Problem set 4

due 10/28/2009

Problem 1 (Rotemberg-Saloner model) Consider an n-firm oligopoly with demand in period t given by $p_t = 1 - \Sigma_{i=1}^{n} q_i + \epsilon$, where ϵ is a random variable observed only at the beginning of period t. Each firm has cost function $c(q) = 0$.

a. Suppose the firms collude (each firm produces fraction $\frac{1}{n}$ of the monopoly quantity) supported by grim trigger Nash reversion. Prove that on the equilibrium path, each firm can earn a payoff of

$$\left(\frac{(1+\epsilon)(n+1)}{4n}\right)^2$$

by deviating and playing her best response to the other $n-1$ firms.

b. Suppose now the firms decide that collusion is unsustainable for high ϵ and so decide to instead play strategies $\tilde{q}(\epsilon)$ on the equilibrium path, where $\tilde{q}(\epsilon) > q^m$ for high ϵ. Show that by deviating, each firm can earn a one-shot payoff of

$$\left(\frac{1+\epsilon-(n-1)\tilde{q}(\epsilon)}{2}\right)^2$$

Problem 2 (Collusion over the business cycle) Suppose that market demand facing an 3-firm oligopoly in period t is given by $p_t = 1 - q_1 - q_2 - q_3 + \epsilon_t$, where $\epsilon_t \sim U[-1, 1]$. ϵ_t is observed by all at the beginning of period t, but not before. Each firm has cost function $c(q) = 0$.

a. Show that if the firms compete as Cournot competitors, they play $q(\epsilon) = \frac{1}{4} + \frac{1}{4}\epsilon$, and the expected per-period profit for each firm is equal to $\frac{1}{12}$.

b. Show that under a collusive agreement in which each firm produces fraction $\frac{1}{n}$ of the monopoly quantity firms play $q(\epsilon) = \frac{1}{6} + \frac{1}{6}\epsilon$, and the expected per-period payoff to each firm $\frac{1}{9}$.

c. Show that, so long as $\delta \geq .8$, collusion supported by a grim trigger punishment path of Nash reversion is a SPE.

d. Now suppose that $\delta = \frac{9}{13}$. Show that while deviating from the collusive strategies outlined in c. will not be optimal in periods of relatively low demand ($\epsilon \leq .5$), in periods of high demand ($\epsilon > .5$), each firm will want to deviate from collusion.

e. Suppose the firms, in the interest of maintaining the cartel, decide each firm should produce the following quantity, as a function of ϵ:

$$\tilde{q}(\epsilon) = \begin{cases} \frac{1}{6} + \frac{1}{6}\epsilon & \text{if } \epsilon \leq 0 \\ \frac{1}{6} + \frac{1}{3}\epsilon & \text{if } \epsilon > 0 \end{cases} \tag{1}$$

Note that this production schedule moves gradually from the monopoly outcome to the Cournot outcome as ϵ moves from 0 to 1.

Show as thoroughly as you can that an equilibrium path of each firm playing $\tilde{q}(\epsilon)$, supported by a punishment path of Nash reversion, is a SPE of a repeated game.

f. Give an expression for price, as a fraction of the monopoly price, as a function of ϵ. Plot this object in a graph over $\epsilon \in [0, 1]$.

g. Consider now the following alternative strategies:

$$\tilde{q}_2(\epsilon) = \begin{cases} \frac{1}{6} + \frac{1}{6}\epsilon & \text{if } \epsilon \leq \frac{1}{2} \\ \frac{1}{2}\epsilon & \text{if } \epsilon > \frac{1}{2} \end{cases} \tag{2}$$

$\tilde{q}_2(\epsilon)$ is similar to $\tilde{q}(\epsilon)$, in that it moves from the monopoly outcome to the Cournot outcome as ϵ moves from $\frac{1}{2}$ to 1. Will a strategy profile which calls for all firms to produce $\tilde{q}_2(\epsilon)$ on the equilibrium path, and to switch permanently to Cournot competition should anyone deviate comprise a SPE? (Hint: you have two options to answer this. You can calculate each firm's incentive constraint and either show it holds for all ϵ or that it does not hold for some ϵ, or you can try to answer this more directly, by thinking about the firms' incentive constraints and $\tilde{q}$ and $\tilde{q}_2$.)

Problem 3 (Tirole, Exercise 7.3) Consider a version of Salop's circular city model (studied in class on 10/21) with quadratic transportation costs. That is, a customer located at y incurs transportation cost $t(y - x_i)^2$ to purchase from a firm located at x_i. Show that under this assumption, equilibrium price if given by

$$p = c + \frac{t}{n^2} \tag{3}$$

while the equilibrium number and optimal number of firms, respectively, are given by

$$n^{eqm} = \left(\frac{t}{f}\right)^{\frac{1}{3}} \qquad n^{opt} = \left(\frac{t}{6f}\right)^{\frac{1}{3}} \tag{4}$$