Merger efficiencies and price effects in differentiated Cournot oligopoly

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Abstract

Suppose differentiated firms compete in quantities. This paper derives a formula for the minimum cost savings that would offset the incentive to increase price created by a merger. The formula depends only on pre-merger information on margins and demand slopes, and is invariant to demand and cost curvature. The paper then develops an algorithm to infer demand slopes – and thus allow calibration of parameterized demand and cost curves – from pre-merger data. While the Cournot model of quantity competition is commonly accompanied by an assumption that rivals' products are interchangeable, the inflexibility of this assumption and its implications opens the model to criticisms. The paper examines the advantages of relaxing the assumption of interchangeability, in particular greater consistency with pre-merger data and greater scope for profitable mergers. An extended numerical example illustrates the application of a differentiated Cournot model to a hypothetical industry.

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1 Introduction

Consider an industry in which firms produce differentiated substitutes, and compete *a la* Cournot by choosing quantities. A merger generically changes the incentives of each merging firm, in that the firm internalizes the negative effect of additional production on the profit of its former rival, and thus optimally decreases its quantity. Should the merger reduce the costs of one or both merging firms, this incentive may be offset to some extent by the now-greater profit margin associated with each sale. In this paper, I derive the minimum cost savings that would completely offset the change in incentive created by the merger when differentiated firms compete in quantity. In keeping with the literature, I refer to this metric as Compensating Marginal Cost Reduction ("$CMCR$").

$CMCR$ depends on pre-merger margins, prices, and demand cross slopes, but is invariant to assumptions on curvature of demand and cost curves. While prices and margins are commonly observable by researchers and practitioners, information on demand slopes may or may not be available. Thus, I also provide a method for calibrating the necessary demand slopes from pre-merger information on margins and diversion ratios. The resulting calibration additionally enables simulation of post-merger prices based on both the calibrated demand slopes and assumptions about the curvature of demand and cost curves away from the pre-merger equilibrium.

I then use merger simulation tools to explore some key differences between the differentiated Cournot model and other models such as the differentiated Bertrand model and the homogeneous Cournot model. In contrast to the homogeneous Cournot model, I show that the differentiated Cournot model can flexibly match per-merger information on margins, thus more plausibly explaining the pre-merger equilibrium. Further, mergers of differentiated Cournot firms appear to be more profitable than mergers of homogeneous Cournot firms, suggesting that the differentiated Cournot model may better capture the incentive to merge. I provide evidence that mergers of differentiated Cournot competitors can result in greater price increases than comparable mergers of Bertrand competitors, but note that this result is not universal. Finally, I provide evidence that when differentiated firms compete in quantities, capacity constraints on merging firms mitigate price effects, while capacity constraints on nonmerging firms exacerbate them, with the former effect being relatively more important.

The industrial organization literature has studied models of differentiated firms competing in quantities for decades. Singh and Vives (1984) argue that when products are differentiated, competition in quantities renders the market "more monopolistic" – and thus higher-priced – than would competition in prices. This result is generalized by Vives (1985), Cheng (1985), Okuguchi (1987), and Qiu (1997), and limited by Qiu (1997), Häckner (2000), Symeonidis (2003), and Alipranti et al. (2014). The intuition for why Cournot industries tend to have higher prices is that when a Cournot firm sets price, it conjectures that all of its rivals will respond by increasing price (so as to maintain a constant quantity). In contrast, when a Bertand competitor sets its price, it conjectures that each rival will not adjust price in response. Hence, the Cournot competitor has an additional incentive to increase price, relative to the Bertrand competitor.

While the literature has extensively studied the relationship between mode of competition and outcome, comparatively little is known about how mode of competition and degree of differentiation affect merger outcomes. For example, Nocke and Whinston (2021) assess the effectiveness of concentration screens as described in the FTC/DOJ *Horizontal Merger Guidelines* (2010), finding that change in the Herfindahl index is a reasonable proxy for merger price effects both when homogeneous firms compete in quantities and when differentiated firms compete in prices. In the same settings, Taragin and Loudermilk (2019) assess the performance of both concentration screens and upward pricing pressure. Neither paper considers a setting in which differentiated firms compete in quantities.

The lack of attention paid to the antitrust implications of the differentiated Cournot model in the academic literature appears to reflect the practices of antitrust practitioners. For example, the FTC employed homogeneous Cournot models in two recent merger challenges: Tronox/Cristal[1] (involving titanium dioxide), and Peabody/Arch[2] (involving coal mined in the South Powder River Basin). Private litigants have employed the homogeneous Cournot model in at least two litigated antitrust challenges.[3] Numerous antitrust matters have involved markets with differentiated goods in which firms appear to compete by setting prices,[4] or by bargaining over price with large customers.[5] In contrast, I am not aware of an instance in which either plaintiffs or defendants argued that firms competed in quantities of differentiated goods.

That antitrust enforcers seem to disfavor the differentiated Cournot model is surprising. First, even apparently similar products sold by distinct firms may be at least modestly differentiated in the presence of different locations and shipping distances, search frictions, customer inertia, branding, or subtle differences in quality or type of product. Second, the competitive interdependence of differentiated firms may well stem from quantity choices, particularly if firms' capacity decisions are difficult to adjust, and firms are incentivized to set prices so as to sell out their capacity.

[1] *See* FTC v. Tronox Limited et al., Case No. 1:18-cv-01622 (D.D.C 2018).

[2] *See* FTC v. Peabody Energy Corp., Case No. 4:20-cv-00317-SEP (E.D.Mo 2020).

[3] *See* Concord Boat Corp. v. Brunswick Corp., 207 F.3d 1039 (8th Cir. 2000) and Heary Bros. Lightning Prot. Co. v. Lightning Prot. Institute, 287 F. Supp. 2d 1038 (D. Ariz. 2003).

[4] *See* FTC v. Whole Foods Market, Inc., 502 F. Supp.2d 1, 39 (D.D.C.2007) (describing how supermarkets, organic and otherwise, compete through differentiation and prices).

[5] *See* Hanner et al. (2016) (describing a bid model used by the FTC when litigating the Sysco/US Foods merger); FTC v. Wilhelmsen and Drew, Civil Action No. 18-cv-00414-TSC, 44 (D.D.C. 2018) (describing a merger simulation model used by the FTC's expert).

Further, standard models of quantity competition in which firms are assumed to be undifferentiated impose a relatively rigid form of competition that implies at least two potentially problematic results. First, standard Cournot models imply that share is proportional to margin, meaning that if firm A has twice the share of firm B, firm A's marginal cost is half of firm B's. Defendants in Tronox/Cristal and Arch/Peabody attacked the FTC's modeling because in their view accounting margins did not match this pattern.[6] In a non-merger matter in private litigation, the court excluded the plaintiffs' Cournot model, and consequently their expert's entire report, on the basis of plaintiffs' use of a Cournot model that assumed firms with different shares had identical marginal costs.[7]

Second, particular implementations of the homogeneous Cournot model often predict that mergers are unprofitable. This fact has been noted in the academic literature by, among others, Salant, Switzer, and Reynolds (1983), Perry and Porter (1985), and Farrell and Shapiro (1990). In litigation, defendants in the Tronox/Cristal matter attacked the perceived unprofitability of the merger under the FTC's Cournot as deligitimizing the entire model.[8]

Finally, as has been pointed out by authors including Salant, Switzer, and Reynolds (1983) and Perry and Porter (1985), when applied to mergers homogeneous Cournot models often require an assumption that the merging firms have increasing marginal cost. The reason is that with constant marginal costs, the merged entity will optimally shutter the higher cost firm post-merger, and Salant, Switzer, and Reynolds (1983) argue that the profit of one firm in an N-firm oligopoly is lower than the combined profits of 2 firms in an $(N + 1)$-firm oligopoly, implying that mergers of Cournot competitors with constant marginal costs are generically unprofitable. Perry and Porter (1985) and Farrell and Shapiro (1990) consider various forms of increasing marginal costs, and antitrust practitioners employing the Cournot model often assume quadratic costs. When products are differentiated, however, the Cournot model allows for constant marginal costs. While there are important applications in which firms have increasing marginal costs, researchers and practitioners may have little insight into the cost structure of a particular firm. Hence, an assumption that firms can replicate their production process in expanding production (at least for marginal units) may be attractive.

As Davis (2002) observes, antitrust practitioners seem to default to Bertrand models when they believe products to be differentiated, and Cournot models when they believe products to be homogeneous. However, this default choice seems to be based on largely technical motivations and analytical

[6] *See* Peabody Opinion, *supra* note 2, at 62, "Defendants object to Dr. Hill's applications of the Cournot model on a number of grounds, [including] that his model's predicted margins do not match observed margins."; Tronox Opinion, *supra* note 1, at 33, "(Defendants] contend[] that [...] use of the Cournot model is not appropriate and leads to results that are inconsistent with market realities.[...] Chemours' marginal cost of producing TiO2 is, according to the model, "more than [redacted]" lower than the "actual" marginal cost as measured by Dr. Hill."

[7] *See* Heary Bros. opinion, *supra* note 3.

[8] *See* Tronox Opinion, *supra* note 1, at 33.

tractability, and not information about the underlying mode of competition. I hope this paper both solves some of the underlying technical issues driving the default choice, and prompts researchers and antitrust practitioners to reconsider the default choice. Not only may the differentiated Cournot model more accurately capture the mode of competition in many differentiated industries, it also produces several technical advantages over the homogeneous Cournot model, as described above.

My paper contributes to a literature examining the effect of merger efficiencies on merger outcomes. While merger efficiencies are a topic of considerable interest to policymakers,[9] both the theoretical and empirical literature on merger efficiencies is thin.[10] My paper most closely resembles Werden (1996) and Froeb and Werden (1998), which derive comparable $CMCR$ metrics for differentiated Bertrand competition and homogeneous Cournot competition, respectively. The Werden (1996) and Froeb and Werden (1998) $CMCR$ metrics are widely used by antitrust practitioners (see Greenfield et al. (2019) for a discussion of $CMCR$ as used in the Tronox/Cristal litigation). My paper establishes a comparable $CMCR$ metric for settings in which competition is differentiated in quantities. Like the Werden (1996) and Froeb and Werden (1998) versions of $CMCR$, the metric I propose is invariant to assumptions on demand and cost curvature.

Section 2 derives $CMCR$ for mergers of differentiated Cournot competitors. Section 3 describes a calibration algorithm to infer demand slopes from pre-merger information on market shares, diversion ratios, and profit margins, and discusses calibration of linear and loglinear demand systems using inferred demand slopes. Section 4 applies differentiated Cournot modeling to a hypothetical industry, and compares the differentiated Cournot model to the homogeneous Cournot model and to the Bertrand model. Section 5 concludes.

2 Model

Assume that N separately-owned single-product firms produce differentiated goods. The firms compete by simultaneously choosing quantities, with each firm then receiving the market clearing price for its quantity. Each firm's price depends on both its own quantity and its rivals' quantities, according to the inverse demand curve $p_i(Q)$, where Q is a vector with generic element q_i. Assume that $p_i(Q)$ is differentiable, with $\frac{\partial p_i}{\partial q_j} < 0$ for all i, j, so that products produced by the N firms are substitutes. Firm i's cost curve is given by $c_i(q_i)$, with $c_i(q_i)$ differentiable and with $c_i' > 0$. Let $m_i = \frac{p_i - c_i'(q_i)}{p_i}$ denote firm i's margin over its cost on its last unit sold.

[9] *See* Wilson (2020) and Blair et al. (2020).

[10] I am aware of two recent empirical papers examining merger efficiencies, An and Zhao (2019) and Osinski and Sandford (2021).

Firm i chooses q_i so as to maximize its profits. Firm i's first-order condition for profit-maximization is:

$$m_i^{pre} = -\frac{q_i}{p_i}\frac{\partial p_i}{\partial q_i} \tag{1}$$

Now suppose that firms 1 and 2 merge. Each merging firm now chooses q_i to maximize the sum of firm 1's and firm 2's profits. Post-merger, firms 1 and 2 have the following first-order conditions:

$$m_i^{post} + \frac{\partial p_j}{\partial q_i}\frac{q_j}{p_i} = -\frac{q_i}{p_i}\frac{\partial p_i}{\partial q_i} \text{ for } i = 1, 2 \tag{2}$$

where m_i^{post} denotes post-merger margin. In general, prices, quantities, and slopes differ before and after a merger. For reasons that will become obvious, we suppress the superscript on all terms other than margin.

If the merger between firms 1 and 2 lowers one or both of these firms' marginal cost, this will raise their margin, all else equal. Proposition 1 derives the amount by which each merging firm's margin would need to increase – and consequently, the amount by which its marginal cost would need to decrease – in order for the merger not to result in a price increase. Following Werden (1996) and Froeb and Werden (1998), I refer to this quantity as firm i's compensating marginal cost reduction ($CMCR$).

Proposition 1. *Following a merger of firms 1 and 2, the amount by which each of the merging firms' marginal costs must decrease so that post-merger quantities and prices are unchanged from pre-merger quantities and prices is:*

$$CMCR_i = \frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}}\frac{m_j\frac{p_j}{p_i}}{1 - m_i} \textit{ for } i, j \in \{1, 2\} \tag{3}$$

Proof. If the post-merger outcome matches the pre-merger outcome, it follows that all terms other than margin are the same in both equations (1) and (2), for firms 1 and 2. The proof proceeds by solving for the implied value of m_i^{post}, as a function of m_i^{pre}.

Multiply the middle term in firm i's post-merger first-order condition (2) by $1 = \frac{\frac{\partial p_j}{\partial q_j}}{\frac{\partial p_j}{\partial q_j}}\frac{p_j}{p_j}$. Then, substitute firm j's pre-merger first-order condition (1) into equation (2), to yield the following:

$$m_i^{post} = m_i^{pre} + \frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}} m_j^{pre}\frac{p_j}{p_i} \text{ for } i = 1, 2,\ j \neq i \tag{4}$$

Following equation (1) of Werden (1996), the relationship between the change in marginal cost and the change in margin is:

$$\frac{c_i^{pre} - c_i^{post}}{c_i^{pre}} = \frac{m_i^{post} - m_i^{pre}}{1 - m_i^{pre}} \tag{5}$$

Substituting the expression for m_i^{post} from (4) into equation (5) yields the expression for $CMCR_i$. ■

$CMCR$ depends only on pre-merger values. The ratio of slopes, $\frac{\frac{\partial p_j}{\partial q_i}}{\frac{\partial p_j}{\partial q_j}}$, measures the closeness of substitutability between goods 1 and 2. If the ratio is high, than an increase in j's quantity q_j affects i's price p_i nearly as much as does an increase in i's own quantity q_i, so i and j are close substitutes. On the other hand, if the ratio is small, than j's price is much less affected by i's quantity than by j's own quantity, and the products are more distant substitutes. The margins m_j and m_i are related to the responsiveness of the merging firms' demand to changes in price/quantity via the pre-merger first-order conditions (1). From inspection of (1), a higher margin implies that the firm's demand is less responsive to changes in price or quantity.

In some cases, the demand slopes in the expression for $CMCR$, $\frac{\partial p_j}{\partial q_i}$ and $\frac{\partial p_j}{\partial q_j}$, may be directly measurable. This is perhaps most likely if price and quantity information spanning contraction and expansion events is available. In other cases, directly estimating these demand slopes may prove difficult. The following section provides an algorithm to calibrate demand slopes using pre-merger information on margins and diversions ratios. The resulting demand slopes are sufficient both to calculate $CMCR$, and to calibrate particular parameterized demand systems to pre-merger information, in order to simulate the merger price effect.

3 Calibration

Researchers and antitrust practitioners often populate the parameters of a demand system using either estimation – meaning econometric identification using variation in a dataset – or calibration – meaning fitting parameters as closely as possible to a relatively small number of observed statistics, commonly margins, diversion ratios,[11] or cost pass through terms.[12] As noted by Miller et al. (2012), antitrust practitioners more commonly use calibration, as confidential information available through subpoenas and discovery commonly suffice to measure diversion ratios and margins, but lack the detail required for econometric identification.

In this section, I discuss what inferences can be made about demand slopes – and thus $CMCR$ – from the margins and diversions ratios that are commonly available to antitrust practitioners. I focus

[11]The diversion ratio from firm i to firm j is commonly defined by D_{ij}, where:

$$D_{ij} = -\frac{\frac{\partial q_j}{\partial p_i}}{\frac{\partial q_i}{\partial p_i}} \tag{6}$$

D_{ij} represents the share of firm i's marginal customers that would switch to firm j were firm i no longer an option.

[12]For a discussion of using observations on cost pass-through to calibrate demand, see Miller et al. (2012).

on demand slopes, rather than assuming functional forms for demand and cost functions, to emphasize both that $CMCR$ is invariant to the curvature of the demand and cost functions and that the calibrated slopes can be fit to a variety of demand and cost curves, depending on the application. For simplicity, I focus on the case of an industry with N firms, for which all N margins and all $(N^2 - N)$ diversions are observable; I note that this case may include industries consisting of publicly traded firms (who generally report profit margins) and for which an assumption of diversion proportional to share is appropriate.

I proceed by adapting existing calibration techniques for fitting demand curves to market observables when competition is in prices. In this setting, calibration chooses own price coefficients so that the margins implied by each firm's first-order condition closely matches observed margins, while choosing both own- and cross-price coefficients so as to match the implied diversion ratios to observed diversion ratios as closely as possible, all while satisfying Slutsky symmetry. If all relevant margins and diversion ratios are observed, calibration is generally overidentified, meaning that there is a tradeoff between more closely matching implied to observed margins, and more closely matching implied to observed diversion ratios. This tradeoff arises because under Slutsky symmetry any vector of own price coefficients implies multiple values of each cross-price coefficient; not only must a calibrated cross-price coefficient match both of these values as closely as possible, but the calibrated own-price coefficients affect the fit of implied to actual diversions, in addition to implying margins.

In setting of this paper, where competition is in quantities, calibration involves the same tradeoff between margins and diversion ratios. Mode of competition determines each firm's first-order condition, and thus the margin implied by a calibrated matrix of demand slopes. However, diversion ratios are a property of the underlying demand system itself, and do not depend on whether competition is in prices or quantities. Because diversion ratios are generally defined with respect to the direct demand system $Q(P)$, I calibrate both a matrix of direct demand slopes $\frac{\partial Q}{\partial P}$ and the corresponding matrix of indirect demand slopes, $\frac{\partial P}{\partial Q} = \left(\frac{\partial Q}{\partial P}\right)^{-1}$.[13]

Before describing the calibration algorithm, one addition to the model is needed. I have described the N firms as a "market." In any market, when customers switch from one firm, some customers go to other firms in the market, a presumably smaller number of customers switch to firms out of the market, and some customers may simply purchase less of the good in question. To capture this, I assume out-of-market diversion equal to $z \geq 0$. Depending on the application, z may be measured econometrically, or may be proxied for. For example, in some matters comparatively little is known about customer switching to Chinese manufacturers; in this case, Chinese manufacturers could be classified as "out-of-market", with out-of-market diversion proportional to a measure of the combined

[13]Slutsky symmetry implies that both $\frac{\partial P}{\partial Q}$ and $\frac{\partial Q}{\partial P}$ are symmetric matrices.

shares of Chinese manufacturers.

3.1 Calibration algorithm

My calibration algorithm first chooses a vector of own price coefficients to populate the diagonal of $\frac{\partial Q}{\partial P}$, the matrix of direct demand slopes. It then chooses off-diagonal elements of $\frac{\partial Q}{\partial P}$ to match implied diversions to observed diversion as closely as possible under Slutsky symmetry. It then inverts $\frac{\partial Q}{\partial P}$ to obtain $\frac{\partial P}{\partial Q}$, the matrix of inverse demand slopes. Then, it calculates an error function increasing in: 1- the squared distance between the diversion ratios as implied by $\frac{\partial Q}{\partial P}$ and observed diversion ratios, and 2- the squared distance between the margins implied by profit maximization and $\frac{\partial P}{\partial Q}$ and observed margins. Finally, the algorithm iterates over different choices of own price coefficients until the error function is numerically minimized.

First, I fix notation. Let B denote an $N \times N$ matrix of calibrated price slopes with generic element b_{ij}. To economize notation, all elements of B are positive, so that $b_{ii} = -\frac{\partial q_i}{\partial p_i}$ and $b_{ij} = \frac{\partial q_i}{\partial p_j}$. Let β denote the corresponding matrix of calibrated quantity slopes, so that $\beta = \overline{B}^{-1} = \frac{\partial Q}{\partial P}$, where $\overline{B}$ is equal to B but with its diagonal elements replaced by $-b_{ii}$, so that $\overline{B} = \frac{\partial Q}{\partial P}$. Let the generic elements of B and β be denoted by b_{ij} and β_{ij} respectively.

Steps 1-5 below construct an error function for any choice of own price coefficients b_{ii}. Calibration of demand slopes then proceeds by choosing $\{b_{ii}\}_{i=1}^{N}$ so as to minimize the error function described by steps 1-5.

1. For a given choice of $\{b_{ii}\}_{i=1}^{N}$, Slutsky symmetry and the definition of diversion (from equation (6)) imply:

$$b_{ij} = b_{ji} = D_{ij} b_{ii} = D_{ji} b_{jj} \tag{7}$$

   It is not generally possible for equation (7) to hold for all i, j. Indeed, a choice of b_{ii} implies that $b_{ij} = b_{ii} D_{ij}$, while a choice of b_{jj} implies $b_{ij} = b_{ji} = b_{jj} D_{ji}$. Calculate b_{ij} to be the weighted average of these two terms, with weights given by relative shares, so that:

$$\hat{b}_{ij} = \hat{b}_{ji} = \left(\frac{s_i}{s_i + s_j} b_{ii} D_{ij} + \frac{s_j}{s_i + s_j} b_{jj} D_{ji} \right) \tag{8}$$

2. A choice of $\{b_{ii}\}$ and $\hat{b}_{ij}$ imply a set of diversion ratios, $\hat{D}_{ij} = \frac{\hat{b}_{ji}}{b_{ii}}$. Scale the off-diagonal coefficients $\hat{b}_{ij}$ so that $\min(\sum_{j \neq 1} D_{1j}, \sum_{j \neq 2} D_{2j}) \leq (1-z)$. That is, $b_{ij} = \hat{b}_{ij} * \min \left\{ \frac{b_{11}(1-z)}{\sum_{j \neq 1} \hat{b}_{j1}}, \frac{b_{22}(1-z)}{\sum_{j \neq 2} \hat{b}_{j2}}, 1 \right\}$

for all $i, j, i \neq j$.[14]

3. Using the initial choice of $\{b_{ii}\}_{i=1}^N$ and the values of $\{b_{ii}\}_{i\neq j}$ implied by step 2, construct $\overline{B}$ whose diagonal elements equal $-b_{ii}$ and whose off-diagonal elements equal b_{ij}. Then, define $\beta = \overline{B}^{-1}$.

4. For each firm i, a choice of b_{ii} implies a margin of $\tilde{m}_i = \beta_{ii}\frac{q_i}{p_i}$ for firm i. Assign weight $\pi * w_i$ to the squared difference between implied margin $\tilde{m}_i$ and the observed margin m_i, with $\sum_{i=1}^N w_i = 1$.

5. The choices of $\{b_{ii}\}$ and $\{b_{ij}\}$ imply values of diversion $\tilde{D}_{ij} = \frac{b_{ji}}{b_{ii}}$. The implied $\tilde{D}_{ij}$ generate $N * (N-1)$ error terms between implied and observed diversion. Assign weight $(1-\pi)\omega_{ij}$ to the squared difference between the implied $\tilde{D}_{ij}$ and the observed D_{ij}, with $\sum_{i,j}\omega_{ij} = 1$.

   The choices of w_i and ω_{ij} I use in my calibration are:

$$w_i = s_i \tag{9}$$

$$\omega_{ij} = \frac{s_i s_j}{1 - \sum_{i=1}^N s_i^2} \tag{10}$$

   It is straightforward that $\sum_{i=1}^N w_i = \sum_{i=1}^N \sum_{j\neq i} w_{ij} = 1$, as required. I have not found that calibrated demand slopes to vary greatly in π, but in the calibrations discussed in section 4, I set $\pi = .99$.

6. Define the error function $\xi(\{b_{ii}\}_{i=1}^N)$ as follows:

$$\xi(\{b_{ii}\}_{i=1}^N) = \pi \sum_{i=1}^N w_i(\tilde{m}_i - m_i)^2 + (1-\pi)\sum_{i=1}^N \sum_{j\neq i} \omega_{ij}(\tilde{D}_{ij} - D_{ij})^2 \tag{11}$$

Given steps 1-5, calibration of demand slopes reduces to:

$$\min_{\{b_{ii}\}_{i=1}^N} \xi(\{b_{ii}\}_{i=1}^N) \tag{12}$$

In practice, I solve equation (12) numerically.[15]. Given a solution to equation (12) consisting of $\{b_{ii}\}_{i=1}^N$, define the matrix B via steps 1-2 above, and the matrix β by step 3.

Lemma 2 incorporates the calibrated matrix β into $CMCR$ as defined in proposition 1. Its proof is immediate.

[14] Other choices of scalar multiple for the off-diagonal terms are possible; the one given in the text is the most conservative, in that it results in the lowest diversion ratios. Another possible scalar is an average of the $\frac{b_{ii}(1-z)}{\sum_{j\neq i}\hat{b}_{ji}}$ terms over $i = 1, 2$.

[15] Matlab code solves the minimization problem in equation (12), given user-inputted values for margins, diversions, out-of-market diversion, and weight π

Lemma 2. *Given a set of calibrated demand derivatives B and β, the amount by which each of the merging firms' marginal costs must decrease so that post-merger quantities and prices are unchanged from pre-merger quantities and prices is following a merger of firms 1 and 2 is:*

$$CMCR_i = \frac{\beta_{ji}}{\beta_{jj}} \frac{m_j \frac{p_j}{p_i}}{1 - m_i} \text{ for } i, j \in \{1, 2\} \tag{13}$$

3.2 Merger simulation

Industrial organization researchers and antitrust practitioners often wish to calculate the counterfactual effect of some event, such as a merger. Doing so generally requires a fully-specified demand system and set of cost curves, and the counterfactual effect will depend on assumptions made about the form and curvature of these functions (unlike $CMCR$, which is invariant to the curvature of the underlying demand system and cost curves).

One counterfactual experiment frequently considered by researchers and antitrust practitioners is merger simulation: resolving profit maximization problems for the merging firms given calibrated or estimated demand and cost curves, and determining the implied merger price effect. While merger simulation requires much stronger assumptions than does $CMCR$, it also produces more granular, and thus potentially more useful, information. Specifically, while $CMCR$ gives the cost reduction for each merging firm that would result in no price increase, it it silent on the amount of any price change should expected merger efficiencies not precisely equal $CMCR$. Second, $CMCR$ can produce an ambiguous answer when applied to a specific merger; suppose that $CMCR_1 = 7\%$, while $CMCR_2 = 3\%$, and both merging firms are expected to reduce their marginal costs by 5%. In this case, $CMCR$ is insufficient to evaluate the merger's total effect on price.

I discuss parameterizing both linear (section 3.2.1) and loglinear (section 3.2.2) demand curves from the matrix of demand slopes β calibrated from pre-merger observables in section 3.1. Section 3.2.3 discusses parameterizations of other demand systems.

3.2.1 Linear demand

First, assume that demand is linear, so that the derivative matrices B and β describe the slope of demand for any values of P and Q, not just those prevailing pre-merger. While merger simulation admits a myriad of potential assumptions on cost curves, for simplicity assume that each firm's cost curve is either linear ($c_i(q_i) = c_i * q_i$) or quadratic ($c_i(q_i) = \frac{\gamma_i}{2} q_i^2$, with γ_i calibrated to match firm i's observed pre-merger margin m_i).

All firms' pre-merger first-order condition are given by equation (1) in section 2. Determine calibrated matrix β using the algorithm in section 3.1. Then, should firms 1 and 2 merge these firms would have the following first-order conditions:

$$\alpha_i - 2\beta_{ii}q_i - \gamma_i q_i - 2\beta_{ij}q_j - \sum_{k=3}^{N}\beta_{ik}q_k = 0 \;\; i,j \in \{1,2\} \text{ (if firm } i \text{ has quadratic cost)}$$

$$\alpha_i - 2\beta_{ii}q_i - c_i - 2\beta_{ij}q_j - \sum_{k=3}^{N}\beta_{ik}q_k = 0 \;\; i,j \in \{1,2\} \text{ (if firm } i \text{ has constant marginal cost)}$$

The non-merging firms have the following first-order conditions:

$$\alpha_i - 2\beta_{ii}q_i - c_i - \sum_{k\neq i}^{N}\beta_{ik}q_k = 0 \;\; i,j \in \{1,2\} \text{ (if firm } i \text{ has constant marginal cost)}$$

$$\alpha_i - 2\beta_{ii}q_i - \gamma_i q_i - \sum_{k\neq i}^{N}\beta_{ik}q_k = 0 \;\; i,j \in \{1,2\} \text{ (if firm } i \text{ has quadratic cost)}$$

If all firms have quadratic costs, the conditions for firms $3, ..., N$ are adjusted accordingly.

Merger simulation solves the system of first-order conditions, and compares the resulting prices and quantities to observed pre-merger prices and quantities.[16]

3.2.2 Loglinear demand

Now assume that demand is loglinear, so that the direct demand curve is given by:

$$\log(q_i) = \gamma_i + \sum_{j=1}^{N}\epsilon_{ij}\log(p_j) \tag{14}$$

where $\epsilon_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}$ is the own elasticity of demand.

Equation (14) can be inverted to produce inverse demand, as follows:

$$\log(p_i) = \eta_i - \sum_{j=1}^{N}\sigma_{ij}\log(q_j) \tag{15}$$

It is direct that $\sigma_{ij} = \frac{\partial \log(p_i)}{\partial \log(q_j)} = \frac{\partial p_i}{\partial q_j}\frac{q_j}{p_i}$. Under the calibration discussed in section 3.1, $\frac{\partial p_i}{\partial q_j} = \beta_{ij}$. Therefore, the vector η and the matrix σ are calibrated from demand slopes β as follows:

$$\sigma_{ij} = \beta_{ij}\frac{q_j}{p_i} \tag{16}$$

$$\eta = \log(P) + \sigma * \log(Q) \tag{17}$$

[16] In practice, I first compute the pre-merger equilibrium prices and quantities implied by the calibrated demand system, which may slightly differ from observed prices and quantities. I then use these implied pre-merger equilibrium values as the baseline. By doing this, any imprecision in the calibration applies equally to pre- and post-merger prices and quantities, so that merger simulation isolates only the effect of the merger. By design, the pre-merger prices and quantities implied by the calibrated demand system closely match observed values.

Under separate ownership, each of N firms simultaneously chooses quantity q_i in order to maximize profits given rivals' quantity choices. A pre-merger-Nash equilibrium satisfies:

$$\Rightarrow p_i = \frac{MC_i}{1-\sigma_i} \text{ for } i = 1, ..., N$$
$$\Rightarrow \sum_{j=1}^{N} \sigma_{ij} \log(q_j) = \eta_i - \log(MC_i) + \log(1-\sigma_i^c) \text{ for } i = 1, ..., N$$
$$\Rightarrow \sigma * log(Q) = K \text{ where } K_i = \eta_i - \log(MC_i) + \log(1-\sigma_i^c)$$
$$\Rightarrow Q = e^{\sigma^{-1}K} \tag{18}$$

Equation (18) provides a closed form solution for the pre-merger equilibrium in quantities. By design, the quantities implied by equation (18) closely match observed quantities.

Following a merger of firms 1 and 2, each merging firm internalizes the effect of its quantity choice on the profits of the other. The resulting post-merger first order conditions are:

$$e^{\theta_1}(1-\sigma_{11}) - e^{\theta_2}\sigma_{21}\frac{q_2}{q_1} - c_1 = 0 \tag{19}$$
$$e^{\theta_2}(1-\sigma_{22}) - e^{\theta_1}\sigma_{12}\frac{q_1}{q_2} - c_2 = 0 \tag{20}$$
$$e^{\theta_j}(1-\sigma_{jj}) - c_j = 0 \text{ for } j = 3, ..., N \tag{21}$$

where $\theta_i = \gamma_i - \sum_{j=1}^{N} \sigma_{ij} \log(q_j)$, so that $e^{\theta_i} = p_i$ equals firm i's price, as a function of logged quantities. This system of first-order conditions in (19)-(21) is solved numerically for q_i, $i = 1, ..., N$. As with linear demand, the solution to system (19)-(21) is compared to the pre-merger solution (18), with the difference (both in quantities and in associated prices) taken to be the effect of the merger.

3.2.3 Other demand systems

The algorithm I describe in section 3 calibrates the slopes of both the direct and inverse demand curves at the pre-merger equilibria. Various assumptions on direct demand curves are used by researchers and practitioners modeling competition in prices, with merger simulation results depending on the assumed form of demand (see Crooke et al. (1999) for a discussion). I chose the linear and loglinear systems as exemplars because of their relative tractability, and because the linear model in particular is commonly used in the industrial organization literature and by antitrust practitioners.[17] However, any direct demand system that is invertible in a large enough neighborhood of the

[17]Papers modeling competition in quantities with differentiated goods include: Alipranti, Millou, and Petrakis (2014); Brander and Spencer (2015); Davis (2002), Häckner (2000), Qiu (1997), Singh and Vives (1984), Symeonidis (2003), Wang and Zhao (2007). Okuguchi (1987) derives results for a class of demand curves satisfying a set of assumptions, without specifying any particular functional form, while Vives (1984) derives results for the class of demand curves satisfying a set of assumptions with linear demand being the only provided example.

Firm	Market share	Margin
1	24.0%	32.0%
2	18.0%	13.0%
3	36.0%	30.0%
4	16.0%	27.0%
5	6.0%	20.0%

Table 1: Observed margins and market shares for a hypothetical industry, and used throughout section 4. I further assume that diversion is proportional to share, with 10% out-of-market diversion, and I use a weighting parameter of $\pi = .99$ in calibrating demand slopes to these observables.

pre-merger equilibrium and which can be calibrated from direct and/or inverse demand slopes can be used to model competition in quantities with differentiated goods. Okuguchi (1986) discusses assumptions on the direct demand system necessary for it to be invertible, and for comparisons between Bertrand and Cournot outcomes to be well-founded.

4 Applications

In this section, I discuss applying the differentiated Cournot versions of $CMCR$, calibration of demand slopes, and merger simulation to a hypothetical industry.[18] I assume that pre-merger information on market shares, margins, and diversion ratios is available for each industry firm. In order to work with a concrete example, I suppose that the industry consists of five firms, with shares and margins as described in table 1. I further assume that diversion is proportion to share, with 10% out of market diversion. Finally, I use weighting parameter $\pi = .99$ in calibrating demand slopes.

Suppose that documentary evidence suggests that firms compete in quantities, and that evidence suggests that while different firms' products are substitutable, they are not interchangeable. I then study the effects of a merger between firms 1 and 2, focusing on different aspects of differentiated Cournot oligopoly in each of five subsections below. First, section 4.1 demonstrates how the differentiated Cournot can flexibly match observed-pre merger margins, in contrast to the homogeneous Cournot model, which implies that margins are proportional to shares. Next, section 4.2 discusses the relative profitability of mergers of differentiated Cournot oligopolists, compared to homogeneous Cournot oligopolists. Section 4.3 derives $CMCR$ and considers the price effects and merger profitability implies by various values of merger cost savings, again demonstrating the greater scope for

[18]To avoid disclosing nonpublic information, the examples do not track any real-world mergers or industries.

profitable mergers under differentiated Cournot demand. Section 4.4 compares merger outcomes under the differentiated Cournot model to those under comparable differentiated Bertrand demand systems, calibrated to the same set of pre-merger observables, and demonstrates that Cournot mergers can, but do not always, result in greater price effects than do Bertrand mergers. Finally, section 4.5 revisits the literature on the interaction between capacity constraints and merger price effects, in the context of the differentiated Cournot model.

4.1 Flexibility in matching observed margins

As discussed in section 1, when applied to industries whose firms are thought to produce homogeneous, interchangeable products, the Cournot model predicts that each firm's market share is linearly related to its profit margin, via its first order condition $m_i = -\frac{s_i}{\epsilon_{ii}}$, where s_i is firm i's share, and ϵ_{ii} its own price elasticity of demand. In practice, observed margins rarely obey the homogeneous Cournot model's prescription, and this incongruence is sometimes used to attack the validity of the Cournot model.[19] Indeed, as Werden (2010) points out, the "key test of a model used to predict the likely unilateral price effects of a merger is how well the model explains premerger pricing." When goods produced by different firms are viewed as differentiated, the Cournot model no longer implies that margins are linearly proportional to share; indeed, it admits any relationship between share and margin, as the various firms have distinct own-price elasticities.

Given the shares and margins in table 1, I calibrate demand slopes using the algorithm described in section 3.1, and then calculate the margins implied by the demand slopes, $\tilde{m}_i = \beta_{ii}\frac{q_i}{p_i}$, using step 4 of section 3.1. Table 2 displayes these implied margins, along with "observed" margins as described in table 1.

The rightmost column of table 2 contains margins implied by calibrating the observables in table 1 to demand slopes under the alternative assumption of homogeneous demand. To obtain the margins implied by the assumption of homogeneous Cournot competition, I first calibrate the market elasticity of demand ϵ that most closely matches the observed margins and shares from table 1.[20] To calibrate ϵ, I choose the value that minimizes the sum of squared errors between implied and observed margins, weighted by share, or:

$$\epsilon^* = argmin_\epsilon \sum_{i=1}^{N} s_i \left(\frac{s_i}{\epsilon} - m_i\right)^2 \tag{22}$$

[19] See footnotes 6 and 7, *supra*, and surrounding discussion.

[20] For the homogeneous Cournot calibration, I do not assume a particular out-of-market diversion, and I do not assume that diversion is proportional to share.

	Observables		Implied margins	
Firm	Market share	Margin	Diff. Cournot	Homog. Cournot
1	24.0%	32.0%	31.9%	23.5%
2	18.0%	13.0%	13.2%	17.6%
3	36.0%	30.0%	30.0%	35.3%
4	16.0%	27.0%	27.0%	15.7%
5	6.0%	20.0%	19.9%	5.9%

Table 2: Observed margins and those implied by calibrating observables to a differentiated Cournot model and a homogeneous Cournot model, respectively.

Then, each firm's implied margin is $m_i = \frac{s_i}{\epsilon^*}$. By inspection of table 2, while the differentiated Cournot calibration can flexibly match observed margins, the homogeneous Cournot calibration implies margins that are quite different from those observed. Notably, neither set of margins depend on underlying assumptions about demand or cost curvature. Instead, a matrix of demand slopes is consistent to calculate margins in the differentiated Cournot model, and the market elasticity from equation (22) is sufficient to calculate margins implied by the homogeneous Cournot model.

4.2 Merger profitability

Section 1 discusses the industrial organization literature documenting the seeming unprofitability of mergers implied by the Cournot model in a broad class of homogeneous goods industries. The section also documents that the purported unprofitability of Cournot mergers is sometimes used by antitrust practitioners to undermine the credibility of the Cournot model.[21]

When different firms' products are viewed as differentiated, mergers of Cournot competitors may be profitable or unprofitable. Because the products are more distant substitutes in the differentiated Cournot model, a reduction in the merging firms' output creates less of an incentive for nonmerging firms to expand production than would be the case were the various firms' products perfect substitutes. Because the expansion of output by nonmerging firms necessarily lowers the profit of the merging firms, differentiated Cournot mergers are thus relatively more profitable than homogeneous Cournot mergers.

Section 4.1 described calibrating demand slopes from the observables in table 1. Using these demand slopes, I now fit linear demand curves to both the differentiated Cournot matrix of demand

[21] *See* footnote 8, and surrounding text.

Firm	Differentiated Cournot Linear demand Linear costs	 Merging quadratic	 Quadratic costs	Homogeneous Cournot Linear demand Merging quadratic	 Quadratic costs
1	3.1%	1.8%	2.1%	1.6%	3.7%
2	4.9%	1.3%	1.5%	1.6%	3.7%
3	1.5%	0.6%	1.0%	1.6%	3.7%
4	1.5%	0.6%	1.1%	1.6%	3.7%
5	1.5%	0.6%	1.1%	1.6%	3.7%
	-1.8%	-0.5%	0.0%	-4.7%	-0.8%
	Profitability of merger to merging firms				

Table 3: Simulated price increases (blue rows) and merger profitability (peach rows) following a merger of firms 1 and 2, for demand calibrated to a differentiated Cournot linear demand system or a homogeneous Cournot linear demand system. Linear cost curves have form $c_i(q) = k_i q_i$, quadratic curves have form $c_i(q_i) = \frac{\gamma_i}{2} q_i^2$. "Merging quadratic" means the merging firms have quadratic costs while nonmerging firms have linear costs.

slopes and the homogeneous Cournot market elasticity. Then, I apply section 3.2.1 to calculate the merger price effects from a merger of firms 1 and 2 (under differentiated Cournot), with a corresponding calculation for homogeneous Cournot. Finally, I calculate the profitability of the merger to the merging firms, $\pi_{1,2}^{post} - (\pi_1^{pre} + \pi_2^{pre})$, for both differentiated and homogeneous Cournot. Results are shown in table 3, for each of three different assumptions on the curvature of cost curves (all firms have linear costs, the merging firms only have quadratic costs while all other firms have linear costs, and all firms have quadratic costs). Because the homogeneous Cournot model does not admit a post-merger equilibrium in which both merging firms continue to exist (see Salant , Switzer, and Reynolds, 1983), I omit merger simulation results for this case under homogeneous Cournot competition.

In this example, a merger under a calibrated homogeneous Cournot demand system is considerably more unprofitable than a merger under a calibrated differentiated Cournot demand system. It follows that there is greater potential for merger cost savings to render the merger profitable when products are modeled as being differentiated.

4.3 Efficiencies

Here, I emphasize both the utility and drawbacks of using $CMCR$ to assess merger cost savings. I first compute $CMCR$ using lemma 2, after calibrating demand slopes assuming that competition is

	Differentiated Cournot	Homogeneous Cournot
Firm	$CMCR$	$CMCR$
1	12.02%	25.49%
2	12.72%	25.49%

Table 4: Values of compensating marginal cost reduction ($CMCR$), given the market shares and margins described in table 2, and the calibration procedures described in section 3.1 (differentiated Cournot) and by equation (22) (homogeneous Cournot).

described by the differentiated Cournot model. I additionally compute $CMCR$ under the alternative assumption that competition is best described by the homogeneous Cournot model, using Froeb and Werden (1998).

From the observables described in table 1, the assumption of diversion proportional to share, and given 10% out-of-market diversion, I calibrate the matrix of demand slopes using the procedure described in section 3.1. This yields:

$$\beta = \frac{\partial P}{\partial Q} = \begin{bmatrix} 0.0133 & 0.0046 & 0.0059 & 0.0068 & 0.0070 \\ 0.0046 & 0.0073 & 0.0040 & 0.0047 & 0.0050 \\ 0.0059 & 0.0040 & 0.0083 & 0.0061 & 0.0063 \\ 0.0068 & 0.0047 & 0.0061 & 0.0169 & 0.0071 \\ 0.0070 & 0.0050 & 0.0063 & 0.0071 & 0.0332 \end{bmatrix} \tag{23}$$

Application of lemma 2 then produces values of $CMCR$. I report these, and values of $CMCR$ obtained by calibrating a homogeneous Cournot demand system using equation (22), in table 4.

I now reconsider the merger simulations of table 3 for various potential values of merger cost savings realized by the two merging firms. To emphasize the flexibility of $CMCR$, I additionally consider the case of differentiated Cournot calibrated to a loglinear demand system. Table 5 lists price increases resulting from a merger of firms 1 and 2 for cost savings of 2%, 6%, and the values of $CMCR$ for both differentiated and homogeneous Cournot demand from table 4. The table also lists the profitability of the merger to the merging firms under each scenario.

		Differentiated Cournot				Homogeneous Cournot	
		Linear demand			**Loglinear demand**	**Linear demand**	
Firm	Cost savings	Linear costs	Merging quadratic	Quadratic costs	Linear costs	Merging quadratic	Quadratic costs
1	2%	2.6%	1.53%	1.77%	23.26%	1.53%	3.43%
2	2%	4.09%	1.11%	1.29%	6.18%	1.53%	3.43%
Merger profitability		4.18%	0.76%	1.19%	-22.81%	-3.32%	0.35%
1	6%	1.56%	0.95%	1.1%	13.88%	1.3%	2.92%
2	6%	2.57%	0.71%	0.82%	2.91%	1.3%	2.92%
Merger profitability		16.87%	3.29%	3.57%	-2.53%	-0.46%	2.75%
1	12.02%	0.0%	0.0%	0.0%	0.34%	0.93%	2.07%
2	12.72%	0.0%	0.0%	0.0%	-0.18%	0.93%	2.07%
Merger profitability		39.29%	7.58%	7.59%	38.80%	4.38%	6.76%
1	25.49%	-3.52%	-2.26%	-2.6%	-19.25%	0.0%	0.0%
2	25.49%	-4.85%	-1.61%	-1.86%	-6.84%	0.0%	0.0%
Merger profitability		94.17%	17.76%	17.03%	75.30%	-4.7%	-0.8%

Table 5: Simulated price effects (blue rows) and merger profitability (peach rows) for firms 1 and 2, for the demand calibrations from table 3 plus an additional calibration to loglinear demand, and for various values of merger cost savings realized by the merging firms. Nonmerging firms do not realize cost savings. Loglinear demand shows a small price change when both firms realize cost savings equal to $CMCR$. I believe this is due to imprecision in the numerical algorithm I use to solve for post-merger equilibrium with loglinear demand.

Once again, while modest cost savings well below the levels of $CMCR$ render a merger of firms 1 and 2 profitable under differentiated Cournot, considerably greater costs savings may be required for profitability of a comparable merger under homogeneous Cournot. Cost savings equal to $CMCR$ result in zero price increase regardless of demand or cost curvature,[22] for both differentiated and homogeneous Cournot.

4.4 Comparison to Bertrand models

As summarized in section 1, a literature starting with Singh and Vives (1984) compares outcomes under Bertrand and Cournot modes of competition when rival firms produce differentiated products. The literature generally finds that Cournot competition is more monopolistic than Bertrand competition. The reason is that when a Bertrand competitor increases price, it reasons that its rivals will hold price constant. In contrast, when a Cournot competitor increases price, it conjectures that each of its rivals will also increase price in response, in order to keep their quantities constant.

To my knowledge, the literature has not studied whether this intuition extends to merger price effects. That is, if Cournot outcomes involve greater markups above cost than do Bertrand outcomes, should our prior be that Cournot merger effects are likely to be *greater* than comparable Bertrand merger effects (since less competitive industries may tend to have greater merger price effects), or should it be that Cournot effects are likely to be *less* than comparable Bertrand effects (since prices are already elevated closer to the monopoly level pre-merger)?

This question is complicated by the fact that mode of competition is, itself, determinative of calibrated demand. That is, while Singh and Vives (1984) and other papers generally compare Cournot and Bertrand outcomes for the same demand curve, the empirical industrial organization literature and antitrust practitioners generally start with a set of observables and fit demand to those observables using the various firms' optimality conditions. Since the optimality conditions depend on the underlying mode of competition, the same observables produce different demand curves depending on whether competition is thought to be in prices or in quantities. Thus, the question I ask here is: for a set of observables, does the demand curve implied by Cournot competition imply greater or lesser merger effects than that implied by Bertrand competition?

For the industry described in table 1, and given parameters z and π and an assumption that diversion is proportional to share, I additionally calibrate a Bertrand demand system, simulate merger outcomes under that system, and compare merger price effects to those described for differentiated

[22]The exception is the small price change predicted under loglinear demand and differentiated Cournot competition, which appears to result from a slight imprecision in the numerical algorithm I use for solving the post-merger equilibrium and the sensitivity of loglinear demand to small changes.

Cournot in table 3.

The Bertrand calibration is identical to the Cournot calibration described in section 3.1, except that the margin implied by any choice of demand slopes reflects the Bertrand first order condition $\tilde{m}_i^{Bertrand} = -\frac{1}{\frac{\partial q_i}{\partial p_i}} \frac{q_i}{p_i}$, rather than the Cournot implied margin $\tilde{m}_i^{Cournot} = \frac{\partial p_i}{\partial q_i} \frac{q_i}{p_i}$. Given this difference, direct demand coefficients b_{ij} are chosen to solve the minimization problem in equation (11), but with $\tilde{m}_i^{Bertrand}$ replacing the corresponding Cournot value. I then use standard techniques to compute the implied Bertrand equilibrium that would follow a merger of firms 1 and 2, for a variety of cost curves. To facilitate comparison with the Cournot simulations in table 3, I consider only linear and loglinear demand calibrations.

Table 6 contains the results. For each type of assumed demand and cost curvature, the calibrated Cournot model predicts a greater merger price effect than does the calibrated Bertrand model. Work to assess Bertrand and Cournot merger price effects across a variety of industries, following the analysis of Greenfield and Sandford (2021), is underway.

It is straightforward to prove that calibrated Cournot demand does not universally imply larger merger price effects than does calibrated Bertrand demand. Consider the special case of two identical (or nearly identical) firms producing homogeneous (or nearly homogeneous) goods. When separately owned, if the firms compete in price, the textbook Bertrand model predicts they will price at cost (or nearly at cost). In contrast, Cournot duopolists producing identical goods earn a markup above cost, such that each firm has margin $m_i = \frac{s_i}{\epsilon}$, where ϵ is the market elasticity of demand.

Now suppose the two firms merge, to monopoly. A monopolist will produce the same quantity and set the same price regardless of whether it was formed by a merger of Cournot duopolists or Bertrand duopolists. Hence, the post-merger outcome is the same across the two scenarios. Since price was lower under the calibrated Bertrand demand system, it follows that the merger price effect is larger under Bertrand competition than under Cournot competition.

4.5 Capacity constraints

A small literature, notably including Crooke, Froeb, and Tschantz (1999) and Greenfield and Sandford (2021), argues that merger price effects are likely to be attenuated should one or both merging firms be capacity-constrained prior to merging, and likely to be amplified should one or more nonmerging firms be capacity-constrained. The literature further suggests that capacity constraints on merging firms are more important determinants of merger price effects than are capacity constraints on non-merging firms. These papers generally consider models of price competition with differentiated goods.

I examine the intuition from this literature in the context of the industry described in table 1.

	Differentiated Bertrand			Differentiated Cournot		
	Linear demand		**Loglinear demand**	**Linear demand**		**Loglinear demand**
Firm	Linear costs	Quadratic costs	Linear costs	Linear costs	Quadratic costs	Linear costs
1	2.8%	1.7%	7.9%	3.1%	2.1%	28.5%
2	3.0%	1.1%	6.8%	4.9%	1.5%	7.7%
3	0.9%	0.7%	0.0%	1.5%	1.0%	0.0%
4	0.8%	0.7%	0.0%	1.5%	1.1%	0.0%
5	0.7%	0.9%	0.0%	1.5%	1.1%	0.0%

Table 6: Simulated price effects under Bertrand and Cournot demand calibrations for the margins and shares in table 2, and assuming diversion proportional to share. Cournot calibration predicts a greater merger price increase than Bertrand calibration for each assumed demand and cost structure.

While Greenfield and Sandford (2021) model a capacity constraint as a discrete jump in a constant marginal cost, they discuss "softer" capacity constraints involving increasing marginal costs. In that vein, quadratic costs are a type of soft capacity constraint, in that it becomes more and more costly for a firm to increase production beyond some economical point. Greenfield and Sandford (2021) model unconstrained firms as having linear costs. Hence, I model a firm with a quadratic cost curve calibrated to the pre-merger equilibrium as being "capacity-constrained," and a firm with a linear cost curve as being unconstrained.

Tables 3 and 5 contain the merger price effects implied by calibrating the industry described in table 1 to various demand and cost curves. Those tables suggest that the results developed in the literature for Bertrand competition also apply to differentiated Cournot competition. For example, in table 3, the merging firms, 1 and 2, raise price by 3.1% and 4.9%, respectively, if all firms are unconstrained (linear costs). If the merging firms – but not the nonmerging firms – are constrained, the corresponding price effects are 1.53% and 1.11%, respectively. Finally, if all firms, merging and nonmerging, are constrained, then the price effects for the merging firms are 1.77% and 1.29%.

5 Conclusion

The differentiated Cournot model seems to have fallen out of favor with academic researchers, and seems to have never caught on at all with antitrust practitioners. Indeed, the latter group appear to default to competition in prices when products are differentiated, and seem to apply the Cournot model only when goods are thought to be reasonably homogeneous. This lack of interest in the differentiated Cournot model is puzzling; clearly, both academic researchers and antitrust practitioners view competition in quantity as an important phenomenon, given the prevalence of academic papers and antitrust litigation premised on the (homogeneous) Cournot model. I do not see a theoretical basis for presuming that competition in quantities ceases to be important once products are differentiated.

Moreover, the homogeneous Cournot model is, by design, inflexible. It assumes that all products are interchangeable, which implies that each firm's profit margin is proportional to its market share; in practice, observed shares rarely follow this dictum. The homogeneous Cournot model's assumption of interchangeability of the different goods implies that nonmerging firms are incentivized to increase production following a merger, suggesting that these nonmerging firms may realize the bulk of the benefits of a merger, and indeed merging firms may have greater total profits when separately owned. Finally, the assumption of interchangeability itself is often empirically suspect, as even commodity products are commonly differentiated by branding, idiosyncratic customer preferences, distance from customer locations to manufacturing plant, and minor differences in quality. Each of these inflexibilities opens the Cournot model to attack when used in academic studies or in antitrust litigation.

The differentiated Cournot model circumvents each of the listed inflexibilities associated with the homogeneous Cournot model, while preserving the (presumably important) setting in which firms compete by choosing quantities, with prices set so as to equate each firm's demand and supply. Perhaps the differentiated Cournot model has fallen into disuse because of technical difficulties associated with applying it to industrial organization and antitrust settings. If so, hopefully this paper will give the differentiated Cournot model new life. As described above, I derive a $CMCR$ metric depending only on pre-merger margins, prices, and demand slopes, and which is invariant to assumptions on demand and cost curvature. I show that if the required demand slopes cannot be measured econometrically, they can be calibrated from pre-merger information on shares and diversions, again without relying on assumptions about demand and cost curvature. Finally, should a researcher or practitioner wish to model merger price effects or other counterfactuals, she can use the calibrated demand slopes to populate the parameters of a system of demand and cost curves.

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