Problem set 3

due 10/14/2009

Problem 1 (Repeated prisoners' dilemma) Consider the following game:

		Player 2: C	D
Player 1	C	8,8	-1,21
	D	21,-1	0,0

i) Draw a picture of all payoffs supportable in a SPE equilibrium of the infinitely-repeated version of this game, provided δ is high enough.
ii) Determine how high δ must be for C, C to be played in every period of a SPE.
iii) What is the highest symmetric payoff that can be achieved in a SPE of the repeated game? Write down strategies that implement this payoff, and determine how high δ must be for your strategies to comprise a SPE. (hints: have the players alternate between C, D and D, C, with permanent reversion to the NE D, D if anyone deviates. Then show that the limit of each player's payoff as $\delta \to 1$ is 10. Alternatively, you can have the players flip a coin to determine who defects first, and then in expectation each will have a payoff of 10.)

Problem 2 (Repeated games and minmaxing) Consider the following game:

		Caliban: a	b	c
	A	1,2	5,1	1,0
Elroy	B	2,1	4,4	0,0
	C	0,1	0,0	0,0

i) Find all Nash equilibria of the one-shot version of this game. Make sure to support your answer by drawing each player's best respondence correspondence and examing all possible supports.
ii) Determine each player's minmax value, and the strategy his opponent would use to minmax him.
iii) Show that a payoff of $(4, 4)$ can be supported in a SPE using a Nash reversion strategy if and only if $\delta \geq \frac{1}{2}$.
iv) Show that for every $\delta \geq \frac{1}{4}$, there is a SPE strategy profile yielding payoffs of $(4, 4)$. (Hint: Nash reversion will not work here.)

Problem 3 (Oligopoly) Suppose market demand is given by $p(q) = a - bq$, and there are two firms, each with a constant marginal costs of c and no fixed cost. The two firms choose quantity simultaneously, and then sell whatever they have produced at the prevailing market price.
i) Determine NE quantities for both firms. Demonstrate that there is only one equilibrium in this game.

ii) Derive the market price, and the profit for each firm. Show that the total quantity produced is greater than the monopoly quantity, but less than the competitive quantity.
iii) How high would δ need to be for there to be a SPE in which firm 1 receives fraction α of the monopoly profit and firm 2 receives fraction $1-\alpha$? Make sure to say how your answer depends on α, including pointing out for what ranges of α no such equilibrium is possible.
iv) Now suppose the game is played only once, but in which firm 1 moves first. Firm 2 moves only after observing the quantity firm 1 chooses. Derive the SPE of this game.
v) Finally, suppose there are J firms serving the market. In the static case, determine NE quantities and profits for each of the J firms. Show that as $J \to \infty$, total production approaches the competitive level, while when $J = 1$, we get the monopoly outcome.

Problem 4 Consider the following game:

		firm 2: S	firm 2: C
firm 1	S	5,2	3,1
	C	6,3	4,4

Note that if this game is played simultaneously, the equilibrium outcome is C, C, while if firm 1 moves first, the outcome is S, S.

Now assume that the game is played sequentially, but instead of observing 1'a action directly, 2 observes a signal $\phi \in \{S', C'\}$ such that $p(\phi = S'|S) = 1 - \epsilon$ and $p(\phi = C'|C) = 1 - \epsilon$, for $\epsilon \in (0, \frac{1}{4})$.
i) Show that the only equilibrium in pure strategies is C, C.
ii) Let λ equal the probability 1 plays S, and let $\eta(S')$ and $\eta(c')$ denote the probability 2 plays S after signals S' and C', respectively. Show that there are exactly two mixed strategy equilibria, one at

$$\lambda = 1 - \epsilon,\ \eta(S') = 1,\ \eta(C') = \frac{1 - 4\epsilon}{2(1 - 2\epsilon)}$$

and one located at

$$\lambda = \epsilon,\ \eta(S') = \frac{1}{2 - 4\epsilon},\ \eta(C') = 0 \tag{1}$$

make sure to point out what player 2's beliefs are in your answer.
iii) Show that one of your mixed strategy equilibria converges to the C, C equilibrium as $\epsilon \to 0$, while the other converges to the S, S equilibrium as $\epsilon \to 0$.