem 3 (Nash equilibria) Compute the set of all rationalizable strategies of the following normal form game, and then find all of its Nash equilibria:

Player 2

		L	C	R
Player 1	T	0,4	5,6	8,7
	B	2,9	6,5	5,1

Problem 4 (price discrimination) Consider a monopolist serving two types of customers, with demands $D_1(p) = a_1 - p$ and $D_2(p) = a_2 - p$. Assume $a_2 > a_1$, and refer to type 1 agents as low types and type 2 agents as high types. Assume there are fraction λ of low types and fraction $1 - \lambda$ of high types in the population.

The monopolist is able to charge different prices for different quantities. He could, for example, charge a total price of \$100 for all quantities less than 20 and a price of \$300 for all quantities between 20 and 30, and charge an infinite price for quantities above 30.

a. What kind of price discrimination is the monopolist practicing? What conditions are favorable/unfavorable to being able to price discriminate in this way?

b. Solve for the monopolist's optimal price schedule, and the quantities each type of customer purchases. Your answer should demonstrate that low types purchase quantity $q_1 = \max\{0, \frac{a_1-(1-\lambda)a_2}{\lambda}\}$ while high types purchase quantity $q_2 = a_2$.

c. Explain intuitively why q_1 depends on λ and a_2 in the way it does. Does this model have anything to say about the folk wisdom that the reason airlines make coach uncomfortable is not to annoy coach passengers, but to motivate first-class passengers to pay extra for their tickets?

Problem 5 (contest) A newspaper runs the following contest: each participant mails in a postcard on which he writes an integer between 0 and 1000 (inclusive). Given the entries, the *largest integer* is defined to be $\frac{9}{10}$ the highest entry, rounding downward if the result is not an integer. All participants who chose the target integer split a $500 prize. If no one chooses the target integer, the closest without going over gets the prize.

Suppose this contest is modeled as a simultaneous move game among 100 players. Using only rationalizability, determine a unique prediction of play. What assumptions do you require for your prediction to hold? Comment on the plausibility of these assumptions.