Upward pricing pressure in mergers of capacity-constrained firms*

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March 25, 2021

Abstract

Merging firms regularly argue that mergers involving capacity-constrained firms are unlikely to be anticompetitive, because a capacity-constrained firm does not represent a meaningful competitive constraint on its rivals. We construct a modified notion of upward pricing pressure called $ccGUPPI$, or capacity-constrained $GUPPI$, which accounts for upward pricing pressure from binding capacity constraints, in addition to standard merger effects. We show that the pricing pressure terms underlying $ccGUPPI$, calculated using pre-merger data, are sufficient to determine whether a merger of capacity-constrained firms will increase price, irrespective of the functional form of demand. Further, using Monte Carlo simulation, we show that $ccGUPPI$ is generally a useful proxy for actual price effects, with lower informational requirements than full merger simulation.

*We thank Sangin Park, Charles Taragin, Ted Rosenbaum, Nathan Wilson, Dave Osinski, Dave Schmidt, Pat DeGraba, Wesley Wilson, and two anonymous referees for helpful comments. The opinions expressed here are those of the authors and not necessarily those of the Federal Trade Commission or any of its Commissioners. This paper previous circulated under the title "Mergers of capacity-constrained firms."

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1 Introduction

Firms with binding capacity constraints increase price and lower quantity relative to their optimal choices absent constraints. Merging firms often argue that this implies that mergers involving capacity-constrained firms are unlikely to increase price, even when there is significant demand substitution between the merging firms' products. The authors have heard such claims in connection with mergers before the FTC in a variety of industries. Merging fitness gyms recently made such arguments to the UK competition authority.[1] Penn State Hershey and PinnacleHealth hospital systems argued that capacity constraints mitigated antitrust concerns in response to the FTC's 2016 effort to block their merger in United States District Court.[2] Merging parties typically argue that since constrained firms would lower price and increase quantity but for the constraint, a merger involving capacity-constrained firms is unlikely to result in higher prices.

While numerous studies point out that mergers involving capacity-constrained firms indeed may increase price,[3] the economics literature lacks tools both to predict which mergers will cause a price increase and to predict the magnitude of any price increase. We aim to fill this gap. Our paper constructs a version of gross upward pricing pressure ($GUPPI$) modified to account for capacity constraints, which we call $ccGUPPI$, or capacity-constrained $GUPPI$. Like other measures of upward price pressure, $ccGUPPI$ relies only on information that is local to pre-merger equilibrium (price, quantity, margins, and demand elasticities of the merging parties' products). It can qualitatively predict whether or not a merger will increase prices, and it can quantitatively predict the magnitude of merger price effects.

Specifically, we employ $ccGUPPI$ to predict whether both merging firms' constraints will continue to bind post-merger, and thus eliminate merger price effects. Used in this way, $ccGUPPI$ provides a diagnostic as to whether a proposed merger between capacity-constrained firms will likely

[1] ''According to the parties, the fact that they operate at or close to capacity indicates that they are not providing a significant competitive constraint on each other, as neither of them is seeking to win new customers.'' See Competition and Markets Authority (2014), ''Anticipated combination of Pure Gym Limited and The Gym Limited,'' paragraphs 141 and 142, found via Neurohr (2016).

[2] Defendants' expert Bobby Willig testified as follows in FTC vs. Penn State Hershey Medical Center and PinnacleHealth System, April 15, 2016: ''But once capacity is taken into account, there can't be substantial diversion of patients from ... Hershey to Pinnacle ... because Hershey just doesn't have the capacity to take on a major influx of patients... So the practical diversion between Pinnacle and Hershey is insignificant due to Hershey's capacity constraint." One of the defendants' briefs contained the following: ''...the combination will alleviate Hershey's capacity constraints and simultaneously allow both hospitals' physicians to treat more people,'' in ''Defendants' Opposition to Plaintiffs' Motion For An Injunction Pending Appeal,'' May 12, 2016.

[3] See Froeb et al. (2003), Higgins et al. (2004), Sandford and Sacher (2016), Neurohr (2016), Oxera (2016), Balan et al. (2017), and Chen and Li (2018).

raise prices, irrespective of the curvature of demand. We further show that $ccGUPPI$ can be used to predict the magnitude of merger price effects. We compare $ccGUPPI$'s predictions to actual price increases calculated via merger simulation, using a version of the Monte Carlo experiment of Miller et al. (2016 and 2017) modified so that some firms are capacity-constrained prior to the merger. We find that $ccGUPPI$ offers excellent predictions of merger price effects when demand is linear or logit, and generally underestimates merger price effects when demand is AIDS. Across all three demand systems, $ccGUPPI$ appears to perform better than the next best alternative predictor, unmodified $GUPPI$. We further show that when used as a screen to identify mergers that will generate a specified minimum price increase, $ccGUPPI$ has a much lower false positive rate than $GUPPI$ under all three demand systems, and a roughly similar false negative rate.

Our paper adds to the somewhat sparse literature on mergers involving capacity–constrained firms. Froeb et al. (2003) simulate the effects of a hypothetical merger in a industry producing differentiated goods, subject to differing capacity constraints on the merging and non-merging firms. Based on their simulations, they argue that capacity constraints on merging firms attenuate merger effects more than capacity constraints on non-merging firms amplify them, and are critical of the 1992 Horizontal Merger Guidelines, which acknowledge the importance of the latter but not the former. The Froeb et al. (2003) paper is commonly cited by merging parties alleging that capacity constraints would eliminate or mitigate merger price effects. Notably, the merging firms are so tightly capacity-constrained in the paper's main example a merger does not increase price at all.[4]

Higgins et al. (2004) discuss a more general model in the same vein as Froeb et al. (2003), and again demonstrate via simulated results that capacity constraints on merging firms may attenuate merger price effects. Chen and Li (2018) argue that in a Bertrand-Edgeworth setting with identical firms, firms play a pure strategy if capacity constraints are low enough or high enough and a mixed strategy for the intermediate range. A merger both expands this intermediate range in both directions and shifts the distribution of prices within the mixed equilibrium to the right. Consistent with the Froeb et al. (2003) example, Chen and Li find that a merger has no effect on price outside of this intermediate range of capacity values. However, a merger in any industry that falls into the pre-merger intermediate range results in a price increase, as does any merger that causes the industry to shift from a pure to a mixed equilibrium. Chen and Li conclude that antitrust authorities should consider the tightness of capacity constraints when evaluating mergers involving such constraints.

Other papers discuss merger price effects when one or both merging firm is constrained in the context of a Cournot model (see Balan et al. (2017), Sacher and Sandford (2016)) or a differentiated Bertrand model (see Balan et al. (2017), Neurohr (2016), Oxera (2016)). All point out that if both

[4]See tables 2 and 3 of Froeb et al. (2003).

merging firms are capacity-constrained pre-merger, positive price effects of the merger result if and only if at least one constraint no longer binds post-merger.

Our paper builds on Neurohr (2016), which constructs a modified notion of upward pricing pressure that is applicable when both merging firms are constrained pre-merger, but neither is constrained post-merger. The intuition underlying our measure of upward pricing pressure is the same as that behind Neurohr's: the tightness of pre-merger capacity constraints determines the extent to which those constraints attenuate the upward price pressure from a merger. Our measure is equivalent to Neurohr's when both merging firms are constrained before the merger and neither firm is constrained after the merger. However, $ccGUPPI$ is a more comprehensive measure of upward pricing pressure in that it also applies to mergers when only one merging firm in constrained before or after the merger. More importantly, $ccGUPPI$ allows for a prediction of which pre-merger constraints will bind post-merger. $ccGUPPI$ is sufficient to determine whether a merger between two capacity-constrained firms will increase prices (i.e., whether at least one firm's constraint no longer binds post-merger), regardless of the form of demand. Further, $ccGUPPI$ allows approximate predictions of which pre-merger constraints will bind post-merger, and of the magnitude of resulting price effects.

The next section presents a leading example, which shows how capacity constraints alter merger price effects. Section 3 describes the modeling framework and derives the effect on pricing incentives of a merger involving one or more capacity-constrained firms. Section 4 describes how we construct $ccGUPPI$. Section 5 describes the Monte Carlo experiment and resulting data. Sections 6 and 7 discuss our results and conclude.

2 Leading example

We first consider a illustrative example of duopoly firms merging to monopoly. Specifically, suppose firms 1 and 2 produce differentiated but substitutable products at constant marginal cost 0, competing *a la* Bertrand by simultaneously setting price. Firms face the following demand system:

$$\begin{aligned} q_1 &= 10 - p_1 + \tfrac{1}{2}p_2 \\ q_2 &= 10 - p_2 + \tfrac{1}{2}p_1 \end{aligned} \tag{1}$$

Absent capacity constraints, the Nash equilibrium of the Bertrand pricing game in which firm i maximizes $\Pi_i = (p_i - c_i)q_i$ is $(p_i, q_i) = (\frac{20}{3}, \frac{20}{3})$ for $i = 1, 2$. Were the two firms to merge the merged entity would jointly choose p_1 and p_2 to maximize $\Pi_1 + \Pi_2$, and post-merger prices and quantities would be $(p_i, q_i) = (10, 5)$ for $i = 1, 2$. Figure 1(a) plots pre-merger best response functions (in red) and post-merger first order conditions for the merged firm (in blue). In both cases, solid lines correspond to firm 1, and dashed lines to firm 2. Since the merged firm recaptures some of the lost sales from a price increase, it has an additional incentive to raise prices that did not exist before the merger, and thus the post-merger first order conditions are bowed out relative to the pre-merger best response functions, so that the post-merger equilibrium has higher prices.

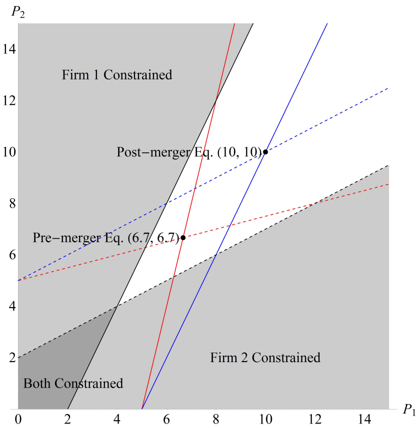
Now, suppose that each firm has K_i units of capacity, with marginal cost constant at zero for $q_i \leq K_i$ and prohibitively high for $q_i > K_i$ such that it would be unprofitable for either firm to produce in excess of its capacity. Figure 1(a) sets $K_1 = K_2 = 8$. We divide each figure into four subsets of the (p_1, p_2) space: where firms 1 and 2 are capacity-constrained, respectively, where both are constrained, and where neither is constrained. In figure 1(a), since both the pre- and post-merger equilibria lie in the region in which neither firm is constrained, the capacity constraints have no effect on either firm.

The example in figure 1(b) is identical, except that $K_1 = K_2 = 4$. This expands the set of prices for which one or both firms is capacity-constrained. Since each of the firms' unconstrained profit-maximizing prices, both pre- and post-merger, would cause demand to exceed capacity, each firm raises price until its demand just equals its productive capacity. Thus, in figure 1(b), each firm optimally sets a price of 12, and sells quantity 4. Here, the constraints are severe enough that the merger has no price effect; each firm is so constrained pre-merger that the incentive to raise price from the constraint exceeds the incentive to raise price stemming from the merger and consequent elimination of competition.

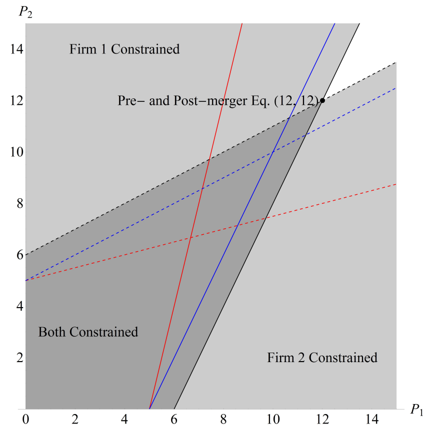
Figure 1(c) considers an example where $K_i = 6$, $i = 1, 2$. Here, each firm's constraint binds before the merger but not after. Before the merger, we have $p_i = 8$, $i = 1, 2$ while post-merger we have $p_i = 10$, $i = 1, 2$. Hence, the capacity constraints attenuate the merger price effect by elevating pre-merger prices, relative to the case in which firms were not capacity-constrained.

Figure 1(d) considers a case with asymmetric capacity ($K_1 = 8$ and $K_2 = 4.5$), so that exactly one firm is constrained, both before and after the merger. Absent the constraints, the pre-merger Nash equilibrium would be located at the intersection of the red best response curves, or $(\frac{20}{3}, \frac{20}{3})$. Since firm 2 is constrained at this point (but not firm 1), firm 2 will increase its price until $q_2 = 4.5$. Since firm 1's best response to a higher p_2 is itself higher, the Nash equilibrium is located at the intersection of firm 1's pre-merger best response curve and the $q_2 = K_2$ locus, or $(p_1, p_2) = (7.3, 9.1)$, with $(q_1, q_2) = (7.3, 4.5)$.

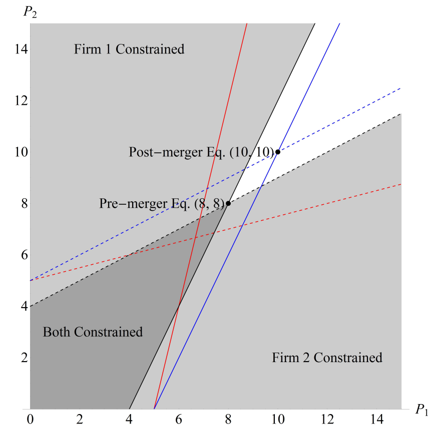
Following the merger, an unconstrained monopolist would set prices of $(p_1, p_2) = (10, 10)$ and $(q_1, q_2) = (5, 5)$; this is the point at which the two blue lines intersect. However, this point is not feasible, as firm 2 would exceed its capacity constraint of 4.5. Hence, p_2 is set so that $10 - p_2 + \frac{1}{2}p_1 =$



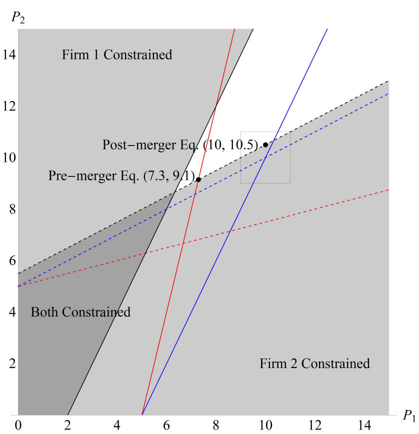
(a) unconstrained pre-and post-merger



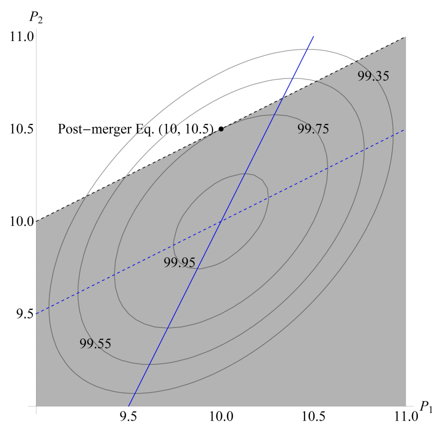
(b) constrained pre-and post-merger



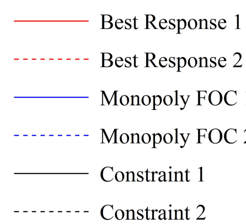
(c) constrained pre-merger, unconstrained post-merger



(d) one constrained pre- and post-merger, one unconstrained



(e) post-merger profit level sets



Legend

Figure 1: Capacity equals $K_1 = K_2 = 8$ in (a), $K_1 = K_2 = 6$ in (b), and $K_1 = K_2 = 4$ in (c), and $K_1 = 8$, $K_2 = 4.5$ in (d) and (e). In each case, duopolists under the demand system 1 merge to monopoly. The presence of the capacity constraints do not affect merger price effects in (a), eliminate price effects in (b), and attenuates price effects in (c)-(e).

4.5, while p_1 is the solution to:

$$\max_{p_1, p_2} p_1 * q_1 + p_2 * K_2 \tag{2}$$
$$\text{s.t.} \quad q_1 = 10 - p_1 + \frac{1}{2}p_2$$
$$p_2 = 5.5 + \frac{1}{2}p_1$$

In solving (2), the merged firm is choosing the point on the $q_2 = K_2$ locus that maximizes $\pi_1 + \pi_2$.

In particular, the monopolist knows that an increase in p_1 will lead to an increase in p_2, since q_2 is increasing in p_1 and decreasing in p_2. The result is that the merged firm sets prices of $(p_1, p_2) = (10, 10.5)$, meaning that $(q_1, q_2) = (5.25, 4.5)$. Figure 1(e) magnifies the area surrounding the point $(10, 10.5)$ and depicts level sets of the function $\Pi_1 + \Pi_2$, with summed profits increasing in the direction of the point $(10, 10)$. Evidently, the maximum achievable profit on the $q_2 = K_2$ locus is at $(p_1, p_2) = (10, 10.5)$.

We can take away several ideas from this example. First, absent any capacity constraints, a merger of firms 1 and 2 would have led to a 50% price increase, and capacity constraints can attenuate or eliminate the merger price effects depending on how tightly they bind. Second, while capacity constraints generally attenuate merger price effects by elevating pre-merger prices, price still increases following the merger, so long as at least one product is unconstrained post-merger. Indeed, even in figure 1(d)-(e), both p_1 and p_2 increase despite firm 2's constraint binding both before and after the merger. The optimization problem of the merged firm changes when one product is capacity-constrained and the other is not, but the merged firm still internalizes increased demand for product 2 following an increase in p_1, and this increased demand allows for a higher p_2.

The next section specifies a general model of differentiated Bertrand competition with capacity constraints. We derive the pre- and post-merger equilibrium conditions and then subsequently use those conditions to construct $ccGUPPI$.

3 Model

We study a standard model of price competition among N firms selling differentiated products, modified to incorporate capacity constraints. Given a vector of prices $\boldsymbol{p}$, firm i's demand is $q_i^D(\boldsymbol{p})$, which is differentiable, decreasing in p_i, and increasing in p_j. Each firm has access to a constant marginal cost production technology capable of producing K_i units (e.g., a factory). We refer to K_i as a firm's *capacity*. Each firm additionally has access to a higher cost production technology of unlimited capacity (e.g., buying or importing the product instead of producing it, or repurposing a factory producing a different good). We refer to this additional production technology as a firm's *flex capacity*. We assume that a firm's marginal cost increases by $\gamma > 1$ once it begins using its flex capacity.[5] Thus, equation (3) describes firm i's total cost:

$$c_i(q) = \begin{cases} c_i q & \text{if } q \leq K_i \\ c_i K_i + \gamma_i c_i (q - K_i) & \text{if } q > K_i \end{cases} \tag{3}$$

[5]Dixit (1980) is the earliest example we know of to include a stepped cost function to model capacity constraints. See also Maggi (1996) and Boccard and Wauthy (2000), each of which uses the same cost function we do.

Firms simultaneously choose price, with each firm maximizing profit taking as given its rivals' prices. We first study the case in which all N firms are separately owned, and then the case in which firms 1 and 2 merge.

Our results depend on characterizing pure strategy equilibrium outcomes using first-order conditions. Hence, we assume that such equilibria exist and are unique, both when firms are individually owned and following a merger of any two firms. Further, we assume that demand and cost curves are such that profit functions are concave in price, and jointly concave in prices following a merger, so that equilibria may be characterized using first-order conditions.[6]

We make two simplifying assumptions about firms' capacity.

Assumption 1: Each firm sells $q_i^D(\boldsymbol{p})$ (no chance to reoptimize over quantity once prices are set).
Assumption 2: γ_i is large enough that firm i does not find it profitable to use flex capacity.

Assumption 1 dictates that firms choose price and then supply whatever quantity the market demands at that price. While this is the usual Bertrand assumption of price-setting behavior, when applied to our model of capacity constraints assumption 1 requires firms to make use of their flex capacity if $q_i^D(\boldsymbol{p}) > K_i$, even if it would not profitable to do so. The alternative assumption, allowing firms to re-optimize over quantity once all prices are set, leads to non-existence of pure strategy Nash equilibria, and any mixed equilibria will depend on an assumed rationing rule.[7] By Dastidar (1997), assumption 1 is justified when prices are set by sealed bid tenders, or when there are large costs to turning away customers. Assumption 1 is ubiquitous in the literature on oligopolies, and appears in models with and without capacity constraints.[8]

[6]Similar assumptions are made, either explicitly or implicitly, in, for example, Froeb et al. (2003), Jaffe and Weyl (2013), Miller et al. (2016 and 2017), Neurohr (2016), and Farrell and Shapiro (2010) (e.g., in Froeb et al., at 52 ''... we assume that the demand and cost functions are such that the profit functions are convex and consistent with the existence of (usually a unique) Nash equilibrium.")

[7]Suppose there were a pure strategy equilibrium in which, after prices were chosen, firms were allowed to sell less than the market demand for their product. In such an equilibrium, each unconstrained firm would set price to equate marginal revenue and marginal cost, while constrained firms would set price so that demand equals capacity. But then any unconstrained firm would have an incentive to increase price slightly, causing the demand for all other firms' products to increase. If the capacity-constrained firms chose not to employ flex capacity to meet this demand increase, then some of the excess demand would be reallocated towards the firm who increased price, according to the assumed rationing rule. As a marginal price increase would have no direct effect on an optimizing firm's profit, the total effect on profit of the price increase plus the additional quantity must be positive. Thus, there can be no pure strategy equilibrium under the alternative assumption. See Shapley and Shubik (1969) for a fuller discussion of potential non-existence of equilibrium.

[8]In addition to Dastidar (1997), see Bulow et al. (1985), Vives (1990), Dixon (1990), Dastidar (1995), and Chen (2009) for examples of oligopoly models that employ analogues of assumption 1 to study capacity constraints. Countless papers employing Bertrand oligopoly models without capacity constraints use assumption 1 in assuming that firms set price, and not quantity.

Assumption 2 ensures that no firm will use its flex capacity in equilibrium. Consequently, in equilibrium, a firm is either *capacity-constrained* ($q_i = K_i$) or *unconstrained* ($q_i < K_i$). Absent assumption 2, a firm with $q_i > K_i$ could also be said to be unconstrained, with marginal cost $\gamma_i c_i$. However, a merger involving this firm could lower its equilibrium quantity to be less than or equal to K_i, thereby lowering its marginal cost. Assumption 2 avoids this this nuisance case. Further, by ensuring that firms set price so that demand does not exceed capacity, assumption 2 implies that a firm's objective function is continuous over the relevant portion of its domain. While we assume the existence of a pure strategy equilibrium, it is possible that no such equilibrium would exist absent assumption 2.[9]

While assumptions 1 and 2 represent a common approach to modeling binding capacity constraints, ours is not the only approach. In particular, smooth but increasing marginal cost functions could closely replicate the "L-shaped" marginal cost curves used in this paper,[10] and would allow for standard pricing pressure analysis while, at least in the limit, producing identical merger price effects. Section 6 explains why $ccGUPPI$ remains an improvement over standard $GUPPI$ even if marginal cost functions are smooth approximations to the "L-shaped" marginal cost functions in our paper.

We proceed by solving the model both before and after a merger of firms 1 and 2. Then, we study how the change in incentives generated by the merger vary in whether or not each merging firm is capacity-constrained prior to the merger.

3.1 Pre-merger pricing pressure

If the N firms are separately owned, each firm i takes other prices p_{-i} as given, and chooses p_i to maximize profits. Under assumption 1, firm i's profits are given by $q_i^D(\boldsymbol{p})p_i - c_i(q_i^D(\boldsymbol{p}))$. Let $q_i^{-1}(K_i, \boldsymbol{p}_{-i})$ denote the price p_i at which $q_i^D = K_i$, and below which $q_i^D > K_i$. Under assumption 2, all firms optimally set price $p_i \geq q_i^{-1}(K_i, \boldsymbol{p}_{-i})$ and so each firm has constant marginal cost of c_i. Thus, firm i's pre-merger maximization problem is:

$$\max_{p_i} q_i^D(\boldsymbol{p})(p_i - c_i) \quad (4)$$
$$\text{s.t. } p_i \geq q_i^{-1}(K_i, \boldsymbol{p}_{-i})$$

If the price p_i which solves (4) exceeds $q_i^{-1}(K_i, \boldsymbol{p}_{-i})$, then the firm is unconstrained, and its first-

[9]We thank an anonymous referee for pointing out the importance of assumption 2 to the existence of a pure-strategy equilibrium.

[10]See, *inter alia*, Ryan (2013) and Miller and Osborne (2014) for examples of such cost functions applied to model capacity constraints.

order condition for (4) is:

$$\frac{\partial \pi_i}{\partial p_i} = q_i(p) + \frac{\partial q_i}{\partial p_i}(p_i - c_i) = 0$$
$$\Rightarrow m_i = -\frac{1}{\epsilon_{ii}} \tag{5}$$

where $m_i = \frac{p_i - c_i}{p_i}$ denotes firm i's margin and $\epsilon_{ii} = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$ denotes firm i's own-price elasticity. Equation (5), relating firm i's margin to its elasticity of demand, is the well-known Lerner condition, and is equivalent to a condition that the marginal benefit of selling one more unit equal the marginal cost of doing so.

On the other hand, if the p_i that solves (4) equals $q_i^{-1}(K_i, \boldsymbol{p}_{-i})$, then the firm is capacity-constrained. In this case, its margin is greater than its inverse elasticity ($\frac{p_i - c_i}{p_i} > -\frac{1}{\epsilon_{ii}}$), so the Lerner condition no longer holds. Let $\lambda_i > 0$ denote the difference between margin and inverse elasticity, or the ''wedge'' between the two sides of the Lerner condition.[11] Then, for any constrained firm, we have:

$$m_i = -\frac{1}{\epsilon_{ii}} + \lambda_i \tag{6}$$

The quantity λ_i measures pricing pressure due to the capacity constraint K_i binding; a greater value of λ_i implies greater pricing pressure from the constraint.[12] As we will see in the next section, λ_i is directly comparable to upward pricing pressure resulting from a merger with another firm (caused by each firm internalizing the effect of its own price increase on its former rival's profits). In particular, a merger involving two capacity-constrained firms results in a price increase if and only if λ_i is less than the post-merger pricing pressure for at least one merging firm.[13]

[11] λ_i is proportional to the shadow price of firm i's capacity constraint, or the rate at which firm i's profits increase as the constraint is relaxed.

[12] In the numerical example of figure 2(b), where both firms are constrained pre-merger, we have that $\lambda_i = \frac{2}{3}$. Given that firm i sells a quantity of 4, a price of \$4 would satisfy firm i's Lerner condition, but firm i in fact charges \$12 due to its capacity constraint. Thus, $\lambda_i = \frac{2}{3}$ represents the fraction of firm i's pre-merger price that is elevated above the price that would satisfy the Lerner condition because of i's capacity constraint. Since $m_i = 1$ and at the pre-merger equilibrium $\epsilon_{ii} = -3$, we have that $\lambda = \frac{2}{3}$, meaning that the capacity constraints result in pricing pressure equal to $\frac{2}{3}$ of the price $p_i = \$12$.

[13] In the example of figure 2(b), we have that $\lambda_i = \frac{2}{3}$ while $GUPPI_i = \frac{1}{2}$, where $GUPPI$ is defined and discussed in section 3.2. This implies that the merger results in less upward pricing pressure than does firm i's capacity constraint. Consequently, the merger does not result in a price increase.

In contrast, in the example of figure 2(c), we have $\lambda_i = \frac{1}{4}$ while $GUPPI_i = \frac{1}{2}$. Thus, the merger produces greater upward pricing pressure than does firm i's capacity constraint, and the merger results in a price increase. In this latter example, λ_i measures the extent to which the merger price effect is muted by the fact that price was elevated pre-merger due to a capacity constraint.

Measuring pricing pressure as the difference between margin and inverse elasticity follows the approach of Farrell and Shapiro (2010) and Jaffe and Weyl (2013), who define a firm's pricing pressure from a merger with a rival as the change in its first-order conditions, evaluated at pre-merger prices. Here, following Neurohr (2016), we extend the same idea to pre-merger pricing pressure from a capacity constraint, defining pre-merger pricing pressure as the change in incentive owing to the capacity constraint.[14]

We formally state the definition of pre-merger pricing pressure in definition 1.

Definition 1. *Firm i's pre-merger pricing pressure is the difference between its margin and its negative inverse elasticity, measured using pre-merger data. Thus,*

$$\textit{firm } i\textit{'s pre-merger pricing pressure } = m_i + \frac{1}{\epsilon_{ii}} = \lambda_i$$

3.1.1 Pre-merger Nash equilibrium

Assumptions 1 and 2, in conjunction with our assumption that demand and cost functions are consistent with the existence of a unique pure strategy Nash equilibrium, allows us to characterize the model's equilibrium as a price vector p that solves equation (5) for unconstrained firms, and that solves equation (6) for some value of λ_i for each constrained firm.

To solve for a Nash equilibrium under a given demand system, first solve the system of N equations described by (5). Then, for any firm i with $q_i^D(\boldsymbol{p}) > K_i$, replace that firm's optimality condition with $q_i^D(\boldsymbol{p}) = K_i$ and resolve. Continue this process until a price vector is reached such that either $q_i = K_i$ or (5) holds, for all firms.

3.2 Post-merger pricing pressure

We now consider a merger of firms 1 and 2, and derive post-merger pricing pressure as the greater of pricing pressure from a capacity constraint and pricing pressure from internalizing the effect of own price on the profit of a former rival. After doing so, we characterize post-merger Nash equilibrium.

Under assumptions 1 and 2, the merged firm jointly chooses prices for products 1 and 2 to maximize $\pi_1 + \pi_2$, subject to the constraint that neither product exceed its capacity.[15] The merged firm

[14] Outside the confines of our model, there are other potential explanations for a wedge between margin and inverse elasticity, such as collusion or a firm misallocating resources, e.g. due a moral hazard problem. While empirical examination of pricing pressure as a measure of pre-merger collusion is a useful avenue for future research, we abstract from causes of pricing pressure other than capacity constraints and mergers in this paper.

[15] We assume that the merged firm cannot reallocate capacity across products. Were it able to do so, merger price effects would be muted, both because the firm could produce both products using the lower-cost production facility, and because

thus solves the following maximization problem:

$$\max_{p_1,p_2}(p_1 - c_1)q_1 + (p_2 - c_2)q_2 \tag{7}$$
$$s.t.\ \ q_1 - K_1 \leq 0$$
$$q_2 - K_2 \leq 0$$

The case in which neither capacity constraint binds is well-studied in the literature. Here, the merged firm's first order conditions are:

$$m_i = -\frac{1}{\epsilon_{ii}} - \frac{\frac{\partial q_j}{\partial p_i}}{\frac{\partial q_i}{\partial p_i}}\frac{p_j}{p_i}\frac{p_j - c_j}{p_j} \text{ for } i = 1,2 \tag{8}$$

Comparing the first-order condition in equation (8) with that for a firm that is unconstrained pre-merger, equation (5), we see that equation (8) contains an additional term relative to (5). This terms captures the additional marginal cost resulting from the merged firm internalizing the effect of an increase in production of one product on the profitability of its other, now commonly-owned, product.

A widely-used technique for measuring the effect of a merger on prices is to evaluate the difference between the pre- and post-merger first order conditions (the term $-\frac{\frac{\partial q_j}{\partial p_i}}{\frac{\partial q_i}{\partial p_i}}\frac{p_j}{p_i}\frac{p_j-c_j}{p_j}$) using pre-merger data. Let $D_{ij} = -\frac{\partial q_j}{\partial p_i}/\frac{\partial q_i}{\partial p_i}$ denote the diversion ratio between firms i and j, or the fraction of firm i's marginal customers who view firm j as their next-best option. Then, let $GUPPI_i = D_{ij}^{pre}\frac{p_j^{pre}}{p_i^{pre}}\frac{p_j^{pre}-c_j^{pre}}{p_j^{pre}}$ denote the pricing pressure from the merger.

The impact of $GUPPI_i$ on firm i's price depends on how the merged firm passes through cost increases to price. Jaffe and Weyl (2013) demonstrate that a vector of $GUPPI$ terms multiplied by a merger pass-through matrix that depends on the curvature of demand and cost functions predicts merger price effects to a first order approximation. Miller et al. (2017) use simulated industries to argue that the identity matrix serves as an acceptable proxy for the merger pass-through matrix when the latter is unknown. As Miller et al. explain, the identity tends to overstate the extent to which firms pass through increases in their own costs through while understating the extent to which they pass through changes in other firms' costs as the market equilibrium adjusts, and the effects roughly balance across an array of simulated industries and demand curvatures. Hence, $GUPPI$ is often interpreted by practitioners as a measure of merger price effects, and is widely used in antitrust en-

excess capacity for one product could alleviate capacity constraints binding production of the other. Such reallocation is less likely if the merged firm's products or production facilities are differentiated. Nonetheless, rationalization of production is a common efficiency claim made by merging parties, and we leave the incorporation of such claims into a model of upward pricing pressure for future research.

forcement.[16] Its terms are intuitive: following a merger with firm j, firm i has an incentive to increase price because some of the customers it loses will be recaptured by its former rival (in proportion to D_{ij}), and the value of these customers depends on relative prices ($\frac{p_j}{p_i}$) and the former rival's margin (m_j).

We extend $GUPPI$ as a measure of pricing pressure to the case where one or both firms is capacity-constrained before and/or after the merger. We call our measure $ccGUPPI$, or capacity-constrained $GUPPI$. We define $ccGUPPI$ analogously to $GUPPI$, as the difference between pricing pressure due to the merger and pricing pressure due to a binding pre-merger capacity constraint, evaluated using pre-merger data.

To calculate pricing pressure due to the merger of firms 1 and 2, we apply the Kuhn-Tucker theorem to the constrained maximization problem in (7), where μ_i is the multiplier on firm i's constraint. The merged firm's first-order conditions are then:

$$p_1 : m_1 = -\frac{1}{\epsilon_{11}} + D_{12}\frac{p_2}{p_1}\frac{p_2 - c_2}{p_2} + \frac{\mu_1}{p_1} - \frac{\mu_2}{p_1}D_{12} \tag{9}$$

$$p_2 : m_2 = -\frac{1}{\epsilon_{22}} + D_{21}\frac{p_1}{p_2}\frac{p_1 - c_1}{p_1} + \frac{\mu_2}{p_2} - \frac{\mu_1}{p_2}D_{21} \tag{10}$$

$$\mu_1 : \quad \mu_1 \geq 0, K_1 \geq q_1, \mu_1(K_1 - q_1) = 0 \tag{11}$$

$$\mu_2 : \quad \mu_2 \geq 0, K_2 \geq q_2, \mu_2(K_2 - q_2) = 0 \tag{12}$$

Analogizing definition 1, we define post-merger pricing pressure in definition 2 as the difference between margin and inverse elasticity, evaluated using pre-merger data.

Definition 2. *Consider a merger of firms i and j, with both firms possibly facing capacity constraints. Firm i's post-merger pricing pressure is the difference between its margin and elasticity, evaluated using pre-merger data. Thus,*

$$\textit{firm } i\textit{'s post-merger pricing pressure} = GUPPI_i + \frac{\mu_i^{pre}}{p_i^{pre}} - \frac{\mu_j^{pre}}{p_i^{pre}}D_{ij}^{pre} \tag{13}$$

Lemma 3 derives closed-form expressions for the multipliers μ_i that depend on which (if any) constraints bind post-merger. It does so by solving the system of first-order conditions (9)-(12).

[16]For example, ''As a general matter, Dollar Tree and Family Dollar stores with relatively low GUPPIs suggested that the transaction was unlikely to harm competition... Conversely, Dollar Tree and Family Dollar stores with relatively high GUPPIs suggested that the transaction was likely to harm competition,'' from "Statement of the Federal Trade Commission In the Matter of Dollar Tree, Inc. and Family Dollar Stores, Inc.," July 13, 2015, accessed on October 2, 2017 from www.ftc.gov/public-statements/2015/07/statement-federal-trade-commission-matter-dollar-tree-inc-family-dollar.

Lemma 3. *The multipliers μ_1 and μ_2 that solve the system of first-order conditions given by equations (9)-(12) are as follows:*

$$(\mu_1, \mu_2) = \begin{cases} (0,0) & \text{if } q_1 < K_1 \text{ and } q_2 < K_2 \\ \left(p_1 * \left(m_1 + \frac{1}{\epsilon_{11}} - D_{12}\frac{p_2}{p_1}\frac{p_2 - c_2}{p_2}\right), 0\right) & \text{if } q_1 = K_1,\ q_2 < K_2 \\ \left(0, p_2 * \left(m_2 + \frac{1}{\epsilon_{22}} - D_{21}\frac{p_1}{p_2}\frac{p_1 - c_1}{p_1}\right)\right) & \text{if } q_1 < K_1,\ q_2 = K_2 \\ \left(p_1 * \left(m_1 + \frac{\frac{1}{\epsilon_{11}} + D_{12}\frac{p_2}{p_1}\frac{1}{\epsilon_{22}}}{1 - D_{12}D_{21}}\right), p_2 * \left(m_2 + \frac{\frac{1}{\epsilon_{22}} + D_{21}\frac{p_1}{p_2}\frac{1}{\epsilon_{11}}}{1 - D_{12}D_{21}}\right)\right) & \text{if } q_1 = K_1, q_2 = K_2 \end{cases}$$

Proof The first case, in which neither constraint binds, follows directly from conditions (11) and (12). The second and third cases, with exactly one constraint binding, result from setting μ_i to zero for the non-binding constraint and solving for μ_i using the first-order condition for price corresponding to the constrained firm. The fourth case, in which both firms are constrained, follows from solving equations (9) and (10) as a system of two equations in two unknowns, μ_1 and μ_2. ■

Definition 2 and lemma 3 imply that post-merger pricing pressure is given by one of three expressions, depending on which capacity constraints bind post-merger. Corollary 3.1 describes the mapping between post-merger constraints and these expressions.

Corollary 3.1. *From definition 2 and lemma 3, post-merger pricing pressure for firm i following a merger with firm j is:*

$$\textit{firm i's post-merger pricing pressure} = \begin{cases} GUPPI_i & \text{if } q_1^{post} < K_1 \text{ and } q_2^{post} < K_2 \\ \theta_i = m_i^{pre} D_{ij}^{pre} D_{ji}^{pre} - \frac{p_j^{pre}}{p_i^{pre}} D_{ij}^{pre} \frac{1}{\epsilon_{jj}^{pre}} & \text{if } q_i^{post} < K_i,\ q_j^{post} = K_j \\ \lambda_i = m_i^{pre} + \frac{1}{\epsilon_{ii}^{pre}} & \text{if } q_i^{post} = K_i \end{cases}$$

While the terms $GUPPI_i$ and λ_i have been studied previously, the term θ_i is novel. It describes the pricing pressure for a firm which is not capacity-constrained post-merger, but whose former rival is. θ_i consists of two terms, both positive (as $\epsilon_{jj} < 0$). The first term captures the value of customers diverted from j to i following an increase in p_i and a consequent increase in p_j. The second term captures the value of the increase in p_j caused by the increase in p_i, holding fixed j's quantity at K_j. Note that the second term is smaller the more elastic j's demand is, reflecting the fact that a smaller increase in p_j would be needed to sell out capacity the more elastic its demand is.[17]

Thus, as corollary 3.1 demonstrates, contrary to arguments made by merging parties discussed in the introduction, the fact that a firm is constrained both before and after the merger does not imply that

[17] In the numerical example of figure 2(d), in which firm 1 is unconstrained and firm 2 is constrained both before and after merging, we have that $\theta_1 = 1 * \frac{1}{2} * \frac{1}{2} - \frac{9.1}{7.3} * \frac{1}{2} * -\frac{1}{2.0} = .56$. From the discussion in section 2, the merger causes firm 1 to increase its price from \$7.30 to \$10, or by 37%.

its former rival's incentives are unaffected by the merger. Instead, an increase in (say) p_1 diverts some of product 1's customers to product 2. If product 2 is at capacity, the merged firm is unable to capture these diverted customers in the form of a greater quantity q_2. Thus, customers diverted to product 2 bid up the price at which product 2 is exactly at capacity, enabling the merged firm to charge a higher price for product 2 to sell the same quantity. Some of product 2's marginal customers will divert to product 1 in response to this price increase, further increasing the merged firm's profits.

3.2.1 Post-merger Nash equilibrium

Assumptions 1 and 2, in conjunction with our assumption that demand and cost functions are consistent with the existence of a pure strategy Nash equilibrium allow us to solve for equilibrium using first-order conditions (9) and (10) and lemma 3.

Post-merger, a Nash equilibrium is a price vector p of length equal to the number of firms such that 1) (p_1, p_2) jointly solve the merged firm's constrained optimization problem (7), given $p_3, ..., p_N$, and 2) p_i solves firm i's profit-maximization problem (4), given p_{-i}, $i = 3, ..., N$. Thus, a price vector $\boldsymbol{p}$ comprises a Nash equilibrium if and only if:

(i) $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \frac{p_j}{p_i} m_j D_{ij}$ if $q_i^D(\boldsymbol{p}) < K_i$ for $i = 1, 2$

(ii) $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}}$ if $q_i^D(\boldsymbol{p}) < K_i, q_j^D(\boldsymbol{p}) = K_j, i, j \in \{1, 2\}$

(iii) $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \lambda_i$ with $\lambda_i \geq 0$ if $q_i^D(\boldsymbol{p}) = K_i$ for $i = 1, 2$

(iv) $\frac{p_l - c_l}{p_l} = -\frac{1}{\epsilon_{ll}} + \lambda_l$, $\lambda_l \geq 0$, for $l = 3, 4, ...$ with $\lambda_l > 0$ iff $q_l^D = K_l$

(v) $q_i^D(\boldsymbol{p}) \leq K_i$ for all i

To calculate Nash equilibrium given knowledge of the demand system, first compute a price vector $\boldsymbol{p}$ satisfying $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} + \frac{p_j}{p_i} m_j D_{ij}$ for each merging firm and $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}}$ for each non-merging firm. Then, for any firm i such that $q_i^D(\boldsymbol{p}) > K_i$ replace firm i's first-order condition with $q_i^D(\boldsymbol{p}) \leq K_i$, and recompute the price vector that satisfies all N first-order conditions. Iterate as necessary until a price vector satisfying (i)-(v) above is reached.

3.3 $ccGUPPI$ is the difference between pre- and post-merger pricing pressure

When firms may be capacity-constrained, some, or even all, of their post-merger pricing pressure may be caused by a binding capacity constraint, and not by the merger. Thus, we distinguish between post-merger pricing pressure (as defined in corollary 3.1) and pricing pressure caused by the merger, which is the difference between post-merger pricing pressure and pre-merger pricing pressure.

Notably, post-merger pricing pressure as described in corollary 3.1 depends on information on pre-merger price, cost, and diversion, and on the constraints that will bind post-merger. The final step in constructing $ccGUPPI$ is to use pre-merger information to predict which constraints will bind post-merger. We do so by comparing pre- and post-merger pricing pressure. For example, if $\lambda_1 > GUPPI_1$, firm 1's post-merger pricing pressure is less than its pre-merger pricing pressure, meaning that firm 1 is likely to continue to be constrained post-merger. If, additionally, $\theta_2 > \lambda_2$, then firm 2's pricing pressure from internalizing its pricing externality on firm 1 exceeds its pre-merger pricing pressure due to its capacity constraint, and so firm 2 is likely to be unconstrained post-merger.

More generally, the ordering of λ_i, $GUPPI_i$, and θ_i generates a qualitative prediction about which capacity constraints will bind post-merger. If λ_i is greatest, pricing pressure from the capacity constraint exceeds that from the merger, and so we predict firm i will remain constrained post-merger. If λ_i is less than $GUPPI_i$ (if firm j is unconstrained so that $GUPPI_j > \lambda_j$) or θ_i (if firm j is constrained so that $\lambda_j > GUPPI_j$), then we predict that one or both firms will be unconstrained.

Our qualitative predictions about which constraints bind post-merger, along with corollary 3.1 and definition 1, yield $ccGUPPI$, or pricing pressure under capacity constraints, calculated using only pre-merger data. Before formally defining $ccGUPPI$, we first state and prove lemma 4, which provides guidance on the possible orderings of $GUPPI$, θ, and λ, and thus simplifies discussion of $ccGUPPI$.

Lemma 4. *The pricing pressure terms λ, $GUPPI$ and θ are ordered as follows:*

1. $GUPPI_i \geq \theta_i \iff \lambda_j \geq GUPPI_j$ *for* $i, j \in \{1, 2\}$
2. $\lambda_i \geq \theta_i$ *and* $\theta_j \geq \lambda_j \Rightarrow \lambda_i \geq GUPPI_i$ *for* $i, j \in \{1, 2\}$

It follows that there are eight possible orderings of the terms λ_i, $GUPPI_i$, and θ_i, $i = 1, 2$.

Proof See appendix. ■

Proposition 5 formally defines $ccGUPPI$ based on the relative sizes of $GUPPI$, θ, and λ, and proves, using lemma 4, that the given orderings of the pricing pressure terms are comprehensive.

Proposition 5. *Allowing for capacity constraints, a firm's pricing pressure from a merger, $ccGUPPI$, is described below:*

$$ccGUPPI_i = \begin{cases} GUPPI_i - \lambda_i & \text{if } GUPPI_i \geq \lambda_i, \text{ for } i = 1, 2 \\ \theta_i - \lambda_i & \text{if } \theta_i \geq \lambda_i \text{ and } GUPPI_j \leq \lambda_j, \text{ with } i \neq j \\ 0 & \text{if } \theta_j \geq \lambda_j \text{ and } GUPPI_i \leq \lambda_i, \text{ with } i \neq j \\ 0 & \text{if } \lambda_i \geq \theta_i \text{ for } i = 1, 2 \end{cases} \tag{14}$$

Proof $ccGUPPI$ is defined to be the difference in pricing pressure caused by the merger, so the values of $ccGUPPI$ do not require proof. That the given orderings of the pricing pressure terms are comprehensive follows directly from the proof of lemma 4. ■

As defined, $ccGUPPI$ depends on both pre-merger pricing pressure created by capacity constraints (the λ_i terms, and, indirectly, the θ_i term), as well as the closeness of the competition that is lost via merger (the $GUPPI_i$ and θ_i terms). Thus, as described in the numerical examples of section 2, binding pre-merger capacity constraints attenuate or eliminate merger price effects, and merger price effects decrease as the pricing pressure caused by a capacity constraint increases.

With one important exception, to be examined in the following section, the qualitative predictions implied by $ccGUPPI$ are approximate. This is because, as with any first-order approximation, $ccGUPPI$ does not capture all feedback effects as prices change following a merger. As an example, consider a case in which both firms are constrained pre-merger, and in which $\theta_1 \geq \lambda_1$ and $GUPPI_2 \leq \lambda_2$. By proposition 5, $ccGUPPI_1 = \theta_1 - \lambda_1$, and $ccGUPPI_2 = 0$. That is, $ccGUPPI$ generates the qualitative prediction that the merger creates more upward pricing pressure for firm 1 than does the pre-merger constraint, and less for firm 2. However, an increase in p_1 spurred by this pricing pressure leads to an increase in p_2, which in turn increases firm 1's demand. This feedback effect is not captured by λ or θ when evaluated at pre-merger equilibrium. For edge cases, it is possible that the increase in demand caused by this feedback effect pushes firm 1 back over its capacity constraint, and thus to be constrained post-merger, even though $\theta_1 > \lambda_1$.

Another feedback effect not captured by $ccGUPPI$ is the interaction between the pricing pressure created by a merger and the capacity constraints of non-merging firms. A price increase caused by a merger, by diverting demand to unintegrated rivals, could conceivably push those rivals above their capacity, which would in turn divert demand back to the merging firms, potentially pushing them above their capacity levels. However, Froeb et al. (2003) hypothesized that capacity constraints on merging firms have a greater impact on merger price effects than do those on non-merging firms, a point for which we provide empirical support using simulated data in section 5.

We evaluate the significance of these feedback effects in section 5 using Monte Carlo simulation, and find that despite this source of error, our qualitative and quantitative predictions perform quite well. Before that, we characterize the post-merger Nash equilibrium of the pricing game using first order conditions.

4 Predicting post-merger prices using $ccGUPPI$

This section contains our main theoretical result, that knowledge of λ and θ is sufficient to determine whether a merger of two capacity-constrained firms will result in a price increase. Then, section 4.1 discusses the appropriate pass-through matrix to use when using $ccGUPPI$ as a predictor of merger price effects.

First, we demonstrate that a merger of two capacity-constrained firms results in no price increase if and only if $\lambda_i \geq \theta_i$. Intuitively, this condition holds if the pricing pressure resulting from pre-merger capacity constraints (λ_i) exceeds that which would otherwise result from a merger of firms i and j, as a result of firm i internalizing the effect of its price on firm j's profit ($GUPPI_i$). Thus, as discussed in the prior section, while $ccGUPPI$ relies on inexact qualitative predictions of which constraints will bind post-merger, the special case of two capacity-constrained firms remaining capacity-constrained post-merger can be predicted without error, using only pre-merger information. Proposition 6 states and proves the result.

Proposition 6. *Suppose that firms 1 and 2 are both capacity-constrained pre-merger, at price vector $\mathbf{p}^*$. Following a merger of firms 1 and 2, $\mathbf{p}^*$ remains an equilibrium if and only if the following condition holds:*

$$\lambda_i \geq \theta_i, \textit{for } i = 1, 2 \tag{15}$$

Proof Given the sufficiency of first-order conditions, it follows from definitions 1 and 2 that firm i's post-merger pricing coincides with its pre-merger pricing if and only if there exist nonnegative multipliers μ_i such that the following holds:

$$\lambda_i = D_{12}^{pre} \frac{p_2^{pre}}{p_1^{pre}} m_2^{pre} + \frac{\mu_i}{p_i^{pre}} - \frac{\mu_j}{p_i^{pre}} D_{ij}^{pre} \text{ for } i = 1, 2,\ j \neq i \tag{16}$$

Equation (16) gives a system of two equations in two unknowns: μ_1 and μ_2. Solving this system, and dropping the superscripts for convenience, we have that:

$$\mu_i(1 - D_{ij}D_{ji}) = D_{ij}p_j\lambda_j - D_{ij}D_{ji}p_i m_i + p_i\lambda_i - D_{ij}p_j m_j \text{ for } i = 1, 2,\ j \neq i \tag{17}$$

Given that $(1 - D_{ij}D_{ji}) > 0$, we have that each μ_i is nonnegative if and only if the right-hand side of (17) is nonnegative:

$$\begin{aligned}
& \mu_i \geq 0 \\
\iff & D_{ij}p_j\lambda_j + p_i\lambda_i \geq D_{ij}D_{ji}p_i m_i + D_{ij}p_j m_j \\
\iff & D_{ij}\frac{p_j}{p_i}(\lambda_j - m_j) + \lambda_i \geq D_{ij}D_{ji}m_i \\
\iff & \lambda_i \geq \theta_i
\end{aligned}$$

The proposition then follows. ■

The conditions of proposition 6 have a simple interpretation: a merger of two capacity-constrained firms results in no price increase if and only if the pricing pressure from the pre-merger capacity constraint (λ_i) exceeds the pricing pressure resulting from the merger for each firm, given that its former rival remains constrained. If we instead had $\theta_i > \lambda_i$, then a unilateral price increase would be profitable for firm i. Under the condition of proposition 6, neither firm has such a unilateral incentive, and hence pre-merger pricing remains an equilibrium outcome following a merger. Proposition 6 is a valuable result for an antitrust agencies charged with predicting whether a particular merger between capacity-constrained firms would raise prices, such as the merging fitness gyms referenced in footnote 1.

4.1 Efficiencies and pass through

Like other measures of upward pricing pressure, $ccGUPPI$ can be compared to the magnitude of any expected cost-saving efficiencies that would result from the merger to determine if the merger's net upward pricing pressure is positive or negative. For example, following Farrell and Shapiro (2010) we say that a merger of one or more capacity-constrained firms has net positive pricing pressure if $ccGUPPI_i > \frac{\Delta c_i}{p_i}$ for merging firm i. Such mergers create net upward pricing pressure, regardless of the particulars of how cost increases are passed through to consumers. More generally, see Werden (1996) and Jaffe and Weyl (2013) for a discussion of comparing upward pricing pressure to efficiencies.

To employ $ccGUPPI$ as a predictor of merger price effects, we pre-multiply the vector of $ccGUPPI$ terms by an $N \times N$ pass-through matrix describing how changes in each firm's costs are passed through to each firm's price. While Jaffe and Weyl (2013) usefully characterize the optimal pass-through matrix as a function of the demand curve, the Jaffe and Weyl pass-through matrix is difficult to implement in practice. For this reason, we follow Miller et al. (2017) in approximating the true pass-through matrix with the identity matrix, irrespective of the demand system. Using an identity pass-through, our prediction for $\frac{\Delta p_i}{p_i}$ is simply $ccGUPPI_i$. Section 5 conducts a series of Monte Carlo experiments in which we compare the identity times $ccGUPPI$ to true merger price effects across a variety of simulated industries. In doing so, we test the usefulness of three approximations: evaluating pricing pressure terms λ, $GUPPI$, θ using pre-merger information, qualitative predictions about which pricing constraints continue to bind post-merger, and the identity as a proxy for pass-through. Overall, the simulations appear to offer strong support for the usefulness of $ccGUPPI$ in predicting merger price effects.

As discussed in Miller et al. (2017), the use of the identity matrix as a proxy for the merger pass-through matrix is not without drawbacks. For example, as shown by equations (5) and (6) of Miller et al. (2017), if firms pass through 100% of own costs and 0% of other firms' costs as implied by the identity matrix, non-merging firms will have a predicted price increase of zero, even if their newly-merged rivals increase price. This prediction is generally incorrect, as non-merging firms will optimally increase price in response to the price increase of the merging firms. Given that the prices of non-merging firms are rarely pivotal in merger analysis, this disadvantage of an identity pass through is typically of minimal importance.

With capacity constraints, an identity pass-through matrix introduces at least one additional bias as a predictor of price effects. When exactly one merging firm (say, firm 2) is predicted to be constrained following the merger, $ccGUPPI_2 = 0$. Despite this, firm 2 would increase price in this scenario, as some of firm 1's lost customers would divert to firm 2, driving up demand and causing it to increase its price to maintain demand equal to capacity. A downward bias in predicting the merger price effect for one of the merging firms is perhaps more serious than one in predicting the price effect of non-merging firms.

However, this latter bias is more easily correctable by tweaking the pass-through matrix used, using information on firm 1's price increase, demand elasticities, and relative prices. Specifically, if pass-through of costs to non-merging firms is zero, a constrained firm 2 would increase price in response to a price increase by an unconstrained firm 1 by approximately $\frac{\partial p_2}{\partial p_1} * \Delta p_1$. The following expression approximates this price increase as a fraction of firm 2's pre-merger price, $\frac{\Delta p_2}{p_2}$, with the derivation given in a footnote:[18]

$$\frac{\Delta p_2}{p_2} \approx -\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} ccGUPPI_1 \frac{p_1}{p_2} \tag{18}$$

Thus, we can adjust pass-through when firm 1 is predicted to be unconstrained and firm 2 constrained post-merger, so that the entry in column 2, row 1 is $-\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}}\frac{p_1}{p_2}$, with similar adjustments in other relevant regions. Using this adjusted pass-through matrix, equation (19), displayed in the appendix, describes revised predictions for $\frac{\Delta p}{p}$ based on $ccGUPPI$ and the pass-through matrix implied by equation (18).

In our Monte Carlo simulations, we do not find that results differ drastically depending on whether we use predicted price increases from equation (14) or equation (19). Hence, in the next section, we present results using only the identity pass-through, and thus using $ccGUPPI_i$ as our predictor of $\frac{\Delta p_i}{p_i}$. The appendix contains a subset of analogous results using the pass-through matrix implied by equation (19). The appropriate choice of pass-through in an antitrust setting is an important and understudied question (with Jaffe and Weyl (2013) and Miller et al. (2017) being the principal exceptions). Our result in section 5 that $ccGUPPI$ times the identity is a reasonable predictor of merger price effects is an empirical result, and, beyond the intuition given in Miller et al. (2017), does not derive directly from economic theory.

5 Monte Carlo Experiment

The previous section developed analytical results balancing upward price pressure from a merger (which occurs because the merged firm internalizes the pricing externality between substitute goods

[18] Firm 2 sets p_2 equal to $q_2^{-1}(K_2, \boldsymbol{p})$, or

$$q_2(p_1, p_2, ...) \equiv K_2$$

Setting second order terms $\frac{\partial p_j}{\partial p_1}$ to zero for $j > 2$, and taking the derivative of both sides of the above equation with respect to p_1, we have:

$$\frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial p_1} + \frac{\partial q_2}{\partial p_1} = 0$$

$$\Rightarrow \frac{\partial p_2}{\partial p_1} = -\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}}$$

We then multiply $\frac{\partial p_2}{\partial p_1}$ by the level of the change in p_1, $ccGUPPI_1 * p_1$ (assuming each element on the diagonal of the pass-through matrix is 1). $-\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_2}{\partial p_2}} * ccGUPPI_1/p_1$ gives the level of price change for firm 2, and dividing by p_2 yields the percent price increase. Symmetric calculations apply to settings in which firm 2 is unconstrained and firm 1 constrained post-merger.

that were previously owned by different firms) and upward price pressure from capacity constraints (which occurs because constrained firms are incentivized to increase price until quantity demanded equals capacity). In this section, we develop Monte Carlo experiments that provide numerical evidence on the extent to which capacity constraints on merging firms attenuate merger price effects and demonstrate that $ccGUPPI$ is a useful tool for predicting the price effects of mergers between capacity-constrained firms.

The experiments generate a dataset where each observation, or random draw of data, represents an industry consisting of four firms. We calibrate three different demand systems (linear, logit, and AIDS) with each draw of data or industry. Each demand system appears to generically conform with the assumptions of section 3, namely that a pure strategy equilibrium exists that is characterized by first order conditions and corresponding conditions for constrained firms.[19] While the demand systems differ in functional form, firms have the same pre-merger prices, quantities, margins, and demand elasticities under each demand system. We also randomly assign capacity constraints to each firm in a given industry. The resulting dataset allows us to examine the price effect of mergers between capacity-constrained firms and the accuracy of $ccGUPPI$ in predicting those effects under a wide range of market conditions.

5.1 Data generating process

Our data generating process is adapted from that of Miller et al. (2016 and 2017) to allow some or all of the firms in a simulated industry to have binding capacity constraints. First, we randomly draw market shares for four firms and an outside good. Next, we randomly assign each firm to be either capacity-constrained or not, excluding draws where all firms are constrained. Then, we draw the margin of a single unconstrained firm.

The margin of the unconstrained firm, market shares, and price (which we normalize to one) are sufficient to calibrate a logit demand system. Then, we calibrate linear and AIDS demand systems using the market shares, prices, and demand slopes from the logit calibration. Each unconstrained firm's marginal cost is implied by its marginal revenue (just as it would in a model without capacity constraints) and each constrained firm's marginal cost is drawn randomly to be below marginal revenue.

Finally, we model a merger between firms 1 and 2, and compute the optimal post-merger price vectors using the calibrated demand systems and marginal cost vector. Thus, each industry, or draw of data, has three post-merger price vectors, one for each demand system.

The specific steps of the data generating process are as follows:

[19] In particular, we impose assumptions 1 and 2 from section 3 on each of our simulated industries.

1. Draw shares s_i for 4 firms and the outside good by drawing $x_i \sim U[0,1]$ for $i = 1, ..., 5$ and setting $s_i = \frac{x_i}{\sum_j x_j}$. Normalize each firm's price to $p_i = 1$.

2. Label each firm as being unconstrained or constrained via 4 independent draws from a Bernoulli distribution with parameter $\frac{1}{2}$. If a firm is constrained, set $K_i = s_i$. If a firm is unconstrained, set $K_i = 100$.[20] If all four firms are capacity-constrained, discard the observation.

3. If firm $\tilde{i}$ is the lowest-numbered firm that is unconstrained, draw firm $\tilde{i}$'s margin $m_{\tilde{i}}$ from a $U[.2, .8]$ distribution.

4. Shares s_i, $i = 1, ..., 4$ and margin $m_{\tilde{i}}$ are sufficient to calculate the five parameters of a logit demand system, following Appendix A of Miller et al. (2017). These parameters determine own and cross elasticities of demand, ϵ_{ii} and ϵ_{ij} for all firms.

5. If firm i is unconstrained, profit maximization implies its margin is given by $m_i = -\frac{1}{\epsilon_{ii}}$. If firm i is constrained, by definition its margin exceeds $-\frac{1}{\epsilon_{ii}}$. In this case, draw margin as follows: $m_i \sim U[-\frac{1}{\epsilon_{ii}}, 1]$. Note that $\lambda_i = m_i + \frac{1}{\epsilon_{ii}}$ for each constrained firm, as defined in section 3.1.

6. The shares s_i and logit elasticities ϵ_{ij} imply unique parameterizations of AIDS and linear demand systems, following Appendix A of Miller et al. (2017). Pre-merger price, quantity, own and cross elasticities, capacity constraints, and margins are identical across all three demand systems.

7. Calculate profit-maximizing prices following a merger of firms 1 and 2 under each of three demand systems by applying the first-order conditions described in section 3.2.

8. Calculate $ccGUPPI$ and $GUPPI$ using pre-merger information on margins, shares, capacities, and elasticities (but not demand parameters).

9. Repeat until 10,000 industries are generated, discarding the small number of industries that fail to calibrate.

This procedure substantively differs from that of Miller et al. (2016 and 2017) only in its treament of capacity constraints in steps 2 and 5. Permutations on the data generating processes described above

[20]Nothing precludes a unconstrained non-merging firm from becoming constrained post-merger, but we set capacity for non-merging firms to be so large that this never happens, for simplicity. Allowing unconstrained non-merging firms to become constrained post-merger would somewhat exacerbate price effects, but would not meaningfully change our results.

produce qualitatively similar results, although we have not explored the effect of demand systems with curvature differing from the logit, AIDS, or linear systems.

The resulting dataset has 10,000 observations, or industries. Per step 7, each observation or industry also has three predicted merger price effects specified by a functional form of demand. Per step 8, each industry has one value of $ccGUPPI$ and one value of $GUPPI$.

Table 1 summarizes the simulated data. It reports order statistics for firm 1's market share, margin, and demand elasticity. The distributions of these variables for the other firms are essentially the same, because the data generating process is the same for all firms. The median market share, 20.2 percent, reflects that there are four firms and an outside good. The median margin is 56 percent and the median elasticity is 2.1. Note that the traditional Lerner index relationship between margins and elasticity does not hold for capacity-constrained firms, as these firms have marginal cost below marginal revenue. The median diversion ratio from firm 1 to 2 is 25.4$\%$.

Table 1: Order Statistics

	p50	p10	p25	p75	p90
Market Share	0.202	0.052	0.117	0.277	0.339
Own-Price Elasticity	2.122	1.384	1.633	2.976	3.977
Diversion Ratio	0.254	0.070	0.149	0.340	0.417
Margin	0.556	0.295	0.411	0.678	0.770
ccGUPPI	0.071	0.000	0.002	0.153	0.239
GUPPI	0.125	0.028	0.064	0.198	0.273
Firm 1 Logit Price Effect	0.070	0.002	0.022	0.142	0.222
Firm 1 Linear Price Effect	0.059	0.002	0.018	0.122	0.198
Firm 1 AIDS Price Effect	0.128	0.003	0.034	0.360	0.863

Table 1 also shows firm 1's upward pricing pressure and simulated price effects resulting from a merger with firm 2. The median $GUPPI$ is 12.5 percent, while the median $ccGUPPI$ is only 7.1 percent. The latter is smaller than the former whenever capacity constraints bind before the merger, putting upward pressure on prices. The median price effects are 7.0, 5.9, and 12.8 percent under logit, linear, and AIDS demand, respectively. The relative sizes of these simulated price effects are consistent with those found by Miller et al. (2016 and 2017) and Crooke et al. (1999). The greater curvature of the AIDS system generates larger price effects, all else equal.

Figure 2 illustrates the empirical distribution of $ccGUPPI$, $GUPPI$, and the simulated price

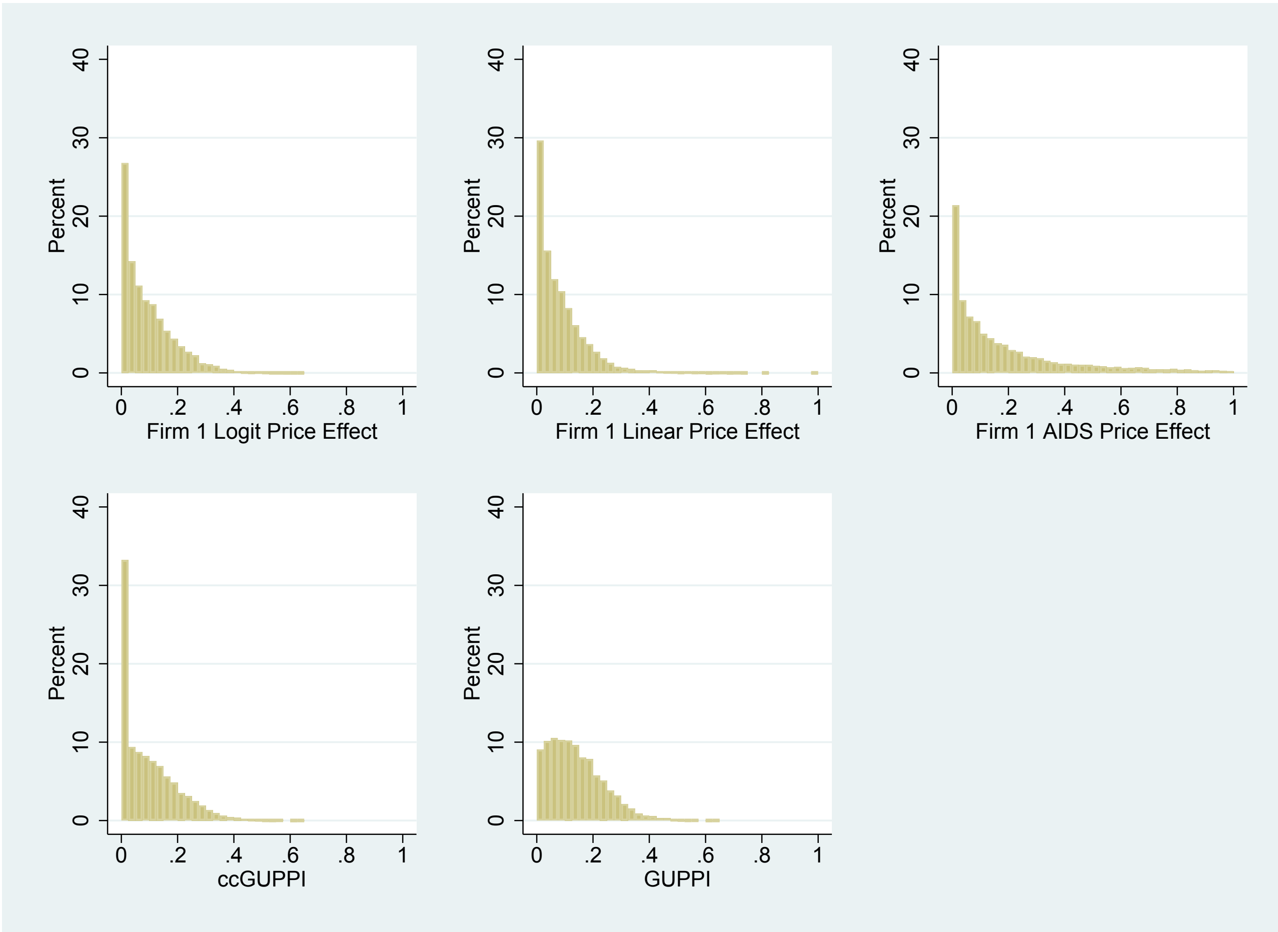


Figure 2: Distribution of $ccGUPPI$, $GUPPI$, and actual merger price effects

effects. The graphs are standard histograms with fixed bin widths of 0.025 over the range 0 to 1. They show the full distribution of the linear and logit price effects, as well as $ccGUPPI$ and $GUPPI$ values. The right tail of the AIDS price effect histogram, which includes about 5 percent of observations, does not appear on the graph.

The histograms confirm that the distribution of $ccGUPPI$ is similar to those of the linear and logit simulated price effects. The distribution of the AIDS price effects has the same general shape but a much longer and thicker right tail. The $GUPPI$ distribution has a fundamentally different shape because $GUPPI$ is strictly positive for every industry, whereas $ccGUPPI$ is 0 for industries with tightly-binding capacity constraints. The center of mass of the $GUPPI$ distribution is also further to the right than that of the $ccGUPPI$ distribution, because none of the $GUPPI$ values are attenuated by the pre-merger upward pricing pressure from binding capacity constraints.

Not apparent from the histograms is the fact that merger price effects for the linear, logit, and AIDS systems all equal zero for the same 743 observations. These observations represent the industries

Table 2: Constraints on Merging Firms and Constraints on Rivals

	Demand System		
	Linear	Logit	AIDS
Constraints on Merging Firms			
Neither	0.087	0.102	0.186
One	0.060	0.072	0.133
Both	0.020	0.025	0.045
Constraints on Merging Firms' Rivals			
Neither	0.056	0.064	0.117
One	0.058	0.069	0.126
Both	0.065	0.079	0.142

where both of the merging firms are capacity-constrained before and after the merger, for which proposition 6 implies neither firm will raise prices. Firm 1's $ccGUPPI$ is also zero for those 743 observations. There are an additional 1,666 observations where $ccGUPPI$ is zero because firm 1 is constrained before and after the merger and firm two is not. In these instances, there is no direct upward price pressure, because firm 1 continues to set a price where demand equals capacity after the merger, but firm 1's price increases because its demand curve shifts out as firm two raises price and reduces output.[21]

5.2 Descriptive Analysis

Before studying the accuracy of $ccGUPPI$, we generate some discriptive statistics from our dataset that highlight the practical importance of accounting for capacity constraints during merger reviews. Part of our motivation is the fact that Froeb et al. (2003) argue that capacity constraints on merging firms attenuate merger effects more than capacity constraints on non-merging firms amplify them, and thus policy makers should be particularly concerned about the former.

Table 2 lists the median price effects for firm 1 based on which firms are capacity-constrained before the merger. The top half of the table lists the median price effects under each demand system after separating the data into three groups of observations based on whether both, one, or neither of the

[21]Note that were we to apply the pass-through matrix described in equation (19), the value of pass-through multiplied by $ccGUPPI$ would equal 0 in the 743 industries with zero simulated price effect, and only these industries.

Table 3: Merger Price Effects and the Tightness of Capacity Constraints

	Demand System		
Lambda	Linear	Logit	AIDS
$\lambda_i < 0.3$	0.029	0.035	0.063
$\lambda_i < 0.2$	0.054	0.062	0.146
$\lambda_i < 0.1$	0.098	0.113	0.333

merging firms are constrained.[22] The bottom half of table 2 lists the median price effects when both, one, or neither of the merging firms' rivals are constrained. Clearly, constraints on the merging firms tend to attenuate merger price effects, while constraints on the merging firms' rivals tend to amplify them. The relative importance of constraints on merging firms versus those on rivals, however, is entirely case-specific.

In our dataset, capacity constraints on merging firms indeed do lower merger price effects more than capacity constraints on non-merging firms raise them, yet this is only an average effect, and not true for each individual industry. Further, even the average effect is entirely dependent on our data generating process. If we changed the data generating process so that the merging firms were less tightly constrained (i.e., smaller values of λ_i, the difference between margin and inverse elasticity), then capacity constraints on merging firms would have a smaller attenuating effect.

Table 3 shows how the median price effects change when we restrict the data based on the values of λ_1 and λ_2, measuring how tightly firms 1 and 2 are constrained before merging. The first row of table 3 shows the median price effects when we restrict the dataset to observations where both of the merging firms are constrained and $\lambda_i < .3$ for $i = 1, 2$. In the second row we restrict the dataset further, to otherwise similar observations where λ_i is less than .2, and we see that the median price effects increase. In the third row, the median price effects increase further still because the data are restricted to observations where λ_i is less than .1. Unsurprisingly, we find that tighter pre-merger capacity constraints result in smaller average merger price effects.

Figure 3 shows the empirical CDF of the linear demand price effects for each subset of data shown in table 3. Figure 3 demonstrates that the average effects identified in table 3 hold more broadly across the distribution of outcomes. Table 3 and figure 3 both illustrate the logic behind $ccGUPPI$: capacity-constrained firms have pre-merger prices that are elevated relative to what they would be absent the

[22]The row labeled as one merging firm constrained includes instances where firm 1 is constrained and firm 2 is not, as well as instances where firm 2 is constrained to firm 1 is not.

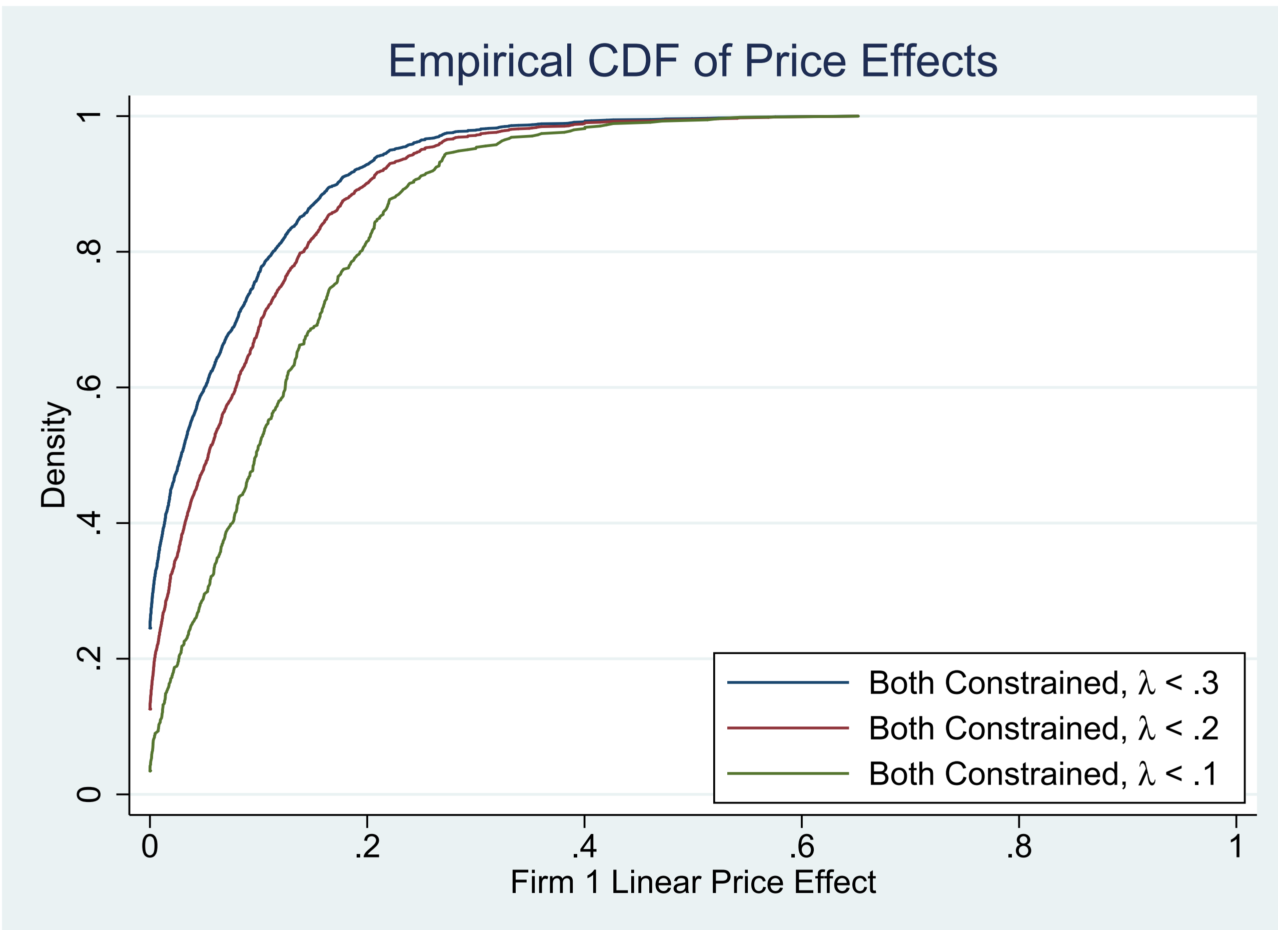


Figure 3: CDF of price effects when both merging firms are constrained.

constraints.

5.3 Accuracy of $ccGUPPI$

This section addresses how well $ccGUPPI$ predicts actual merger effects under different functional forms of demand. We find that it does a better job predicting merger effects when demand is linear or logit than it does when demand is AIDS, where $ccGUPPI$ tends to under predict the magnitude of price effects. Overall, we find that $ccGUPPI$ offers more accurate and precise predictions of merger price effects than $GUPPI$, based on our simulated dataset. We assess accuracy based on median absolute prediction error and precision based on the standard deviation of prediction errors.

First, we evaluate $ccGUPPI$'s accuracy as a predictor of merger price effects graphically, using the identity pass-through. The graphs in figure 4 each plot either $ccGUPPI$ or $GUPPI$ on the vertical axis and the simulated merger price effect under a specific demand systems on the horizontal axis. A 45-degree reference line indicates exact predictions, where $ccGUPPI$ or $GUPPI$ equals the simulated price increase.

Under logit and linear demand, $ccGUPPI$ is quite accurate with the dots tightly dispersed around the 45-degree line.[23] In contrast, standard $GUPPI$ is systematically biased upward, with the dots dispersed above the reference line. The line of dots along the vertical axis represents observations where the merging firms are constrained before and after the merger. Under AIDS demand, $ccGUPPI$ under-predicts simulated price effects, consistent with Miller et al. (2016 and 2017) who find that standard $GUPPI$ and the identity pass-through matrix tend to under-predict price increases with AIDS demand absent capacity constraints. Under AIDS demand, $GUPPI$ predictions are widely dispersed, as $GUPPI$ over-predicts the actual price increase when capacity constraints bind tightly and under-predicts prices effects capacity constraints do not bind. The appendix contains a replication of figure 4 using the alternative pass-through matrix described in section 4.1. The results do not differ substantially.

Next, we evaluate $ccGUPPI$ and $GUPPI$'s accuracy as a predictor of merger price effects numerically. Define the prediction error as $ccGUPPI$ or $GUPPI$ minus the analytically derived price increase under a specific demand system. Table 4 shows the median prediction error, the standard deviation of the prediction error, and the median absolute prediction error of $ccGUPPI$ and $GUPPI$ relative to each demand system.

The median prediction error confirms that $ccGUPPI$ is a good predictor of price effects under

[23]The line of dots clustered along the horizontal axis represent observations where firm 1 is constrained before and after the merger but firm 2 is not. These dots shift up closer to the 45-degree line if we use the alternative pass through matrix contemplated in equation (19) in the appendix.

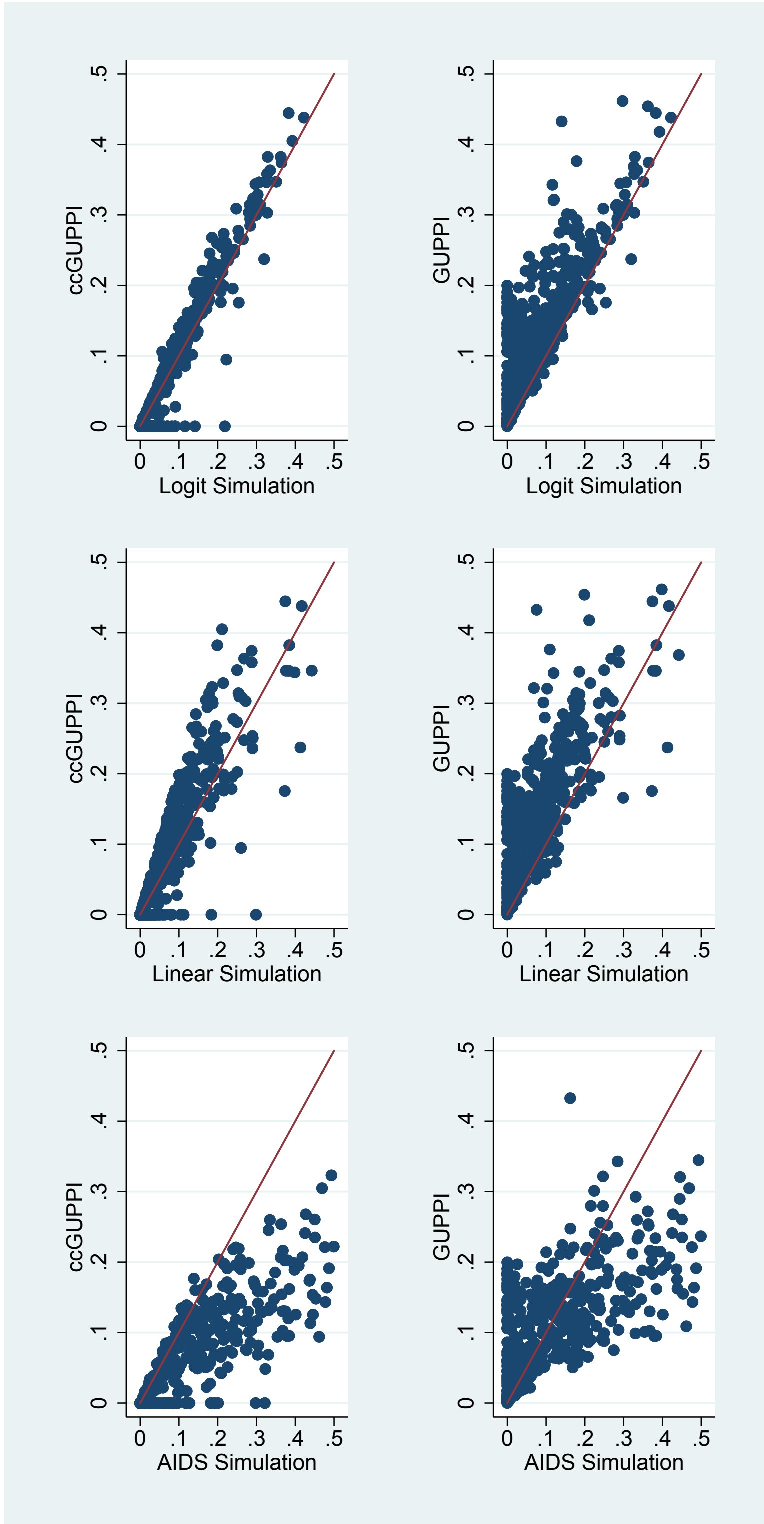


Figure 4: $ccGUPPI$ and $GUPPI$ price predictions (y-axis) versus simulated price effect (x-axis).

Table 4: Prediction Error

	Demand System		
	Linear	Logit	AIDS
Median Prediction Error			
ccGUPPI	0.007	0.003	–0.052
GUPPI	0.046	0.028	–0.019
Standard Deviation of Prediction Error			
ccGUPPI	0.052	0.028	5.655
GUPPI	0.063	0.053	5.656
Median Absolute Prediction Error			
ccGUPPI	0.021	0.009	0.052
GUPPI	0.049	0.029	0.068

linear and logit demand, but tends to under-predict price increases under AIDS demand. In addition, the prediction error of $GUPPI$ has a higher standard deviation than that of $ccGUPPI$ under linear of logit demand, and is roughly the same under AIDS.[24] Perhaps more importantly, if the underlying demand is either linear, logit, or AIDS, one can be confident that $ccGUPPI$ is either relatively accurate (under linear or logit) or under-predicts price effects (under AIDS). By comparison, there is no way to predict the likely sign of the prediction error with standard $GUPPI$. Finally, the median absolute prediction error statistics again suggest that $ccGUPPI$ outperforms $GUPPI$. Under all demand systems, $ccGUPPI$ has a lower median absolute error than standard $GUPPI$.

Antitrust agencies may also want to flag mergers whose price effects will likely be greater than a specified threshold. Following Miller et al. (2017) we consider a test to screen out all mergers likely to generate a price increase greater than 10 percent, as predicted by $ccGUPPI$ or standard $GUPPI$. For each observation in the simulated data, we determine whether $ccGUPPI$ and standard $GUPPI$ exceed ten percent. A false positive, or Type II error, means that the $ccGUPPI$ or $GUPPI$ of at least one of the merging products is greater than ten percent while the actual price effect of both merging products is less than ten percent. A false negative, or Type I error, means that the $ccGUPPI$ or $GUPPI$ of both products is less than ten percent and the actual price effect of at least one product is greater than ten percent.

[24]The standard deviation of the $ccGUPPI$ prediction error under AIDS demand is 0.4 if we exclude the top 1 percent of prediction error values.

Table 5: Threshold Merger Screen Accuracy

	Demand System		
	Linear	Logit	AIDS
ccGUPPI			
Type 1 Error (False Positive)	0.197	0.046	0.011
Type 2 Error (False Negative)	0.001	0.002	0.070
GUPPI			
Type 1 Error (False Positive)	0.398	0.247	0.167
Type 2 Error (False Negative)	0.000	0.001	0.024

Table 5 summarizes the frequency of type I and type II errors. The prevalence of type I errors is clearly lower for $ccGUPPI$ than standard $GUPPI$. This is obviously because $GUPPI$ over-predicts price effects when firms are capacity-constrained. The prevalence of type II errors is similar for $GUPPI$ and $ccGUPPI$ under linear and logit demand, and lower for $GUPPI$ under AIDS. This is in part explained by instances where $GUPPI$ generates larger price effects because it does not account for capacity constraints. In essence, standard $GUPPI$ generates fewer false positives by mistake, because it does not account for capacity constraints. Overall, $ccGUPPI$ generates substantially fewer total type I and II errors under logit, linear, and AIDS demand.

6 Discussion

We believe that $ccGUPPI$ will be a useful tool for antitrust practitioners, including staff economists at government agencies and consulting economists hired by merging firms. We are aware of one proposed merger involving capacity constraints in which $ccGUPPI$ was employed to analyze the impact of capacity constraints on merger price effects. Like other measures of upward pricing pressure, $ccGUPPI$ provides an intuitive description of the first order impact of a merger between differentiated product rivals. It also provides useful predictions of actual merger price effects. Perhaps most importantly, it provides a comprehensive prediction, independent of the form of demand, on whether a merger between capacity-constrained rivals will increase prices.

$ccGUPPI$ is most applicable to industries where short run marginal costs are approximately constant until a level of output where further production becomes infeasible or much more expensive. An alternative modeling approach to such industries is suggested by Ryan (2013) and Miller and Osborne

(2014); there, marginal cost is constant up to a point, after which it increases steeply, but smoothly. Under such a framework, practitioners could apply conventional $GUPPI$ to calculate pricing pressure, as there would be no capacity constraints that ''bind'' in the sense discussed in this paper.

However, in such a setting, conventional $GUPPI$ would be a poor measure for $\frac{\Delta p}{p}$. Suppose two firms who have just reached the ''steep'' part of their respective marginal cost curves propose to merge. It may be that both firms are likely to decrease production by a discrete amount, with both returning to the ''flat'' part of their marginal cost curves. In this case, merger price effects will be unmoored from $GUPPI$, at least when using an identity pass through matrix. Instead, the key determinant of the merger price effects is the degree to which the increase in marginal costs affects pre-merger pricing, which is best approximated by this paper's λ. Even if marginal costs are known to increase smoothly but steeply, this paper's framework is likely to be superior to conventional $GUPPI$, absent information on demand and cost curvature which could improve pass through estimation.

Another consideration when implementing $ccGUPPI$ is that capacity constraints are necessarily a short run phenomena, when capital and potentially other factors of production are fixed. Antitrust agencies will surely want to consider both short- and long-run merger effects. For example, capacity constraints may bind so tightly in the short-run that a merger would generate no short-run price effects, yet in the long-run the merged firm would have the incentive to reduce capacity and increase prices.

Implementing $ccGUPPI$ requires one additional piece of information not required for a traditional $GUPPI$ calculation. Specifically, one needs to know the price elasticity of demand in addition to margin, or, equivalently, the difference between marginal revenue and marginal costs (λ). Identifying the price elasticity of demand econometrically is difficult given it requires exogenous variation in price. Doing so when firms are capacity-constrained can be even more challenging given some consumers may face a truncated choice set (see for example Conlon and Mortimer (2013)). Nevertheless, antitrust agencies can supplement econometric evidence with deposition testimony, documents from industry participants regarding the price sensitivity of demand, and accounting data on costs.

Accounting information on the rental cost of capital may be particularly helpful for discerning the gap between marginal revenues and short run marginal costs. For example, consider a Leontief production function y = min(k, l), which generates short-run marginal costs $c(q) = w$ s.t. $q < \bar{k}$ and long-run marginal costs $c(q) = w + r$, where w is the wage and r is the rental cost of capital. In the long run, we would expect firms to choose a level of capacity where marginal revenue equals $w + r$, thus the gap between marginal revenue and short-run marginal costs (λ) would equal r. If the industry appears to be close to long-run equilibrium, one could infer (λ) from the rental cost of capital. $ccGUPPI$ is particularly applicable to such industries, as we would expect to observe binding pre-merger capacity constraints.

When analyzing industries where it may not be feasible to precisely measure demand elasticity separately from margin, practitioners could identify the range of demand elasticities for which $ccGUPPI$ would exceed some critical level, such as the level of expected efficiencies. Firm i's upward pricing pressure due to its capacity constraint, λ_i, is all else equal increasing in the absolute value of its own price elasticity of demand, ϵ_{ii}. Consequently, more elastic demand for firm i's product implies lower merger price effects from a merger involving firm i, because prices will already be elevated be firm i's capacity constraint prior to the merger. Similarly, more elastic demand for the product of firm j implies lower merger price effects for firm i, if i and j merge. This is because all else equal θ_i is decreasing in the absolute value of firm j's own price elasticity. Thus, the combination of high margins and elastic demand in the presence of capacity constraints should be seen as reducing the likelihood of substantial merger price effects.

7 Conclusion

This paper provides antitrust practitioners with a simple tool to evaluate mergers involving one or more capacity-constrained firms. Our analysis generates two main results. First, as shown by proposition 6, knowledge of the pricing pressure terms underlying $ccGUPPI$ is sufficient to determine whether a merger between capacity-constrained firms will increase prices. Like other binary upward price pressure tests (e.g. Werden (1996) and Farrell and Shapiro (2010)), this diagnostic does not depend on the functional form of demand. Second, simulated data from our Monte Carlo experiments suggest that $ccGUPPI$ performs better than standard $GUPPI$, and is a quite accurate predictor of merger price effects when demand is linear or logit, and a lower bound on price effects under AIDS demand.

Our paper contributes to closing a gap related to the evaluation of mergers involving capacity constraints. Froeb et al. (2003) demonstrated that because firms with binding capacity constraints set price so that their demand equals capacity, such firms' prices may be less responsive to competitive conditions than the prices of firms with no such constraints. While that paper – and others – correctly pointed out that merger price effects may be eliminated, attenuated, or essentially unaffected by capacity constraints, depending on the degree to which they were binding, little practical guidance existed for gauging the relevance of capacity constraints to a given merger's price effects. We extend a commonly-used model of upward pricing pressure to incorporate pricing pressure resulting from both a merger and from capacity constraints. We show empirically, using simulated data, that $ccGUPPI$ multiplied by the identity matrix as a proxy for pass through, is a useful predictor of merger price effects across a variety of demand systems consistent with the same pre-merger data.

Both the theoretical underpinnings of $ccGUPPI$ and our empirical results suggest that a limited amount of pre-merger information – on margins, diversion, and elasticities – suffices to generate a useful metric of the price effect of a merger in the presence of capacity constraints. Thus, in addition to closing a theoretical gap on the analysis of mergers with capacity constraints, $ccGUPPI$ may close a gap in the analysis of such mergers by practitioners. As discussed in the introduction, merging parties may argue that binding pre-merger capacity constraints obviate any reasonable concern that such a merger could harm consumers. While such an argument is not supported by the literature, antitrust practitioners lacked a framework for assessing the extent to which capacity constraints in a particular merger attenuate merger price effects. By providing such a framework in $ccGUPPI$, such disputes need not be settled on an all-or-nothing basis, in which capacity constraints either eliminate merger price effects, or are irrelevant.

Appendix

This appendix contains three items: a proof of lemma 4 (describing conditions on the ordering of pricing pressure terms λ, θ, and $GUPPI$), equation (19) (estimates of $\frac{\Delta p_i}{p_i}$ based on an alternative pass-through matrix implied by equation (18) and $ccGUPPI$), and a revised version of figure 4 using the predictions of equation (19). We begin with the proof of lemma 4.

Proof of lemma 4

Item 1:

$$
\begin{aligned}
& GUPPI_i \geq \theta_i \\
\iff & \frac{p_j}{p_i} m_j D_{ij} \geq m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij} \frac{1}{\epsilon_{jj}} \text{ (using the definitions of } GUPPI \text{ and } \theta) \\
\iff & m_j \geq \frac{p_i}{p_j} m_i D_{ji} - \frac{1}{\epsilon_{jj}} \\
\iff & \lambda_j \geq GUPPI_j \text{ (using the definitions of } \lambda \text{ and } GUPPI)
\end{aligned}
$$

Item 2:

$$
\begin{aligned}
&\lambda_i \geq \theta_i \\
\iff &\lambda_i \geq m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij}(\lambda_j - m_j) \text{ (using definitions of } \theta, \lambda) \\
\Rightarrow &\lambda_i \geq m_i D_{ij} D_{ji} - \frac{p_j}{p_i} D_{ij}(\theta_j - m_j) \text{ (given } \theta_j \geq \lambda_j) \\
\iff &\lambda_i \geq m_i D_{ij} D_{ji} + GUPPI_i - \frac{p_j}{p_i} D_{ij} \left(m_j D_{ij} D_{ji} - \frac{p_i}{p_j} D_{ji} \frac{1}{\epsilon_{ii}} \right) \text{ (using definitions of } \theta, GUPPI) \\
\iff &\lambda_i \geq m_i D_{ij} D_{ji} + GUPPI_i - \frac{p_j}{p_i} m_j D_{ij}^2 D_{ji} + D_{ij} D_{ji}(\lambda_i - m_i) \text{ (using definition of } \lambda) \\
\iff &\lambda_i (1 - D_{ij} D_{ji}) \geq GUPPI_i (1 - D_{ij} D_{ji}) \text{ (using definition of } GUPPI) \\
\iff &\lambda_i \geq GUPPI_i
\end{aligned}
$$

Tautologically, there are six possible ways of ordering the quantities λ_i, θ_i, and $GUPPI_i$ for $i = 1, 2$, meaning there are 36 combinations of orders across the two merging firms. The first part of this lemma rules out 26 of these combinations, while the second part rules out an additional two. The remaining eight possible combinations of orders are:

1. $\lambda_1 > \theta_1 > GUPPI_1$ and $GUPPI_2 > \lambda_2 > \theta_2$
2. $GUPPI_1 > \lambda_1 > \theta_1$ and $\lambda_2 > \theta_2 > GUPPI_2$
3. $\lambda_1 > GUPPI_1 > \theta_1$ and $\lambda_2 > GUPPI_2 > \theta_2$
4. $GUPPI_1 > \theta_1 > \lambda_1$ and $\lambda_2 > \theta_2 > GUPPI_2$
5. $\lambda_1 > \theta_1 > GUPPI_1$ and $GUPPI_2 > \theta_2 > \lambda_2$
6. $\theta_1 > \lambda_1 > GUPPI_1$ and $GUPPI_2 > \theta_2 > \lambda_2$
7. $\theta_1 > GUPPI_1 > \lambda_1$ and $\theta_2 > GUPPI_2 > \lambda_2$
8. $GUPPI_1 > \theta_1 > \lambda_1$ and $\theta_2 > \lambda_2 > GUPPI_2$

■

Equation (19) describes our alternative estimates of $\frac{\Delta p}{p}$ using $ccGUPPI$ and the pass-through matrix implied by equation (18). It should be contrasted with equation (14).

$$
PT * ccGUPPI_i = \begin{cases} GUPPI_i - \lambda_i & \text{if } GUPPI_i \geq \lambda_i, \text{ for } i = 1, 2 \\ \theta_i - \lambda_i & \text{if } \theta_i \geq \lambda_i \text{ and } GUPPI_j \leq \lambda_j, \text{ with } i \neq j \\ -\frac{\frac{\partial q_i}{\partial p_j}}{\frac{\partial q_i}{\partial p_i}} \frac{p_j}{p_i} (\theta_j - \lambda_j) & \text{if } \theta_j \geq \lambda_j \text{ and } GUPPI_i \leq \lambda_i, \text{ with } i \neq j \\ 0 & \text{if } \lambda_i \geq \theta_i \text{ for } i = 1, 2 \end{cases} \tag{19}
$$

Finally, figure 5 reproduces figure 4 using the predictions for $\frac{\Delta p}{p}$ from (19). This removes the cluster of points along the horizontal axis in figure 4 where $ccGUPPI$ is zero because firm 1 is constrained before and after the merger, yet still raises price because firm 2 is not constrained after the merger (see footnote 23). Otherwise the joint distributions of $ccGUPPI$, $GUPPI$, and the three simulated price increases appear to be substantively identical to those in the main body of the paper, which use an identity pass-through.

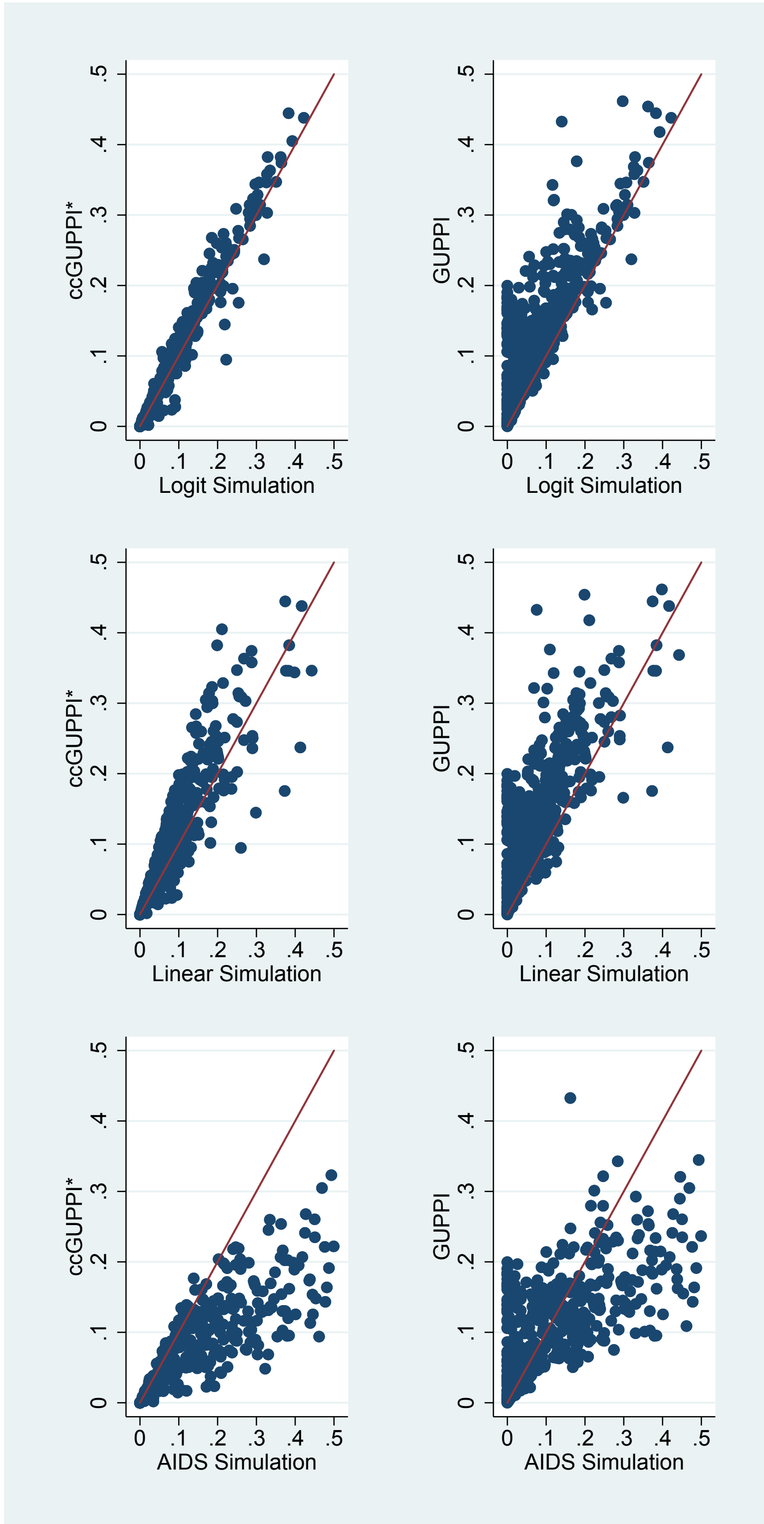


Figure 5: $ccGUPPI$ and $GUPPI$ price predictions using the revised pass-through matrix described in equation (19) (y-axis) versus simulated price effect (x-axis).

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