Competition and Endogenous Impatience in Credence-Good Markets

by

Jeremy Sandford*

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In markets for credence goods, such as doctor visits, customers sample a firm for a few periods, before deciding whether to retain or fire that firm. In our model, customers have endogenously determined patience in tolerating bad outcomes from credence-good providers. The more competitive the market, the more options customers have away from a firm, and so the less tolerant of bad outcomes she will be. Competition thus increases equilibrium firm effort, as providers work harder to impress impatient customers. Higher effort raises customer surplus and helps balance the informational advantages providers enjoy in credence-good markets. (JEL: C73, D82, L14, L15)

1 *Introduction*

A recent undercover government investigation into EZ Lube, a chain of about 75 southern California auto repair shops, found 640 instances of fraud, including customers being charged for services that were never performed and being advised to order unnecessary repairs.[1] A 2006 investigation of Los Angeles-area Jiffy Lube stores by the television station KNBC showed workers charging for transmission flushes and other services that were not done, evidently at the behest of the area district manager.[2] *The Economist* speculates that as much as one-third of medical spending in the U.S. is on "irrelevant tests, unproven procedures, and unnecessarily pricey drugs and devices" (see *The Economist*, February 2, 1999, p. 89). A Federal Trade Commission (1980) study of the optometry industry found evidence suggestive of optometrists systematically prescribing unnecessary treatment (via Wolinsky, 1993).

This paper is concerned with *credence good* markets, in which an *expert* firm supplies goods or services to a customer who can only partially evaluate quality, even ex post. For example, a tourist charged $50 for a cab ride from the airport to a hotel can at best speculate whether she was overcharged. New parents are often poorly equipped to

*University of Kentucky, Lexington. I thank Larry Samuelson, Bill Sandholm, and two anonymous referees for helpful comments.

[1] See California's Department of Consumer Affairs press release from September 20, 2006.

[2] See www.nbclosangeles.com/Is_Your_Mechanic_Cheating.html.

judge the medical necessity of a C-section.[3] A pet owner will never know if expensive veterinary treatment was the most appropriate response to an illness, or if rest and time would have produced the same results.

With payment divorced from unobserved quality, what incentive do credence-good providers have to honestly diagnose the severity of a problem and to do their best to correct it? This paper adds to the existing literature on credence goods by demonstrating that customers will be less patient in absorbing bad outcomes the more firms there are in the market. In the context of the previous examples, a *bad* outcome may be either a dog that remains ill for several days after a vet visit, or a dog that is healthy and lively after a $500 vet visit, while a *good* outcome is a healthy dog *and* a low bill. Similarly, a bad outcome may be a C-section birth, while a good outcome may be a nonsurgical birth. In both examples, the bad outcomes are not, by themselves, indicative of poor or unscrupulous service; more expensive treatment is often medically preferable for good reason. But, all else equal, the bad outcomes are worse than the good outcome in which minimal medical action is necessary for good health. Pet owners and patients often have very limited information about the medical necessity of tests; thus, tolerance, or *patience*, for bad outcomes depends greatly on the trust they have in the provider.

The contribution of this paper is to endogenize customer patience for bad outcomes. In particular, as product market competition increases, customers tolerate fewer bad outcomes before switching to a new firm, as the opportunity cost of switching – the possibility that a customer is giving up on a highly qualified expert who was unlucky enough to produce several bad outcomes – decreases as the number of firms in the market increases. This incentivizes better service for two reasons: one, firms have fewer chances to convince customers to continue with them when the market is more competitive, and two, if customers do decide to try other credence-good providers in town, then not only is it less likely that they will ever return, but it will be farther in the future if they do. Therefore, mechanics who repeatedly overcharge customers (for example) will eventually lose business to their competitors. Doctors who routinely provide C-sections regardless of medical necessity will develop a reputation for doing so and find it hard to attract patients.

The results of our paper apply to any credence-good market in which reputation is important, either because of repeat visits (e.g., mechanics, accountants, general practitioners) or because of word-of-mouth reputation (e.g., "everyone I know who has used that ob-gyn ended up getting a C-section"). The mechanism we describe has no bite in markets where providers have little or no reputational concerns, like the market for taxi rides or for restaurants located at highway rest stops.

Interestingly, the competitive effect described here *depends* on quality being imperfectly observed by customers. Were quality fully observable, customers could always immediately fire firms providing them with low quality, meaning all firms would provide good service in equilibrium. With quality imperfectly observed, customers update

[3]See Gruber and Owings (1996), which provides empirical evidence that ob-gyns respond to reduced income caused by lower fertility by providing more C-section deliveries, which are reimbursed at a higher rate.

their beliefs about the quality of a provider after each outcome; the less competitive the market, the less strong this belief needs to be before switching is optimal.

Beyond the effect on customer patience mentioned above, we also consider comparative statics on the gains from trade and discount factors. Surprisingly, if a customer's discount factor increases, credence-good providers may produce *worse* outcomes. The reason for this is that while customers with higher discount factors are likely to stick around for longer, thus making their retention more valuable and incentivizing good service, longer-tenured customers will have, on average, higher tolerance for bad outcomes, eroding provider incentives. The quality of service is increasing in firms' discount factor, as well as in the price paid by customers.

Much work has been done examining how reputational concerns can induce expert firms to exert high effort (for example, Mailath and Samuelson, 2001, Sandford, 2010, Liu, 2011, and Ely and Välimäki, 2003). There has been little work on how the competitiveness of these markets affects incentives for high effort. Two exceptions stand out: Rob and Sekiguchi (2006) and Hörner (2002).

Rob and Sekiguchi (2006) argue that competitive pressure can incentivize scrupulous service in non-Markovian repeated-game equilibria with two firms. Exactly one firm plays high effort in each given period. All customers patronize the high-effort firm, but observe only a signal imperfectly correlated with effort. If this signal is low, all customers switch to the other firm, and that firm plays high effort in the next period. This switching continues indefinitely. Owing to the all-or-nothing nature of the competition, this model does not speak to what happens as the degree of competitiveness varies, as does our paper. Indeed, customers do not learn anything upon getting bad outcomes from firms; the resulting anonymity ensures that even with only two firms, there is an effectively limitless supply of new firms to switch to.

In Hörner (2002), a continuum of firms provide credence goods, with each either an inept type or a normal type. Customers patronize only those firms with unblemished histories, and so the threat to firms of losing all of their customers if caught shirking motivates competent firms to put forth high effort. While Hörner's model also relies on the perpetual threat of customers being stolen away by the competition, with a positive measure of firms there is no way of varying the competitiveness of the market, as there is in our paper with a finite number of firms.

Unlike those two papers, we assume a finite number of heterogeneous firms. These assumptions are crucial to our results on the relationship between competition and frequency of good outcomes. Our paper is the first to study the link between competition and customer patience, and the resulting quality of firm service.

Other papers have suggested separation of diagnosis and repair as a partial remedy for credence-good incentive problems (see Emons, 1997, 2001, Taylor, 1995, and Wolinsky, 1993, 1995), though they note that this is unappealing if there are economies of scope in combining the two, as with the services of a car mechanic, doctor, lawyer, or accountant. A small literature exists on the relationship between market structure and quality when quality is observable only ex post.[4] Kranton (2003), Bar-Isaac (2005), and Dana and

[4]Goods sold in such markets are often called "experience goods."

Fong (2011) point out that as markets become more competitive price tends to fall, which lowers firm margins and decreases the attractiveness of trying to maintain a reputation by providing high-quality goods, offsetting the competitive effects induced by having firms compete for market share. A nonmonotonic relationship between competition and quality arises. The effect identified in this paper, that customers become less patient with firms as the market becomes more crowded, is novel to this literature; indeed, to isolate this effect, this paper holds the price of a firm's service constant in the number of firms. In a similar vein, Pesendorfer and Wolinsky (2003) conclude that price competition between credence-good providers can lead to poor outcomes when the only way to verify firm effort is via second opinions. These papers do not examine the relationship between competition and customer patience studied here.

This paper is also related to the substantial industrial organization literature on tenure dependence in customer choice of firms. One empirical constant from this literature is that departure probabilities decline with increasing customer tenure. A natural question is whether these declining probabilities are the result of customers having a stronger taste for tenured firms or the result of selection, in which customers who deem themselves to be ill suited for a particular firm leave over time. Israel (2005) uses exogenous price variation in insurance plans to empirically distinguish between these two effects and finds that while both are important, selection seems to play a much larger role. The effect modeled in this paper is quite similar, in that customers prefer some firms to others, and so a decision to switch is made when a customer deems it sufficiently likely that she can get a better match elsewhere. Long-tenured customers are then those who have selected into being long-tenured, i.e., those who have sampled the services of a firm and decided that they are well matched. This paper also suggests a mechanism through which customers can learn about firms they have hired over time in order to inform the tenure decisions discussed in the tenure-dependence literature.

2 Model

2.1 Market Structure

Consider a market in which each of N firms repeatedly provides credence goods to a continuum of customers. Each customer is matched with at most one firm, and while customer–firm relationships stretch over many discrete time periods, they can be discontinued at any time by dissatisfied customers, who can then switch to a new firm at cost $s \geq 0$. Suppose that each period of a match between a firm and a customer produces either a good outcome (G) or a bad outcome (B). A good outcome for a customer might be a car that has no further mechanical problems after visiting a mechanic, or a patient who feels better after visiting a doctor's office. The probability of a good outcome is increasing in firm effort $e \in [0, E]$, with per-customer cost $c(e)$ strictly increasing, convex, and differentiable. Think of effort as a firm's investment in its business, such as hiring and training competent employees or purchasing and maintaining high-quality equipment. Effort captures both the scrupulousness of firms (e.g., in that higher effort means more honest diagnoses) and their literal effort invested in fixing customers' problems.

A firm's effort is chosen anew each period, and is not customer-specific; effort is the same for all customers, regardless of their history with the firm. Time extends infinitely forward and backward.

The quality of a match between a customer and firm is heterogeneous; each pairing is either a good match or a bad match, with match quality being drawn "independently" of all other matches at the time the match is formed, so that a fraction μ of all new matches are good.[5] A good customer–firm match occurs when, for example, a mechanic is particularly well suited to diagnose and repair the specific problems a customer's make and model of car is likely to have, or a doctor has a bedside manner or specific expertise that particularly meet a patient's needs. A more general structure would see match quality correlated across customers of a given firm, but for tractability we assume uncorrelated match quality. An oft-used alternative assumption (see Mailath and Samuelson, 2001, Hörner, 2002, Ely and Välimäki, 2003) is that match quality is *perfectly* correlated across customers, with each firm being innately "good" or "bad." Given the finite number of firms in this model, this paper's structure allows for stationarity in the total measure of good and bad matches.

Neither customers nor firms can directly observe match quality, though both share a (correct in equilibrium) belief about the fraction of all matches that are good. Customers do not observe the effort their matched firm puts forth, just whether they get a good or a bad outcome in each period. A customer updates her belief that she is in a good match upward or downward after each good or bad outcome, respectively. Specifically, a good match generates a good outcome in each period with probability $f(e)$, where $f : [0, E] \to [0, 1]$ satisfies $f(0) = 0$, $f'(e) > 0$, and $f''(e) \leq 0$ for all $e \in (0, E)$, while a bad match generates a bad outcome with probability 1. All customers share a belief about firm effort $t = P(G \mid \text{good match})$, which is correct in equilibrium (i.e., $t = f(e)$). Outcomes are independent across all customers and firms; effort e produces a good outcome for a fraction $f(e)$ of a firm's good matches. It follows that a customer who believes himself in a good match with probability μ updates this belief to $\mu_G = 1$ after a good outcome and $\mu_k = (\mu(1-t)^k)/(\mu(1-t)^k + (1-\mu)) < \mu$ after k consecutive bad outcomes.[6] Firms are aware of aggregate statistics regarding their customers (i.e., the total measure of those who received a bad outcome last period), and condition their effort choice on this.

Once a customer has experienced a good outcome, she has no reason to ever switch firms, and will remain permanently matched with the firm that produced the good outcome. However, a customer who has realized only bad outcomes from a match may

[5] Simply requiring that the fraction μ of new matches be good would avoid the well-known problems with taking the distribution of outcomes of a continuum of i.i.d. random variables to be the same as that of the random variable itself (see Judd, 1985, Feldman and Gilles, 1985, and Al-Najjar, 1995). However, it is helpful to present the model in terms of independent draws.

[6] Throughout, we slightly abuse notation, as μ denotes both the (objective) fraction of all matches that are good, and the (subjective, but correct in equilibrium) beliefs that both customers and firms have over the fraction of all matches that are good.

choose to pay search cost s and switch firms at any time. Let the term *probationary customer* refer to any customer still active in the market who has not yet received a good outcome from any firm. Finally, a customer may also switch costlessly to an outside option at any time – for example, performing maintenance on her own car or eschewing doctors completely.

Surplus π is generated each period in which a customer receives a bad outcome (B); all of this surplus goes to the firm. Surplus $\pi + A$ is generated in a period with a good outcome (G), of which π goes to the firm and A to the customer. This structure amounts to firms charging a constant price π, regardless of the competitiveness of the market.[7] Were price to decrease in number of firms, as seems reasonable, the effect would be identical to lowering the exogenous π, discussed in section 4.3.3, and therefore would partially offset the direct competitive effects of reducing customer patience. Customers receive per-period payoff $R < A$ from their outside option. Suppose that a fraction $1 - \delta$ of customers leave the market for exogenous reasons each period, so that each customer discounts future payoffs at rate δ. Firms discount future profits at rate β.

Informally, a customer's choice variable is a *patience level*, the number of consecutive bad outcomes she will tolerate before switching away from a given firm. This paper's central point is that this patience level varies in a customer's history. A customer who is new to the market and therefore matched with her first firm may have a different patience level with that firm from that of one who has already tried and switched away from several firms. Specifically, we will show that patience is decreasing in the customer's payoff upon switching. As this payoff decreases with increasing negative information she has about the firms in the market, a customer will tend to be more patient the more firms she samples. Similarly, a customer will be more patient the fewer firms there are in the market, as her payoff to switching away from a firm is decreasing in the number of firms left to try out. Intuitively, someone living in a small town with only two or three doctors will tolerate a lower quality of care than will a patient living in a big city where there are hundreds of available doctors.

A firm's incentive to exert effort comes from the desire to retain customers. If customers are very patient, a firm has little incentive for costly effort. With very impatient customers, who might switch away from a firm after only one bad outcome, firms have a strong incentive to invest in effort in order to impress their customers within the limited window during which they have them.

In this environment, we show that firm effort is increasing in the number of the firms in the market. This happens for two reasons. One, the more firms, the less patient the average customer will be in absorbing bad outcomes, incentivizing firms to exert higher effort. Two, the more firms, the longer it will take for a customer who switches away from a firm to (eventually) return, after finding other firms in the market to be equally unsatisfactory. The following two numerical examples illustrate both of these

[7]It is easy to construct collusive repeated-game equilibria with this feature, in which defecting firms are punished by an extended period of mutually destructive behavior. See the introduction for a discussion of papers examining the tension between price competition and incentives for high quality in credence-goods markets.

mechanisms.

2.2 Two Numerical Examples

2.2.1 No Search Costs, $s = 0$

Suppose that switching is costless. Here, any customer who has never experienced a good outcome will have patience 1, switching away from a match after one B outcome, while remaining permanently matched with the first firm to produce a G outcome for her. Therefore, a hypothetical unlucky customer who only receives bad outcomes will cycle through the market's N firms for one period each, lowering her belief that each firm is a good match to μ_1. After sampling each firm for one period, she will sample each firm again; after two bad outcomes, her posterior belief that each firm is a good match is $\mu_2 = (\mu(1-t)^2)/(\mu(1-t)^2 + 1 - \mu)$. She will continue cycling through all firms in the market until she has experienced k bad outcomes from each firm, where k is determined endogenously by the unique integer such that the value of matching with a firm believed to be a good match with probability μ_k is less than the value of the outside option, $(R)/(1-\delta)$.[8]

How do firm incentives change as the market becomes more competitive and N increases? Unlike the case with positive search costs, customer patience is unaffected by an increase in N. However, firms' incentives change along a different dimension. With, for example, $N = 2$, a firm knows that many of its customers who switch after receiving a bad outcome will return 2 periods later after also getting a bad outcome from the firm's rival. If, however, there are 20 firms in the market, it is much less likely that a customer who switches away from a firm will ever return, and it will take a longer time if she does. Therefore, the more firms there are in the market, the more urgency there is for a firm to retain its customers, and hence the greater the incentive to exert high effort.

Consider the numerical example of Table 1. We show in the Appendix that if there are two firms in this market, both exert effort $e^* = 5$, while if there three firms, all three exert effort $e^* = 6.55$ in the model's symmetric steady-state equilibrium. In either case, a new customer samples each firm sequentially two times, stopping at the first G outcome or with her outside option after $2N$ periods.[9] Firms take customer patience as given and equate the marginal cost of effort with the marginal benefit of effort. The latter is, informally, equal to the marginal increase in permanently matched customers multiplied by the value of such customers, $(\pi)/(1-\delta)$, minus the marginal decrease in number of returning customers, multiplied by the value of a returning customer.

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[8] We defer a more formal description of customer behavior under these, or any, circumstances to section 3.1.

[9] With two firms ($t = 0.5$), after two B signals at any firm, the payoff to returning to that firm for one more period is $\mu_2 t(A + (\delta t A)/(1-\delta)) + (1-\mu_2 t)\delta(R)/(1-\delta) = 75.75$, while the payoff to leaving immediately is $(R)/(1-\delta) = 100$. After only one bad outcome, the payoff to sampling a firm one more time is $\mu_1 t(A + (\delta t A)/(1-\delta)) + (1-\mu_1 t)\delta(R)/(1-\delta) = 126.3$. A similar calculation shows that with three firms ($t = 0.655$), customers also have patience 2.

Table 1
Example Parameters under which Effort Increases in Competition even with Zero Search Cost

Model object	Object name	Value
Search cost	s	0
Pr(good match)	μ	0.5
Payoff to G outcome	A	15
Value of outside option	R	1
Customer discount factor	δ	0.99
Firm discount factor	β	0.99
Firm per-customer profit	π	12.065
Pr(G | good match)	$f(e)$	$e/10$
Firm cost function	$c(e)$	$0.07e^2$

In the next example, customers choose not to sample any firm more than once, yet we get a very similar effect. The reason is that even with only one sample, customers are less patient in absorbing bad outcomes the more firms there are, incentivizing high effort.

2.2.2 $s > 0$: Customers are Less Patient the More Firms they Try Out

Now consider the case where $s = 2.2$, that is, any customer wishing to switch firms must pay a switching cost of 2.2 to do so. Assume that it is not necessary to pay this switching cost to switch to the outside option. Under the model parameters in Table 2, unlucky customers who never get a good outcome cycle through all the firms in the market once, with increasing patience, before permanently switching to their outside options.

Table 2
Example Parameters under which Customers Pay a Positive Switching Cost and Endogenously Choose to Visit Each Firm Once, with Patience Increasing in Firms Sampled

Model object	Object name	Value
Search cost	s	2.2
Pr(good match)	μ	0.5
Payoff to G outcome	A	1.8
Value of outside option	R	1
Customer discount factor	δ	0.99
Firm discount factor	β	0.99
Firm per-customer profit	π	25
Pr(G | good match)	$f(e)$	$e/10$
Firm cost function	$c(e)$	$0.2323e^{1.2}$

First, suppose that $N = 1$. We show that there is an equilibrium in which the market's one firm plays effort 6.5. To do this, we show that if customers have the belief $t = 0.65$ (a 65% chance of a good outcome conditional on a good match), they optimally have patience level 3 with the firm, switching permanently to their outside option after three consecutive bad outcomes, while if customers play patience level 3, the firm optimally plays effort $e^* = 6.5$.

Given a belief $t = 0.65$, customers choose their patience level as follows. After each bad signal, they compare the payoff of permanently choosing their outside option, $R/(1-\delta) = 100$, with that of remaining matched for one more period, $\mu_k t(A+(\delta tA)/(1-\delta)) + (1-\mu_k t)\delta R/(1-\delta)$. The patience is the largest number k for which the former is larger than the latter, so after k bad outcomes, a customer prefers his outside option. After two bad outcomes, the payoff to remaining for one more period is $\mu_2 t(A+(\delta tA)/(1-\delta))+(1-\mu_2 t)\delta R/(1-\delta) = 100.32$, while the payoff to leaving immediately is $R/(1-\delta) = 100$. Similarly, after three bad outcomes, the payoff to leaving immediately is 100, while the payoff to giving the firm one more period to produce a good outcome is $\mu_3 t(A+(\delta tA)/(1-\delta))+(1-\mu_3 t)\delta R/(1-\delta) = 99.50$. Therefore, each customer optimally chooses a patience level of 3.[10]

If all new customers have patience 3, the firm compares the marginal cost of effort ($c'(e) = 0.27876e^{0.2}$) against the marginal benefit, which is the increase in present discounted lifetime profits from convincing a customer with patience 3, 2, or 1 to remain matched in perpetuity, multiplied by both the marginal increase in probability of a good outcome with more effort, and the measure of customers with patience 3, 2, and 1, respectively. In a steady-state equilibrium, in which the measure of each type of customer is constant in time, equation (6) in section 3.2.2 formally solves for the equilibrium marginal benefit. Jumping ahead to that result, we find that, given a customer patience of 3, we have $c'(e) = mb(e)$ at $e = 6.5$, and so new customers having patience 3 and the single firm playing $e = 6.5$ comprise a steady-state equilibrium.

Now suppose that N increases to 2. We show that both firms playing effort level 7.4 in every period is an equilibrium. Given customers' belief $t = 0.74$, customers will have patience 2 with the first firm they match with, and patience 3 with the second firm. As we will show that no customer who switches away from a firm will ever return, a customer's patience level at his second matched firm is necessarily the same as that when there is only one firm, which is 3. The patience level at the first matched firm is calculated in the same way, except the value of switching is not equal to $R/(1-\delta) = 100$, but the value of being newly matched with the second firm, given explicitly in Proposition 3, in section 3.1.1. Applying the result of Proposition 3, a customer's payoff to switching firms at search cost $s = 2.2$ is 112.83. It is then direct to calculate that a customer prefers to switch away from her first matched firm after two consecutive bad outcomes. The lower patience level at a customer's first firm reflects this higher continuation payoff. Once at the second firm, her outside option is the higher of the value of returning to the first firm (namely, $\mu_2 t(A + (\delta tA)/(1-\delta)) + (1-\mu_2 t)\delta R/(1-\delta) - s = 98.62$) and that of permanently switching to the outside option (namely, $R/(1-\delta) = 100$).

Given that customers have patience 2 and 3 at their first and second firms, respectively, in a steady state firms optimally choose effort $e = 7.4$. Again, this follows from Proposition 4 in section 3.2.3. Intuitively, since the average patience level among a firm's

[10] This follows directly from Proposition 2 in the sequel.

customers is lower when $N = 2$ than when $N = 1$, there is a greater benefit to sending a customer a good signal, and so the marginal benefit of effort increases, increasing equilibrium effort.[11]

It is important to understand the reason that customers do not immediately switch away from a firm upon seeing a bad outcome. This is because there is an option value to staying with that firm for another period or two that is forfeited by switching, since returning to that firm will be too costly. In the case with $N = 2$, consider the problem of a customer who has just received one bad outcome. If she were to switch immediately, her payoff upon returning would be $\mu_1 t(A+(\delta t A)/(1-\delta))+(1-\mu_1 t)\delta R/(1-\delta)-s = 99.9395$, which is worse than her outside option, so a customer who leaves, even after only one period, will never return. To switch, then, is to permanently give up on the firm, despite the chance that it is a good match that got unlucky in producing one bad outcome. The customer therefore decides to stay one more period to see if that produces a good outcome. If not, she deems it sufficiently unlikely that the firm is a good match that she is willing to give up and move on.

3 Solving the Model

The examples of sections 2.2.1 and 2.2.2 demonstrate how to solve the model for general s. If s is very low, as in section 2.2.1, customers cycle endlessly through firms until getting a good signal, at which point they stop permanently. If s is higher, as in section 2.2.2, a customer tries each firm for a few periods, leaving after several bad outcomes, never to return, and remaining permanently matched after the first good outcome.

For a general s, other possibilities exist besides those from the two examples. If s is small but positive, for example, customers might try each firm in the market twice, for a few periods each, with their patience increasing as they cycle through firms with only bad outcomes. For analytical tractability, we focus on the customer behavior exhibited in section 2.2.2: s is high enough that customers do not return to already fired firms, even after uniformly bad outcomes from all other firms. Letting $V[\mu_k]$ describe the payoff of matching with a firm believed to be a good match with probability μ_k, the following is a sufficient condition for a customer to never return to already fired firms, but to still be willing to sample each firm once for several periods, stopping upon her first good outcome:

$$s \in \left[V(\mu_1) - \frac{R}{1-\delta}, V[\mu] - \frac{R}{1-\delta}\right]. \quad (1)$$

An alternative assumption to (1) that produces the same behavior is that $s = 0$ the first time a firm is switched to and $s = \infty$ the second time. As the exposition is vastly simpler in this case, we will refer to this alternative assumption throughout the rest of the paper: customers are simply assumed not to return to already fired firms, behavior

[11]Formally, the increased profit from producing a G outcome for a patience 1 customer is $(\beta\pi)/(1-\beta)$, while the increased profit from producing a G outcome for a customer with patience $p > 1$ is $(\beta^p\pi)/(1-\pi)$.

that arises endogenously in section 2.2.2. After trying every firm in the market once, for a few periods each, customers who receive only bad outcomes permanently switch to their outside option.[12]

Two effects of competitiveness in a credence-good market are described in sections 2.2.1 and 2.2.2. One, as the number of firms increases, it takes customers longer to return to already fired firms, raising the urgency of keeping them and thus increasing firm effort. Two, as the number of firms increases, average customer patience decreases, meaning that firms have less time in which to impress customers before they leave after a series of bad outcomes. The second effect is driven by customers becoming more patient as they run out of options, a phenomenon described in section 3.1. The first effect does not appear under the assumption that customers do not return to already fired firms, though the Appendix demonstrates how to solve the model if switching is cheap enough so that customers do return.

We now solve a general model in which customer patience is endogenous, varying across firms as they are sampled in sequence. We solve for the model's stationary, symmetric equilibria, in which firms and customers play identical strategies across time, all state variables are constant across time, and each customer and firm optimizes given the behavior of all other agents in the market (see section 3.3 for a full definition of equilibrium). We begin in the next section by examining optimal customer strategies.

3.1 Customers

A customer receives payoff A, 0, or R upon a good outcome, upon a bad outcome, and from her outside option, respectively. Her goal is to maximize the total discounted value of the sum of these payoffs over her lifetime. A match believed to be good with probability μ generates a good outcome and payoff A with probability μt, and a bad outcome and payoff 0 with complementary probability. A prior belief of μ gets downgraded to $\mu_j = \mu(1-t)^j/(\mu(1-t)^j + 1 - \mu)$ upon j consecutive B outcomes, and upgraded to 1 upon a G outcome.

As a customer's payoff to remaining matched is decreasing in her belief about her firm's being a good match, there exists some cutoff belief below which firing is optimal. Importantly, this might not be where her one-period payoff is higher at another firm or out of the market. For example, if there is only one firm in the market, even if $\mu t A < R$, a customer may choose to remain with the firm for a few more periods, because of the chance that the firm actually is a yet undiscovered good match. The same logic applies to a market with more than one firm: because customers do not return to a firm they have already fired, customers may remain with an underperforming firm even if they

[12] One possible criticism of a model in which $s = 0$ the first time a customer considers switching to a firm, but $s = \infty$ the second time he considers switching, is that the number of "free" searches given to a customer is increasing in N, and thus results on the how firm effort varies in the competitiveness in the market may be driven by cheaper search, and not increased competitiveness. However, this assumption is isomorphic to $s \in [V(\mu_1) - R/(1-\delta), V[\mu] - R/(1-\delta)]$ for all visits. As the example of section 2.2.2 demonstrates, here an increase in the competitiveness of the market spurs firms to increase their effort in order to retain customers.

could get a higher one-period payoff elsewhere, because of the informational value of the firm's outcomes, which are only accessible to a customer while matched with that firm.

As μ_j is decreasing in the number of bad outcomes experienced, each customer chooses how many bad outcomes she will absorb from a given firm before ending the match. She may be more patient with the second firm she tries than with the first, and so on. A complete description of her strategy is a set of N numbers, describing her patience level at the first firm she visits, the second firm, and so on up to the Nth firm.[13]

A customer switches firms when her payoff from being matched with that firm, including the option value resulting from the possibility that that firm is a good match, falls below her *continuation payoff*, defined as the highest payoff a customer can get upon leaving her current firm, and denoted by Γ. Proposition 1 establishes that a customer's optimal strategy is fully characterized by the following condition on her posterior belief μ:

$$\text{remain matched if } \mu \geq \frac{\Gamma(1-\delta)}{t\big[A + \frac{\delta t A}{1-\delta} - \delta\Gamma\big]},$$

$$\text{switch if } \mu < \frac{\Gamma(1-\delta)}{t\big[A + \frac{\delta t A}{1-\delta} - \delta\Gamma\big]}.$$

Proposition 1 *A customer with continuation payoff Γ has a cutoff belief of*

$$\bar{\mu}(\Gamma) = \frac{\Gamma(1-\delta)}{t\big[A + \frac{\delta t A}{1-\delta} - \delta\Gamma\big]};$$

switching is optimal if and only if her posterior belief is below $\bar{\mu}(\Gamma)$.

Proof Let $W[k,\mu,\Gamma]$ be the payoff of being newly matched with a firm believed to be a good match with probability μ, with continuation payoff Γ and patience k.

Straightforward calculations give that

$$W[1,\mu,\Gamma] = \mu t\left[A + \frac{\delta t A}{1-\delta} - \delta\Gamma\right] + \delta\Gamma \geq \Gamma \iff \mu \geq \frac{\Gamma(1-\delta)}{t\big[A + \frac{\delta t A}{1-\delta} - \delta\Gamma\big]},$$

so switching is preferred only if $\mu < \bar{\mu}$. A simple inductive argument gives that if $\mu < \bar{\mu}$, then $W[1,\mu,\Gamma] > W[k,\mu,\Gamma]$ for $k \in \{2,3,\ldots\}$, and so switching immediately is preferred to some higher patience level. *Q.E.D.*

Using Proposition 1 and Bayes's rule, Proposition 2 gives an explicit relationship between a customer's continuation payoff Γ and her patience level p.

[13] In addition to the order in which she tries the firms, a customer could also usefully condition her strategy on a firm's state variables, such as the number of customers each firm has. However, as we eventually solve for symmetric, stationary equilibria, these state variables are identical across all firms, and so can be ignored without cost.

Proposition 2 *Optimal patience is given by the decreasing function* $p(\Gamma)$*, where*

$$p(\Gamma) = \left\lceil \left\lceil \frac{\log\left(\frac{\bar{\mu}(\Gamma)(1-\mu)}{\mu(1-\bar{\mu}(\Gamma))}\right)}{\log(1-t)} \right\rceil \right\rceil$$

where $\lceil\lceil x \rceil\rceil = \{x, x+1\}$ *if* $x \in \mathbb{Z}$*, and is a standard ceiling function otherwise.*

Proof By Proposition 1, a customer with continuation payoff Γ waits p periods upon a new match with a firm if $\mu_{p-1} \geq \Gamma(1-\delta)/(t(A+\delta tA/(1-\delta)-\delta\Gamma))$ and $\mu_p < \Gamma(1-\delta)/(t(A+\delta tA/(1-\delta)-\delta\Gamma))$.

Define $\mu : \mathbb{R}_+ \to [0,1]$, with $\mu(x) = \mu(1-t)^x/(\mu(1-t)^x + 1 - \mu)$. Direct calculation gives that $\mu(x) = \bar{\mu}(\Gamma)$ if and only if

$$x = \frac{\log\left(\frac{\bar{\mu}(\Gamma)(1-\mu)}{\mu(1-\bar{\mu}(\Gamma))}\right)}{\log(1-t)}.$$

As $\mu(x)$ is clearly decreasing in x, the result follows from Proposition 1. *Q.E.D.*

$p(\Gamma)$ is generically single-valued, but takes on two values on the countable set where $[\log([\bar{\mu}(\Gamma)(1-\mu)]/[\mu(1-\bar{\mu}(\Gamma))])]/[\log(1-t)]$ is an integer. The resulting indifference over two patience levels guarantees the existence of equilibrium (see sections A.1.1 and A.1.2).

In a symmetric, stationary equilibrium with all firms playing the same strategies, once a good match has been found there is no reason to ever switch to a new firm, and a customer becomes permanently matched. Likewise, once a customer chooses her outside option, she gains no new information and so has no reason to reenter the market. A customer's strategy is therefore completely described by the function $p(\Gamma)$.

We now calculate continuation payoff Γ, as a function of the number of unsampled firms remaining.

3.1.1 Customers' Continuation Payoffs Vary in Number of Firms

When a probationary customer is matched with the last available firm in the market, her continuation payoff is the value to taking her outside option in perpetuity, $R/(1-\delta)$. When matched with the second-to-last available firm, it is the value to being matched with the last firm, and so on. Numbering the firms she tries backwards from N (the first firm she tries) to 1 (the last firm in the market she tries), we can say that her continuation payoff when matched with firm k is the value to being matched with firm $k-1$.

Proposition 3 provides a closed-form expression for the payoff to a new match when the continuation payoff is Γ, denoted by $Z[\Gamma]$. Note that $Z[\Gamma] = W[p(\Gamma), \mu, \Gamma]$.

Proposition 3 *A customer with continuation payoff* Γ *has expected payoff* $Z[\Gamma]$ *at the beginning of a match:*

$$Z[\Gamma] = \mu T\left(A + \frac{\delta t A}{1-\delta}\right) + \sum_{k=1}^{p(\Gamma)-1} \mu_k t\left(A + \frac{\delta t A}{1-\delta}\right)\prod_{j=0}^{k-1}\delta(1-\mu_j t) + \Gamma \prod_{k=0}^{p(\Gamma)-1}\delta(1-\mu_k t).$$

Proof A probationary customer's payoff upon receiving a good outcome is $A+\delta tA/(1-\delta)$, discounted appropriately. The probability of receiving this payoff in the first period of a new match is μt, in the second period is $\mu_1 t(1-\mu t)$, in the third period is $\mu_2 t(1-\mu t)(1-\mu_1 t)$, and so on. A probationary customer receives $p(\Gamma)$ consecutive bad outcomes with probability $(1-\mu t)(1-\mu_1 t)\ldots(1-\mu_{p(\Gamma)})$, in which case she gets payoff Γ, weighted by her discount factor $\delta^{p(\Gamma)}$. Weighting her possible payoffs $(A+\delta tA/(1-\delta)), \delta(A+\delta tA/(1-\delta)), \ldots, \delta^{p(\Gamma)-1}(A+\delta tA/(1-\delta)), \delta^{p(\Gamma)}\Gamma$ by their probabilities and summing gives us the expression in the proposition. *Q.E.D.*

By Proposition 3, a customer's payoff upon being matched with her last firm is $Z[R/(1-\delta)]$, her payoff to being matched with her second-to-last firm is $Z[Z[R/(1-\delta)]]$, and so on. Letting $Z^k[\Gamma]$ denote the kth iteration of the function Z on continuation payoff Γ, we find $Z^k[R/(1-\delta)]$ as the value to a customer of being on her kth-to-last firm. Clearly, $Z[\Gamma] > \Gamma$ for any $\Gamma > 0$, so that expected payoff is increasing in the number of firms left to sample.

Propositions 2 and 3 describe a customer's strategy. For example, a customer with 7 firms left to sample has continuation payoff $Z^6[R/(1-\delta)]$, and therefore patience $p_7 = p(Z^6[R/(1-\delta)])$. Generally, since $p(\Gamma)$ is weakly decreasing, a customer is less patient the more firms there are serving the market, becoming gradually more patient as she samples firms unsuccessfully.

3.2 Firms

Choosing high effort is costly and confers no immediate benefit on a firm, but makes it more likely that firm will be able to identify itself as a good match to its probationary customers, which in turn leads to longer-tenured customers.[14] This section studies the trade-off between concerns for long-run customer retention and short-run profit.

Two state variables are relevant to the effort decision. First, an element of the vector $\vec{x} = [x_1, x_2, \ldots, x_{p_1}]$ is the measure of probationary customers who are well matched but who will nonetheless fire the firm after k bad outcomes, for $k = 1, \ldots, p_1$. The dimension of $\vec{x}$, which is $p_1 < \infty$, is determined endogenously as the patience level of the most patient customers a firm has: those on their last firm. Second, let m denote the measure of a firm's permanently matched customers. A third state variable describing the total measure of customers a firm has, $T = \vec{1}'\vec{x} + U + m$, where U denotes the total

[14] Playing a higher effort level has no effect on a firm's permanently matched customers, as they already believe their current match to be high quality with probability 1. However, effort is assumed to apply to all customers.

measure of poorly matched probationary customers, can be described in terms of $\vec{x}$ and m, as U is exogenous to a firm's effort decision. Finally, let each element of the vector $\vec{eta} = [\eta_1, \eta_2, \ldots, \eta_{p_1}]$ denote the measure of new customers arriving each period who have patience k. In a symmetric equilibrium, $\vec{eta}$ is determined by the effort levels of other firms.

A firm with state variables $(\vec{x}, m, T)$ chooses effort e to maximize its discounted infinite string of profits. A straightforward application of the contraction mapping theorem provides for the existence of a differentiable value function depending on state variables, following Stokey, Lucas, and Prescott (1989). Therefore, let $V[\vec{x}, m]$ equal the value to a firm of having state variables $(\vec{x}, m)$. Then, letting $\vec{x}'$ and m' represent one-period-ahead values, we have

$$
\text{(2)} \qquad V[\vec{x}, m] = \max_{e, \vec{x}', m'} T(\pi - c(e)) + \beta V[\vec{x}', m']
$$

$$
\begin{aligned}
\text{subject to} \quad m' &= m\delta + f(e)\delta \vec{1}'\vec{x}, \\
x'_k &= x_{k+1}\delta(1 - f(e)) + \mu\eta_k, \quad k = 1, 2, \ldots, p_1 - 1, \\
x'_{p_1} &= \mu\eta_{p_1}.
\end{aligned}
$$

A firm's per-period profit is $T(\pi - c(e))$; it receives a fee of π and pays a cost of $c(e)$ for each customer it has, and its total measure of customers is given by T. Customers that have patience level k in one period will have patience level $k - 1$ in the next period if they get a bad outcome and will be permanently matched if they get a good outcome. Thus x'_k, the measure of tomorrow's customers with patience level k, is given by $\delta(1 - f(e))x_{k+1}$, the measure of today's customers with patience level $k + 1$ who both get a bad outcome and survive, plus $\mu\eta_k$, the measure of well-matched new customers with patience k.

Assigning the first $p_1 + 1$ constraints, in order, multipliers $\theta_0, \theta_1, \ldots, \theta_{p_1}$, the first-order conditions of (2) are

$$
\begin{aligned}
e : c'(e)T &= f'(e)\delta \left(\theta_0 \vec{1}'\vec{x} - \sum_{k=1}^{p_1 - 1} \theta_k x_{k+1} \right), \\
m' : \beta V_{m'}[\vec{x}', m'] &= \theta_0, \\
x'_k : \beta V_{x'_k}[\vec{x}', m'] &= \theta_k \text{ for } k = 1, 2, \ldots, p_1 - 1.
\end{aligned}
$$

The envelope theorem gives the derivatives of the value function with respect to the state variables:

$$
\begin{aligned}
x_k : V_{x_k}[\vec{x}, m] &= \pi - c(e) + \theta_0 \delta f(e) + \theta_{k-1}\delta(1 - f(e)), \\
m : V_m[\vec{x}, m] &= \pi - c(e) + \theta_0 \delta,
\end{aligned}
$$

where $\theta_0 = 0$, reflecting the fact that the value to a firm of a customer with patience 1 who receives a bad signal is 0.

Combining the first-order condition for m' with the envelope condition for m gives us the recursive relationship $\theta_0 = \beta(\pi - c(e) + \delta\theta'_0)$, where θ'_0 is the one-period-ahead value of the multiplier θ_0. Repeatedly substituting this expression for θ_0 into the envelope condition for m gives us that

$$V_m[\vec{x}, m] = \frac{\pi - c(e)}{1 - \beta\delta},$$

and so the benefit to a firm of adding to its pool of matched customers is that it receives $\pi - c(e)$ from any matched customer for as long as that customer lives, given discount rate β and death probability $1 - \delta$. Similarly, the value to adding a well-matched customer with patience k is equal to V_m discounted by the probability that the customer will leave after receiving k bad outcomes:[15]

$$V_{x_k}[\vec{x}, m] = \frac{\pi - c(e)}{1 - \beta\delta}(1 - [\beta\delta(1 - f(e)]^k). \tag{3}$$

Combining the first-order condition for effort e and the above expressions for V_{x_k} and V_m produces an equation determining the steady-state effort that is free of Lagrange multipliers:

$$c'(e) = \frac{1}{T} f'(e)\beta\delta \frac{\pi - c(e)}{1 - \beta\delta}\left[\vec{1}'\vec{x} - \sum_{k=1}^{p_1 - 1} x_{k+1}\big(1 - [\beta\delta(1 - f(e))]^k\big)\right]. \tag{4}$$

If (4) holds, then the marginal benefit of effort e equals the marginal cost of effort e in the steady state, given that all other firms are also playing steady-state strategies of e and customer strategies are given by Proposition 2.

3.2.1 Stationary State Variables

In any stationary, symmetric equilibria, the state variables in the equality (4) can be calculated explicitly. First consider pure equilibria, in which customers do not mix over patience levels; allowing customers to mix is important mainly for the existence of equilibrium.

Let $\gamma(e) = \delta[1 - f(e)]$ describe the fraction of well-matched customers who both live and get a bad outcome each period. Let $a(p, e) = \mu\gamma(e)^p + (1 - \mu)\delta^p$ describe the fraction of customers arriving at a firm with patience p who will eventually fire that firm. Then, if there are N firms, we have $\eta_{p_N} = 1/N$, $\eta_{p_{N-1}} = \eta_{p_N} a(p_N, e)$, and so on. Generally,

$$\eta_{p_k} = \frac{1}{N} \prod_{l=k+1}^{N} a(p_l, e).$$

For fixed effort level e, $T_N(e)$ obeys the following dynamic:

$$\Delta T_N(e) = -(1 - \delta)T_N(e) + \sum_{k=1}^{N} \eta_{p_k} - \sum_{k=1}^{N} \eta_{p_k} a(p_k, e),$$

[15] In deriving equation (3) from the first-order conditions, we specify that state variables and effort are stationary, which anticipates the equilibrium condition that state variables must be in the steady state.

where $(1-\delta)T_N(e)$ is the measure of a firm's customers who die each period. $\sum_{k=1}^{N}\eta_{p_k}$ is the total measure of new customers a firm gets each period, some new to the market, some coming from other firms. $\sum_{k=1}^{N}\eta_{p_k}a(p_k,e)$ is the measure of customers each firm has who permanently fire that firm after an unsuccessful stay each period. Setting $\Delta T_N(e)$ to 0 yields

$$T_N(e) = \frac{\sum_{k=1}^{N}\eta_{p_k}(1-a(p_k,e))}{1-\delta}. \tag{5}$$

Equation (5) describes the total number of customers each of N firms has, given a steady state with effort e.

3.2.2 Marginal Benefit of Effort when $N = 1$: $mb(p,e)$

When $N = 1$, all customers have patience $p_1 = p(R/(1-\delta))$. We have $\eta_{p_1} = 1$ and $\eta_k = 0$ for all $k \neq 1$, as all new customers go to the market's one firm, leaving the market only after $p_1 B$ outcomes. The equilibrium state vector $\vec{x}$ has elements $x_{p_1-k} = \mu[\delta(1-f(e))]^k$ for $k = 0, 1, \ldots, p_1 - 1$.

Let $mb(p,e)$ refer to the marginal-benefit side of the firm's optimality condition (4) under constant effort, with one firm, and all customers having patience p. From equation (5), substitute $T = (1-a(p_1))/(1-\delta)$. With patience p,

$$\vec{1}'\vec{x} = \mu\frac{1-\gamma(e)^p}{1-\gamma(e)}, \quad \sum_{k=1}^{p-1} x_{k+1} = \mu\frac{1-\gamma(e)^{p-1}}{1-\gamma(e)},$$
$$\text{and} \quad \sum_{k=1}^{p-1} x_{k+1}[\beta\gamma(e)]^k = \mu\beta\gamma(e)^{p-1}\frac{1-\beta^{p-1}}{1-\beta}.$$

Substituting each of these into (4) gives

$$mb(p,e) = \frac{1-\delta}{1-a(p,e)}\mu f'(e)\beta\delta\frac{\pi-c(e)}{1-\beta\delta}\gamma(e)^{p-1}\frac{1-\beta^p}{1-\beta}. \tag{6}$$

The set of interior, nontrivial equilibria with $N = 1$ is then $\{e \in (0,E) : c'(e) = mb(p_1,e)\}$.

3.2.3 Marginal Benefit when $N \geq 2$ is a Weighted Average of $mb(p,e)$ Curves

With $N \geq 2$ firms in the market, customers sampling the kth-to-last firm have patience $p_k = p(Z^{k-1}(R/(1-\delta)))$, for $k = 1, 2, \ldots, N$.[16] Proposition 4 establishes that the marginal benefit to a firm when facing customers with different patience levels is a weighted average of the marginal benefit to facing each type of customer individually, with weights increasing in how many customers of that type a firm sees.

[16] Let $Z^0[x] = x$.

Proposition 4 *For any $N > 0$, the set of pure, interior, nontrivial equilibria is $\{e \in (0, E) : c'(e) = \sum_{k=1}^{N} \alpha_k mb(p_k, e)\}$, where $\sum_{k=1}^{N} \alpha_k = 1$.*

Proof With patience levels $p_1, p_2, \ldots, p_N$, we have $x_k = \mu \sum_{j=k}^{p_1} \eta_j \gamma(e)^{j-k}$. Thus,

$$\vec{1}'\vec{x} = \mu \sum_{j=1}^{N} \eta_{p_j} \frac{1-\gamma(e)^{p_j}}{1-\gamma(e)}, \quad \sum_{k=2}^{p_1} x_k = \mu \sum_{j=1}^{N} \eta_{p_j} \frac{1-\gamma(e)^{p_j-1}}{1-\gamma(e)},$$

$$\text{and} \quad \sum_{k=2}^{p_1} x_k (\beta\gamma(e))^{k-1} = \mu \sum_{j=1}^{N} \eta_{p_j} \beta\gamma(e)^{p_j-1} \frac{1-\beta^{p_j-1}}{1-\beta}.$$

Substituting these three equalities into the marginal-benefit side of (4) and solving yields a marginal benefit of

$$N \frac{1-\delta}{1-\prod_{k=1}^{N} a(p_k, e)} \mu f'(e) \beta\delta \frac{\pi - c(e)}{1-\beta\delta} \sum_{j=1}^{N} \eta_{p_j} \gamma(e)^{p_j-1} \frac{1-\beta^{p_j}}{1-\beta}, \tag{7}$$

which reduces to $\sum_{k=1}^{N} \alpha_k mb(p_k)$, where

$$\alpha_k = \frac{(1-a(p_k, e)) \prod_{l=k+1}^{N} a(p_l)}{1-\prod_{k=1}^{N} a(p_k, e)}.$$

It is direct that $\sum_{k=1}^{N} \alpha_k = 1$. *Q.E.D.*

3.3 Definition of Equilibrium

In the symmetric, stationary equilibria we solve for, state variables and effort choices are identical across firms and across time, and both firms and customers optimize:

Definition 1 *A symmetric, stationary equilibrium is an effort level $e \in [0, E]$, patience levels $\vec{p} = [p_1, p_2, \ldots, p_N]$, state variables $\vec{x} = [x_1, x_2, \ldots, x_{p_N}]$, m, U, and $\vec{eta}$, and a belief t for customers such that:*

(1) Each customer's patience level is optimal given belief t and outside option Γ, as described in Propositions 2 and 3.

(2) Firm effort e is optimal given state variables m and $\vec{x}$, and vector $\vec{eta}$. In particular, any interior equilibrium effort level e satisfies the firm's first-order condition given in equation (4).

(3) Effort e and state variables $\vec{x}$, m, U, and $\vec{eta}$ are stationary and symmetric across all N firms.

(4) Each customer's belief t is correct, i.e., $t = f(e)$.

An interior equilibrium is an equilibrium involving effort $e \in (0, E)$.

Throughout, we refer to equilibria in which customers are never indifferent over different patience levels and so do not mix as *pure* equilibria, and call those that involve customers mixing *mixed* equilibria.

4 Results

We now turn to analyzing the model and presenting results, beginning with two illustrative numerical examples. We then discuss the existence of equilibria, and comparative-statics results on N, δ, β, π, and A. The paper's main result, that firm effort is increasing in the number of firms because customers are less patient in more competitive markets, is discussed in section 4.3.1. The section concludes with a discussion of welfare in section 4.4.

4.1 Two Numerical Examples

Adopt the shorthand $mb_N = \sum_{k=1}^{N} \alpha_k mb(p_k, e)$ for the steady-state equilibrium marginal benefit of effort when there are N firms active, and $mc = c'(e)$ for the marginal cost of effort. Figure 1 illustrates equilibrium incentives, here for $N = 1$. All customers have patience p_1, which decreases from 14 to 11 over $e \in [2.8, 4]$ (the domain of the graph), where e is the steady-state effort level. Then mb_1, the bold line on the graph, equals $mb(14, e)$ for the leftmost portion of the graph, $mb(13, e)$ for the second part, and so on, with its discontinuities occurring where the optimal customer patience jumps. e_2 and e_3 are locations of equilibria; e_1, e_4, and all other points are not. If, for example, all customers play patience 12, the firm's profit-maximization problem (2) has a unique solution at e_3, and given firm effort e_3, customers optimally choose patience 12. On the other hand, if customers have patience 11, the firm's profit-maximization problem is solved by e_4; however, at e_4, customers optimally choose patience 12, and so e_4 cannot be the location of an equilibrium.

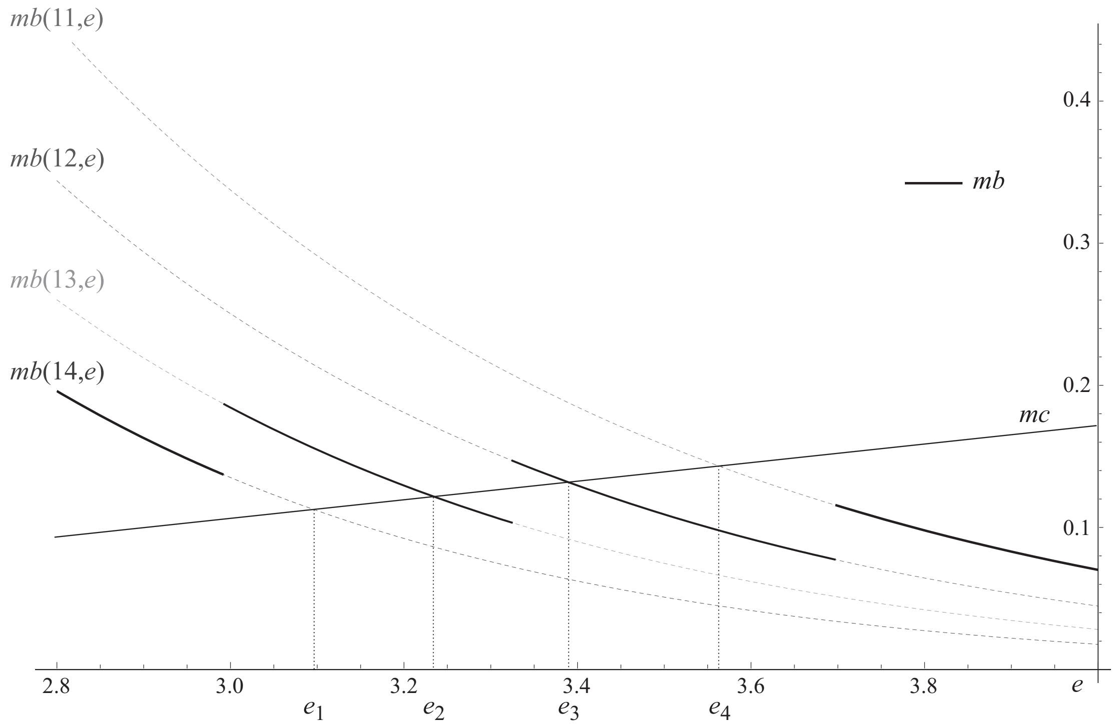
Figure 2 depicts mb_2 and mc for parameters $(\delta, \beta, \mu, \pi, A, R, E) = (0.99, 0.99, 0.4, 50, 20, 1, 10)$ and functions $f(e) = e/10$ and $c(e) = 0.09e^2$. The mb curve now is discontinuous both where p_1 changes and where p_2 changes. The discontinuities associated with changes in p_2 are relatively larger, and those associated with changes in p_1 relatively smaller, as there are more customers matched with their first firm than their second. Figure 2 has two effort levels where $mc = mb_2$: $e = 3.207$ and $e = 3.416$. The former has customers playing strategies $p_1 = 13$ and $p_2 = 9$, while the latter has $p_1 = 12$ and $p_2 = 8$. Both are locations of pure equilibria.

4.2 Existence of Equilibrium

A trivial equilibrium exists for any N, in which customers believe all firms to be worthless ($t = 0$) and thus never hire anyone and so are never proven wrong. In this equilibrium, customers believe that neither a good match nor a bad match would produce a good outcome, and therefore a customer's payoff to entering the market is 0 while her payoff to immediately taking her outside option is $R/(1-\delta) > 0$. Having no customers, playing 0 effort is clearly optimal for the firms, and so customer beliefs are correct.

Nontrivial equilibrium generically exists if customers are allowed to mix over patience levels over which they are indifferent and if a customer's payoff to a good outcome is high enough relative to her outside option. The latter condition ensures that a firm's marginal

Figure 1
The Marginal Benefit of Effort with $N = 1$ is Constructed from Individual $mb(p, e)$ Curves



benefit of effort is greater than its marginal cost for very low values of effort, while the former bridges the discontinuities in marginal benefit stemming from the discreteness of customer strategies.

Proposition 5, proven in the Appendix, gives sufficient conditions for the existence of a nontrivial equilibrium.

Proposition 5 *For any parameters* $\vec{\Psi} = (\delta, \beta, \mu, \pi, N, E)$, *there exists a constant* $K(\vec{\Psi})$ *such that if* $A/R > K(\vec{\Psi})$ *a nontrivial equilibrium exists. If, furthermore,* $c(E) \geq \pi$, *an interior equilibrium exists.*

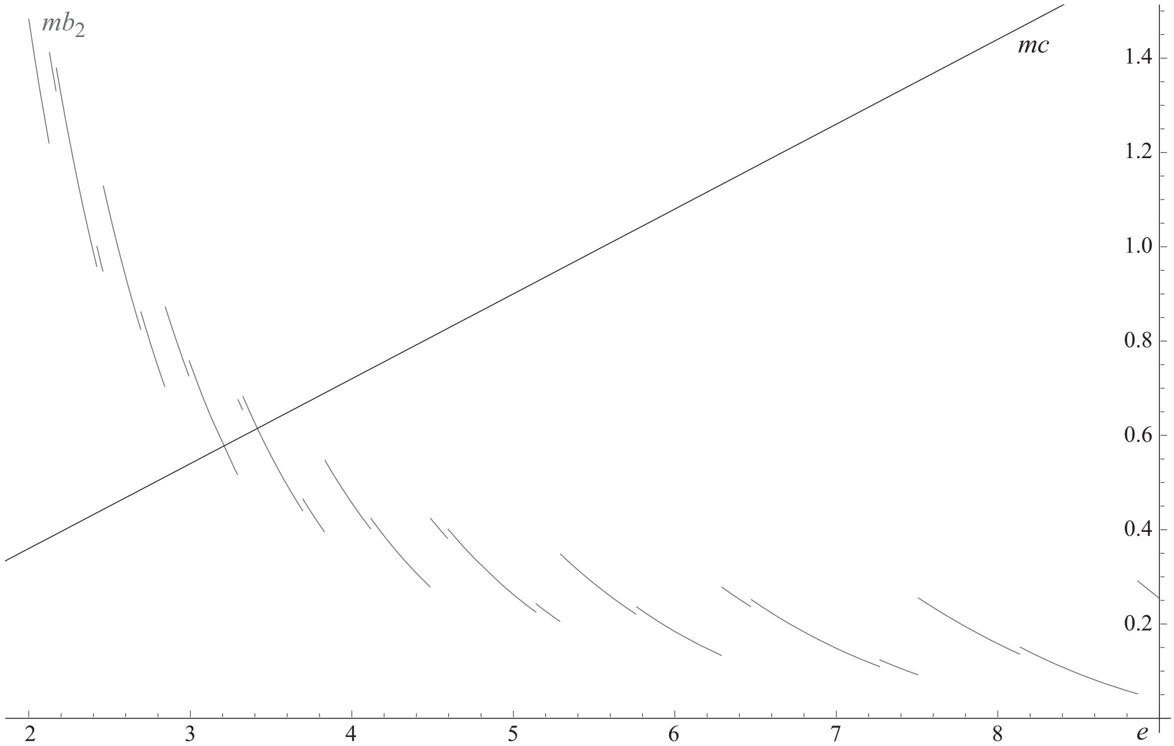
Owing to the nonmonotonic nature of a firm's equilibrium marginal benefit of effort, equilibria are not generally unique. For instance, the example of Figure 2 has two pure and one mixed equilibrium. It is generally true that equilibria are "close," i.e., clustered around one or two jumps in the equilibrium marginal benefit curve. The Appendix contains a more detailed discussion on existence of equilibria.

4.3 Comparative-Statics Results

Since multiple nontrivial equilibria are possible, we consider comparative-statics calculations under Definition 2.

Definition 2 *A parameter change increases (decreases) equilibrium effort if both the*

Figure 2
With $N = 2$ Firms, Marginal Benefit of Effort Jumps when Both p_1 and p_2 Change



Note: The jumps associated with changes in p_2 are relatively larger, since there are more customers at firm 2 than at firm 1.

minimum and maximum effort equilibrium increase (decrease) as a result of the parameter change.

4.3.1 Number of Firms: More Competitors Means More Effort

Our most important result is that firm effort is increasing in the number of firms in the market. This is because customers are less patient the more competitive the market is, which makes retaining customers relatively more urgent to firms, prompting them to exert more effort in doing so. We have the following general result.

Proposition 6 *Equilibrium effort is increasing in the number of firms, N.*

Proof We show that $mb_N = \sum_{k=1}^{N} \alpha_k mb(p_k, e)$ is increasing in N; the proposition then follows.

First, the function $mb(p, e)$ is decreasing in p. Direct inspection yields

$$mb(p,e) > mb(p+1,e) \iff$$
$$\Delta = \frac{1-\beta^p}{1-\mu\gamma(e)^p-(1-\mu)\delta^p} - \frac{\gamma(e)(1-\beta^{p+1})}{1-\mu\gamma(e)^{p+1}-(1-\mu)\delta^{p+1}} > 0.$$

We show that Δ is decreasing in β; as $\Delta|_{\beta=1} = 0$, clearly $\Delta \geq 0$ for all β, and thus $mb(p,e)$ is decreasing in p. Now $\partial\Delta/\partial\beta \leq 0$ if and only if

$$\text{(8)} \qquad \frac{p}{1-\mu\gamma^p-(1-\mu)\delta^p} \geq \frac{(p+1)\beta\gamma}{1-\mu\gamma^{p+1}-(1-\mu)\delta^{p+1}}.$$

(8) holds for all $\beta \in (0,1)$ if and only if it holds for $\beta = 1$, or if

$$\text{(9)} \qquad (1-(1-\mu)\delta^{p+1}) + \frac{1}{p}\mu\gamma^{p+1} - \frac{p+1}{p}\gamma(1-(1-\mu)\delta^p) \geq 0.$$

As the left-hand side of (9) is decreasing in γ, (9) holds for all $\gamma \in [0,\delta]$ if it holds for $\gamma = \delta$, or if

$$\text{(10)} \qquad \frac{1}{p}\delta^{p+1} - \frac{p+1}{p}\delta + 1 \geq 0.$$

Since the left-hand side of (10) is decreasing in δ, the fact that (10) holds for $\delta = 1$ therefore establishes that $\partial\Delta/\partial\beta \leq 0$. Since $\Delta|_{\beta=1} = 0$, we therefore establish the claim that $mb(p,e) > mb(p+1,e)$.

Second, via Propositions 3 and 2, when the number of firms increases from N to $N+1$, customer patience p_k is unchanged for $k \leq N$, while $p_N \geq p_{N+1}$. Therefore, from equation (6), $mb(p_k,e)$ is unchanged for $k = 1,2,\dots,N$, while $mb(p_{N+1},e) \geq mb(p_k,e)$ for $k = 1,2,\dots,N$.

Finally, it is direct that if the number of firms increases from N to $N+1$, the weights from Proposition 4,

$$\alpha_k = \frac{(1-a(p_k))\prod_{l=k+1}^N a(p_l)}{1-\prod_{k=1}^N a(p_k)},$$

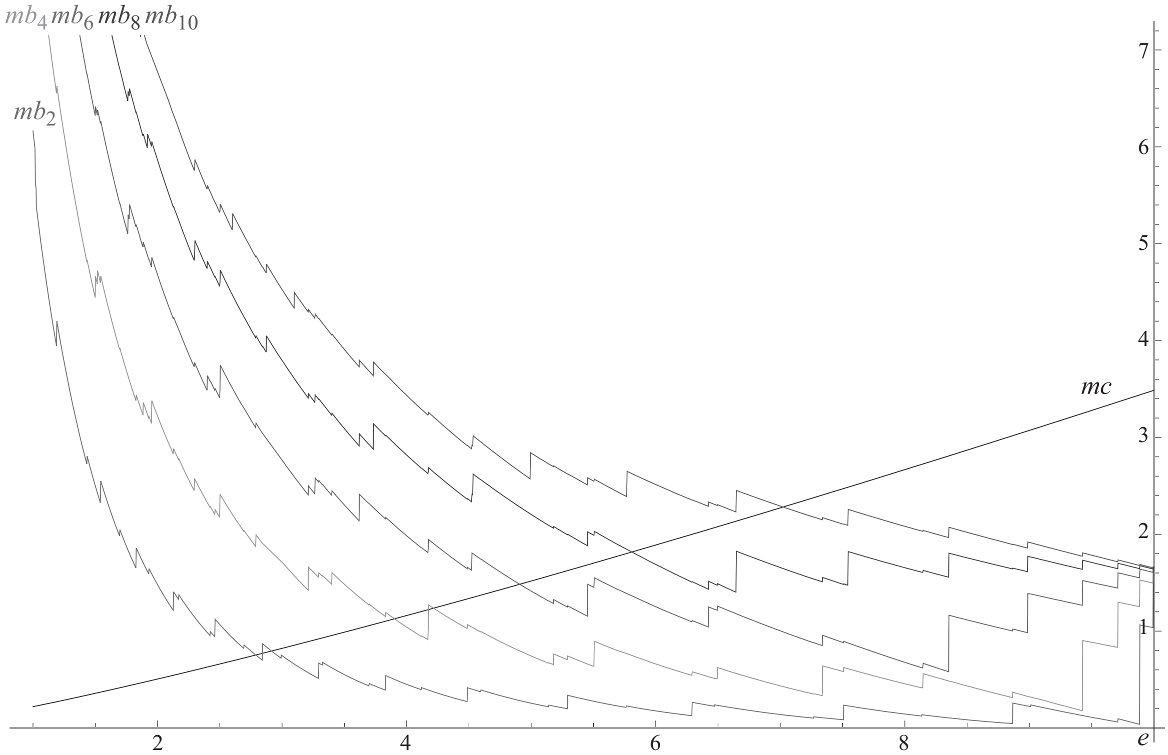
decrease for $k = 1,2,\dots,N$. That is, some of the weight assigned to $mb(p_k,e)$ in the weighted average mb_N is transferred to $mb(p_{N+1},e)$ for each $k = 1,2,\dots,N$. Given that $mb(p_{N+1},e) \geq mb(p_k,e)$ for $k = 1,2,\dots,N$, we have $mb_N < mb_{N+1}$.

Conclude that $mb_N = \sum_{k=1}^N \alpha_k mb(p_k,e)$ is increasing in N, and so equilibrium effort is also increasing in N. *Q.E.D.*

Increasing competition thus raises the marginal benefit to effort for firms. As the mc curve is upward sloping and continuous, shifting the mb_N curve upwards must then increase the effort level at the points of intersection between the two curves. Increasing N does not affect the decision of a customer to enter or not enter the market, so the full result is that if there is a nontrivial equilibrium when there are N firms, the equilibrium effort when there are $N+1$ firms is greater.

Example: Increasing Number of Firms Increases Effort. Figure 3 depicts a marginal cost curve and separate mb_N curves for $N \in \{2,4,6,8,10\}$. Discontinuities of the mb_N curves are connected with vertical line segments, allowing for customer mixtures. Model parameters are fixed at $(\delta,\beta,\mu,\pi,A,R,E) = (0.99, 0.99, 0.4, 50, 20, 1, 10)$, the probability

Figure 3
Equilibria for $N \in \{2, 4, 6, 8, 10\}$



Note: Competition increases equilibrium effort.

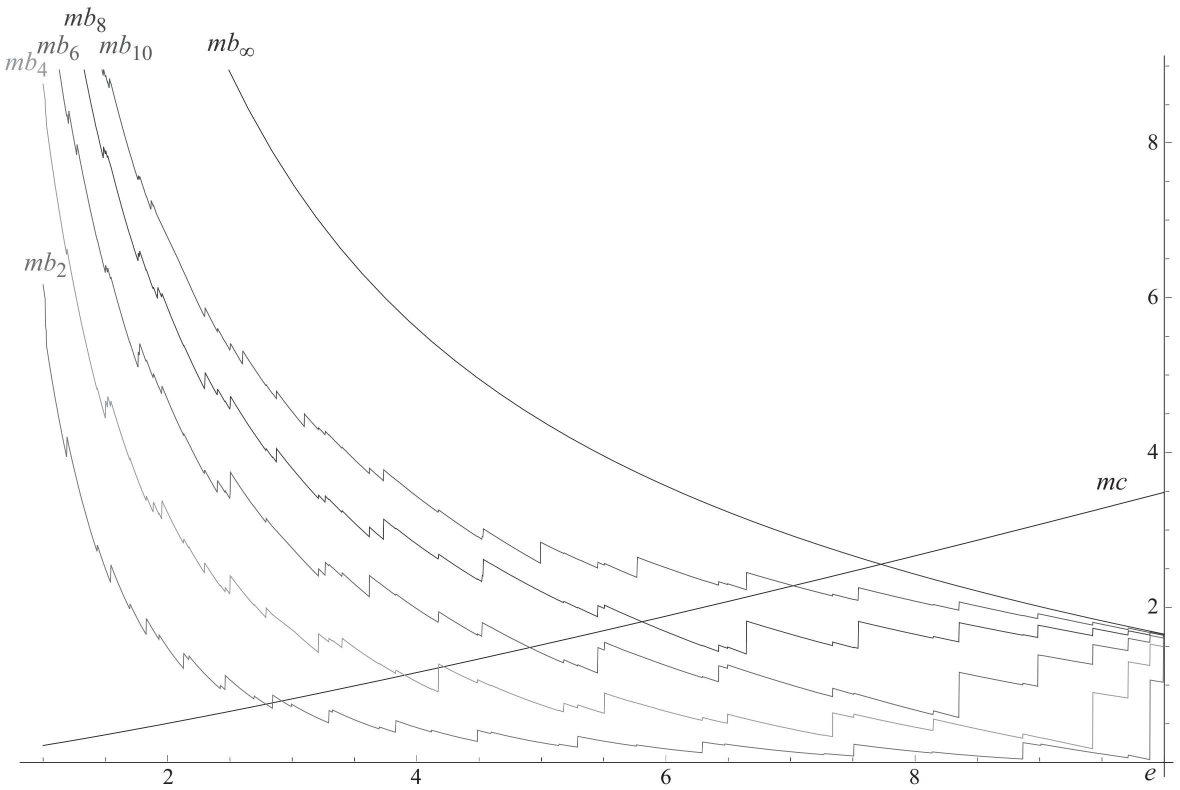
of a good outcome conditional on a good match is $f(e) = e/10$, and the cost function is here taken to be $c(e) = 0.1e^{2.2}$, so that the graphed marginal cost curve is given by $c'(e) = 0.22e^{1.2}$. In Figure 3, there are three equilibria when $N = 2$, located around $e = 2.8$, while the unique equilibrium for $N = 10$ is located at about $e = 7.05$.

Example: Competitive Limit. What happens in the limit as $N \to \infty$? We know $p_{k+1} \leq p_k$ for all $k \in \{1, 2, \ldots, N\}$, but $p_k \geq 1$ for any k. Indeed, as the number of firms increases, the measure of customers with a patience other than 1 decreases to zero. In the limit, all customers fire any firm unless they see a good outcome in the first period of the match. This creates an effective incentive for high effort for firms. Figure 4 redraws Figure 3, adding a marginal benefit curve mb_∞ for $N = \infty$. The sequence $\{mb_N(e)\}_{N=1}^{\infty}$ converges pointwise to mb_∞ as $N \to \infty$. The equilibrium in the limit is located at $e = 7.6$.

4.3.2 Discount Factors: Long-Lived Customers Erode Incentives for Effort

Customer patience as described in Proposition 2 is weakly increasing in survival probability δ. However, this fact alone is insufficient to say what happens to firms' mb curve and hence to equilibrium effort. For example, if $N = 3$, and customers have patience

Figure 4
Equilibria for $N \in \{2, 4, 6, 8, 10\}$ and in the Limit as $N \to \infty$



Note: With an infinite number of firms, all customers have patience 1 at every firm.

vector $\vec{p} = (p_1, p_2, p_3)$, should an increase in δ cause p_3 to increase by 1, it is ambiguous whether mb would increase or decrease. Increasing p_3 has the direct effect of making customers less patient, which lowers mb, and also has the indirect effect of lowering the proportion of a firm's customers who are on their second or third firm and hence are more patient; lowering this proportion increases mb. Either effect can dominate.

However, as δ increases to 1, $mb_N \to 0$ for all $e \in (0, E)$. From equation (7), mb_N is directly proportional to $1 - \delta$, which goes to 0 as $\delta \to 1$. Proposition 7 establishes that for high enough δ, the only equilibrium is located at $e = 0$.

PROPOSITION 7 *For any $R > 0$ and any parameters $(\beta, \mu, \pi, A, R, E, N)$, $\exists\ \delta^* < 1$ such that no nontrivial equilibria exist for $\delta > \delta^*$.*

PROOF From inspection of (7), $mb_N \to 0$ for any e as $\delta \to 1$. As $c(e) > 0$ for all e, δ can be made big enough that $mb_N < c'(e)$, and thus e is not an equilibrium location, for any e. *Q.E.D.*

As δ becomes large, customers become longer-lived, and so the measure of a firm's probationary customers relative to the same firm's matched customers in any equilibrium becomes vanishingly small. As firms do not care which outcome matched customers experience, their incentive to exert high effort is eroded.

Lowering customers' survival probability thus encourages higher effort. However, market participation involves an investment, in sampling firms until a good match is found. Customers sample even when they could get a higher payoff from their outside option. Specifically, new customers prefer market participation to their outside option if and only if

$$\mu t A + \mu t \frac{\delta(tA - R)}{1-\delta} \geq R. \tag{11}$$

From equation (11), a decrease in δ then raises the cutoff belief $t_{\min}$ of Proposition A1 below which customers choose their outside option over market participation, and thus lowering δ sufficiently can eradicate nontrivial equilibria entirely. A marginal decrease in δ, however, will not eliminate an equilibrium unless (11) holds with equality. That said, in many examples it is an intermediate level of δ that yields the highest effort equilibrium. Very high δ erode firm incentives; low δ prompt customers to forgo market participation.

Similarly, a firm's incentive to exert high effort increases as its discount factor β increases. Any positive effort level costs the firm $c(e)$ for each customer it has, confers no immediate benefit, yet sends a good outcome to some customers that will cause them to remain with the firm in the future until death. However, given that customers die each period with probability δ, letting β increase to one makes effort more attractive for firms and raises the mb curve, equation (7). Given $\delta < 1$, the effect is bounded.

4.3.3 Gains from Trade

In this section, we examine the relationship between the equilibrium firm effort and the parameters π and A, respectively the firms' payoff and customers' payoff to a good outcome.

π, Firms' Payoff. Equilibrium effort is increasing in both π, a firm's per-customer, per-period payoff, and A, a customer's payoff to getting a good outcome. From equation (7), mb_N increases at each $e \in (0, E)$, and so increasing π raises equilibrium effort, for fairly obvious reasons. If a nontrivial equilibrium exists, equilibrium effort approaches E as π increases, all else equal. If a nontrivial equilibrium does not exist, increasing π eliminates the type of failure of existence seen in Figure A1. If there is a nontrivial equilibrium located at $e < E$, the set of nontrivial equilibria will increase in π. These results are formalized in Proposition 8.

Proposition 8 *As π increases, equilibrium effort increases. If customers prefer market participation to their outside option for belief $t^* = f(e^*)$, then any $e \in (e^*, E)$ can be supported as an equilibrium if π is made large enough, all else equal.*

Proof From equation (7) in section 4.1, mb_N is increasing in π for any e at which customers prefer market participation to their outside option. At such an e, varying π

from 0 to ∞ varies mb_N from 0 to ∞, continuously, and so there exists some value of π such that $mb_N = c'(e)$. *Q.E.D.*

A, Customers' Payoff to a Good Outcome. Customer patience is weakly increasing in A; indeed, p_1 increases to ∞ as $A \to \infty$. However, for $k > 1$, p_k is bounded above in A. This is because when there remain unsampled firms, an increase in A increases both a customer's expected payoff to remaining with a firm and her payoff to switching to a new firm. On her last firm, a customer's continuation payoff is fixed at $R/(1-\delta)$, and so an increase in A only raises the expected value to remaining with that firm, causing patience to increase unboundedly. Proposition 9 states and proves our result.

PROPOSITION 9 *(1) p_1 increases in A unboundedly. (2) For $k > 1$, p_k is increasing in A but bounded above by a finite number.*

PROOF From Proposition 2,

$$p(\Gamma) = \left\lceil \left\lceil \frac{\log\left(\frac{\bar{\mu}(\Gamma)(1-\mu)}{\mu(1-\bar{\mu}(\Gamma))}\right)}{\log(1-t)} \right\rceil \right\rceil, \quad \text{where} \quad \bar{\mu}(\Gamma) = \frac{\Gamma(1-\delta)}{t\left(A + \frac{\delta t A}{1-\delta} - \delta\Gamma\right)}.$$

p depends on A only through

$$\frac{\bar{\mu}(\Gamma)}{1-\bar{\mu}(\Gamma)} = \left(\frac{t(A + \frac{\delta t A}{1-\delta})}{\Gamma(1-\delta)} - \frac{\delta t}{1-\delta} - 1 \right)^{-1} \tag{12}$$

and is decreasing in $\bar{\mu}(\Gamma)/(1-\bar{\mu}(\Gamma))$. Consider a customer on firm k, and adopt the shorthand $\bar{\mu}_k/(1-\bar{\mu}_k)$, where it is understood that $\Gamma_k = Z^{k-1}(R/(1-\delta))$. Per Proposition 3, Γ_k depends on A both directly and through the terms $p_1, p_2, \ldots, p_{k-1}$. First, note that outside of a set of measure zero, the terms $p_1, p_2, \ldots, p_{k-1}$ are invariant in A. Second, note that when a marginal increase in A causes p_j, $j < k$, to change by one, Γ_k is unchanged, as a customer is indifferent between these two adjacent patience levels per Proposition 2.

Therefore, the effect of a change in A on $p(\Gamma)$ can be computed directly from (12) and Proposition 3. Let

$$\lambda(p) = 1 + \frac{\sum_{k=1}^{p-1} \mu_k \prod_{j=0}^{k-1} \delta(1-\mu_j t)}{\mu},$$

and let

$$\tau(p) = \prod_{k=0}^{p} \delta(1-\mu_k t).$$

Substituting the value of Γ_k from Proposition 3 into equation (12) gives

$$\frac{\bar{\mu}_m}{1-\bar{\mu}_m} = \left(\left((1-\delta)\mu \sum_{k=1}^{m-1} \lambda(p_k) \prod_{j=k+1}^{m-1} \tau(p_j) + \frac{R}{t\left(A + \frac{\delta t A}{1-\delta}\right)} \prod_{k=1}^{m-1} \tau(p_k) \right)^{-1} - \frac{\delta t}{1-\delta} - 1 \right)^{-1}. \tag{13}$$

The only expression in (13) that varies in A is $R/(t(A+\delta tA/(1-\delta)))$, which is decreasing, which in turn implies that $\bar{\mu}_m/(1-\bar{\mu}_m)$ is decreasing in A and thus that p_k is increasing in A. Clearly, $\lim_{A\to\infty}\bar{\mu}_1/(1-\bar{\mu}_1)=0$, and so $\lim_{A\to\infty}p_1=\infty$. By the same logic, as $A\to\infty$, $\bar{\mu}_m/(1-\bar{\mu}_m)$ decreases, but is bounded away from 0 for $m\geq 2$. *Q.E.D.*

If $N=1$, increasing A will first decrease the effort played in nontrivial equilibria, eventually pushing any such equilibria to zero effort. However, if there is more than one firm, increasing A will have a smaller effect, with even large changes in A becoming irrelevant to the location of equilibria. For example, if A is increased from 20 to 2×10^{100} in the example of Figure 3, the unique nontrivial equilibrium with $N=8$ shifts from $e^*=5.8077$ in the former case to $e^*=5.8066$ in the latter. When $A=20$, $(p_2,\ldots,p_8)=(5,4,3,3,2,2,2)$ in both cases, while p_1 is 7 when $A=20$, and 270 when $A=2\times 10^{100}$.

4.4 Welfare

Increasing the number of firms increases the equilibrium effort level. Higher effort is unambiguously better for customers, and unambiguously worse for firms. Firms, in choosing effort, ignore the benefit of higher effort to customers, and therefore equilibrium effort may be below the efficient level. On the other hand, the reputational concerns compelling firms to exert themselves have, like advertising, no social value, and so equilibrium effort could be above the efficient level. The effect of increasing competition on total surplus then depends on which effect dominates.

Let $T_N^m(e)$ denote the measure of customers a firm has who are well matched; the total surplus per firm is given by $T_N^m(e)Af(e)+\pi-T_N(e)c(e)$. To examine the efficiency of an equilibrium e^*, consider, for that given $(T_N(e^*),T_N^m(e^*))$ pair, what effort level would maximize total surplus. If $f(e)=e/E$, the efficient effort $\hat{e}$ is given by $\hat{e}=c'^{-1}[AT_N^m(\hat{e})/ET_N(\hat{e})]$. Then $T_N^m(e)/T_N(e)=\mu(1+\epsilon)$, where $\epsilon\in(0,(1-\mu)/\mu)$ equals

$$\frac{(1-\mu)\sum_{k=1}^{N}(\delta^{p_k}-\gamma^{p_k})\prod_{l=k+1}^{N}a(p_l,e)}{1-\prod_{k=1}^{N}a(p_k,e)}.$$

Consider the example of section 4.1 and Figure 3; when $N=2$, one equilibrium is located at $e=2.92$. In equilibrium, $T_N^m(e)/T_N(e)=0.86$, and the corresponding efficient effort level is 0.82. Firms thus exert a higher effort than the efficient level in equilibrium; the same is true for the cases of $N=4,6,8,10$. Indeed, as $T_N^m(e)/T_N(e)$ can be no greater than 1, the location of the efficient effort level can be no greater than $c'^{-1}(A/E)$, or 0.86 under the example's parameters, and so increased competition decreases total surplus; the deadweight loss of firms competing on reputation dominates the gain customers get from improved outcomes.

However, for parameters $(\delta,\beta,\mu,\pi,A,R,E)=(0.99,0.994,0.4,20,75,1,10)$, when $N=2$, the equilibrium effort is 1.47 and the efficient effort would be 2.34; when $N=4$, the equilibrium effort is 2.20 and the efficient effort 2.55; when $N=6$, the equilibrium effort is almost exactly at the efficient level of 2.611. The equilibrium effort increases away from the efficient level as N increases from 6. Still other examples could be constructed where even the equilibrium effort with an infinite number of firms is inefficiently low; in these example, competition would uniformly increase the total surplus.

A, a customer's gain from a good outcome over a bad outcome, has only a negligible effect on the location of equilibria (see section 4.3.3), yet is crucial in determining the efficient effort. Indeed, raising (lowering) A sufficiently would destroy equilibria only in knife-edge cases and move them only slightly, while making any interior equilibrium effort level inefficiently low (high).

Finally, while searching has no explicit cost here, the increase in customer welfare resulting from an increase in the number of firms might be at least partially offset by the experimentation costs resulting from more frequent switching.

5 *Conclusion*

In a 2005 statement on the closing of its investigation into an allegedly anticompetitive market structure in the Vermont home health care industry, the U.S. Department of Justice (2005) claimed that "competition motivates providers to improve quality to attract customers and referral sources, invest in new technology, and train qualified staff." The investigation did uncover evidence suggesting that home health providers in Vermont, which operated in a virtually monopolistic environment, might not have provided the same quality of service as their counterparts in neighboring states with more competitive markets (see *Rutland Herald*, November 24, 2005).

This paper identifies proconsumer effects of competition amongst credence-good providers; it may help explain why Vermont home health customers experienced poor outcomes, and provide an additional argument for more lax policies on entering such markets. As the DOJ suggested, firms in markets with few competitors are less motivated to exert themselves in serving customers who have bleak prospects away from that firm, while firms in highly competitive markets try harder to impress customers who have little patience for poor results. As the number of firms in a market increases, the effort played in equilibrium increases as well. Thus, customers are served most diligently by experts in a competitive market.

This analysis suggests at least two empirical projects. One, if the model discussed here is relevant, then customers who describe themselves as dissatisfied with, say, their mechanic should be longer tenured in that relationship the less competitive their market is, whereas there should be no relationship between average tenure length and market competitiveness for satisfied customers. Two, in more competitive markets we expect greater firm effort, but less-patient customers. The relationship between average tenure with a firm and market competitiveness is thus ambiguous, and an empirical question. It could, in fact, be the case that markets in which competition among firms is high see relatively short average tenure among customers, as the effect of competition on customer impatience dominates that on higher firm effort.

Appendix: Existence of Equilibria

A.1 Proofs

Here we prove the results on the existence of equilibrium mentioned in the body of the paper. First, we allow customers to mix between patience levels if indifferent. Doing so is important for the existence of equilibria, as the firms' equilibrium marginal benefit of effort is discontinuous.

A.1.1 Mixed Equilibria

If $N = 2$ and at effort level $\hat{e}$ a fraction ζ of customers play (p_1, p_2) while a fraction $1-\zeta$ play $(p_1, \tilde{p}_2)$, then letting

$$M(p,e) = \mu f'(e)\beta\delta\frac{\pi - c(e)}{1-\beta\delta}\gamma(e)^{p-1}\frac{1-\beta^p}{1-\beta} = mb(p,e)\frac{1-a(p)}{1-\delta},$$

the marginal benefit to effort at $\hat{e}$ is

$$mb(\zeta) = \frac{(1-\delta)(\zeta M(p_2,\hat{e}) + (1-\zeta)M(\tilde{p}_2,\hat{e}) + (\zeta a(p_2) + (1-\zeta)a(\tilde{p}_2))M(p_1,\hat{e}))}{\zeta(1-a(p_1)a(p_2)) + (1-\zeta)(1-a(p_1)a(\tilde{p}_2))}.$$

Clearly, $mb(\zeta)$ ranges continuously from $\alpha(p_1)mb(p_1,\hat{e})+\alpha(p_2)mb(p_2,\hat{e})$ to $\alpha(p_1)mb(p_1,\hat{e})+\alpha(\tilde{p}_2)mb(\tilde{p}_2,\hat{e})$, and thus spans the discontinuity.

A.1.2 Existence of Nontrivial Equilibria

Interior equilibria lie at the intersection of the mb_N and mc curves; the former is decreasing and continuous in effort except at a finite number of points, which mixed strategies effectively bridge; the latter is increasing and continuous in effort. It is obvious from inspection that if $p_k > 0$ for $k \in \{1, 2, \ldots, N\}$, then $mb_N > 0$ for low values of e while $mc(0) = 0$, and it is easy to parameterize the model so that $mb_N|_{e=E} \leq 0$ and $c'(E) > 0$. However, it may be the case that for low levels of firm effort customers prefer not to enter the market, instead taking their outside option in each period. $mb_N = 0$ for any effort level at which customers do not participate in the market.

Specifically, there exists some number $t_{\min}$ such that customers enter the market for beliefs $t \in (t_{\min}, 1]$ and opt out for $t \in [0, t_{\min}]$. Then, if $e_{\min} = f^{-1}(t_{\min})$, the mb_N curve will take the value 0 over the range $[0, e_{\min}]$, and the mb_N curve may or may not lie above the mc curve at $e_{\min}$. If, however, $e_{\min}$ is very close to 0, then at $e_{\min}$ we have $mb_N > mc$, and thus firms find it optimal to invest in at least some effort. Proposition A1 provides for the existence of this cutoff belief $e_{\min}$.

Proposition A1 *For any model parameters $(\delta, \beta, \mu, A, \pi, R, N, E)$, there exists some $t_{\min} \in (0, 1]$ such that if customers have a belief $t > t_{\min}$, they optimally choose to enter the market, while for beliefs $t \leq t_{\min}$ not entering is optimal.*

Proof From Proposition 1 in section 3.1, customers prefer market participation to their outside option if and only if $W[1,\mu,\Gamma] \geq \Gamma$. The "if" direction is obvious; to see the "only if" direction, a direct calculation yields $W[k,\mu,\Gamma] \geq W[k-1,\mu,\Gamma] \Rightarrow W[k-1,\mu,\Gamma] \geq W[k-2,\mu,\Gamma]$, which in turn implies that if $k > 1$ is the optimal patience, then $W[1,\mu,\Gamma] > \Gamma$. From Proposition 1,

$$W[1,\mu,\Gamma] \geq \Gamma \iff \mu > \frac{R}{t\left[A + \frac{\delta t A}{1-\delta} - \frac{\delta R}{1-\delta}\right]} \iff$$

$$A\left(\mu t + \frac{\mu\delta t^2}{1-\delta}\right) - R\left(\frac{\mu\delta t}{1-\delta} + 1\right) > 0. \tag{A1}$$

Direct calculation gives that the derivative of the left-hand side of (A1) with respect to t is given by $A(\mu + 2t\mu\delta/(1-\delta)) - R\mu\delta/(1-\delta)$, which is greater than zero if $A(\mu + t\mu\delta/(1-\delta)) - R\mu\delta/(1-\delta) > 0$, a condition implied by (A1). Therefore, if a belief t merits entering the market, then so does any belief $t' > t$. It is obvious that for very low beliefs, not entering is optimal; the proposition follows. *Q.E.D.*

Equilibria may not exist if $mb_N|_{e=e_{\min}} < c'(e_{\min})$ and the mb_N curve lies everywhere below the mc curve. Figure A1 gives an example in which this happens. If $mb_N|_{e=e_{\min}} > c'(e_{\min})$, as is guaranteed if $e_{\min}$ is sufficiently low, a nontrivial equilibrium does exist. However, if $mb_N|_{e=E} > c'(E)$, there may fail to be an interior equilibrium. Figure A2 gives such an example. Here, there is a nontrivial equilibrium, at $e = E$, but it does not solve the firm's problem as described in previous sections.

Figure A1
No Nontrivial Equilibrium; Customers Enter the Market Only if $e > 0.42$

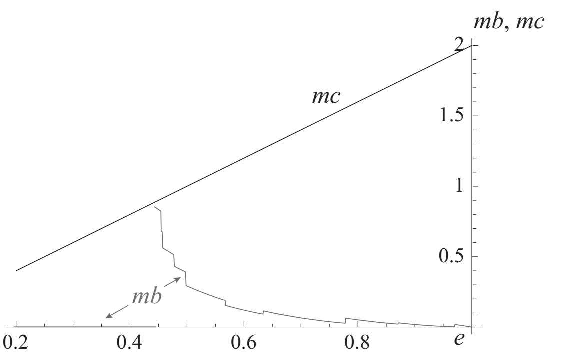
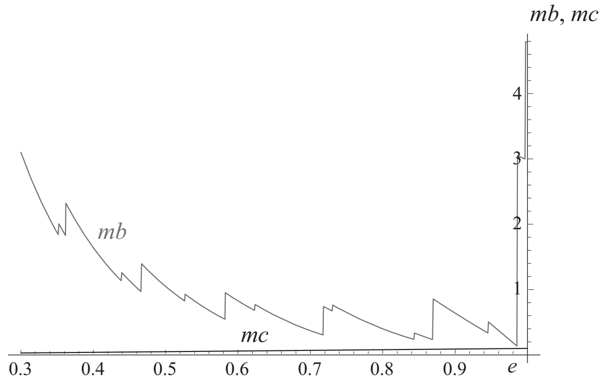


Figure A2
No Interior Equilibrium



In light of Figures A1 and A2, Proposition 5 says that if the good outcome achievable in the market (A) is high enough relative to the customers' outside option (R), a nontrivial equilibrium exists. If, also, the cost curve is sufficiently steep to make very high levels of effort unattractive to the firms, the solution is interior.

Proposition A2 (Proposition 7 as a corollary to Proposition A1) *For any parameters $\vec{\Psi} = (\delta, \beta, \mu, \pi, N, E)$, there exists a constant $K(\vec{\Psi})$ such that if $A/R > K(\vec{\Psi})$ a nontrivial equilibrium exists. If, furthermore, $c(E) \geq \pi$, an interior equilibrium exists.*

PROOF From Proposition A1, customers prefer market participation to their outside option if equation (A1) holds. Thus, for any $t > 0$, customers enter if A/R is high enough; if customers enter, $mb_N > 0$. Because $mc \to 0$ as $e \to 0$, there is some $\tilde{e} > 0$ such that $mb_N|_{\tilde{e}} > mc(\tilde{e})$ if customers enter. The fact that $c(E) \geq \pi$ ensures that $mb_N|_{e=E} \leq 0$, as the marginal benefit is directly proportional to $\pi - c(e)$. *Q.E.D.*

Equilibria may still exist even for low A/R, particularly if effort is not very costly to firms. Other existence results could be derived in which discount factors and the firms' gains to trade are considered.

A.2 Customers are Allowed to Return to Firms

This section resolves the paper's main model while relaxing the assumption that it is prohibitively costly to return to an already fired firm. The results of this section are used in the example of section 2.2.1.

Suppose there are two firms, A and B, and that all new customers have patience 2. Let p_A describe the set measure of customers who are a good match for firm A and who first sample firm A, let p_{AB} be the measure of probationary good matches for firm A who first sampled A and are now at B, and so on. The problem of firm A is then

$$V[p_A, p_B, p_{AB}, p_{BA}, p_{ABA}, p_{BAB}, p_{BABA}, m] = \max_{e, p'_{AB}, p'_{BAB}, m'} T(\pi - c(e)) + \beta V[\cdot']$$

$$\text{subject to} \quad \begin{aligned} p'_A &= \frac{1}{2}\mu, \\ p'_B &= \frac{1}{2}\mu, \\ p'_{BA} &= p_B \delta(1 - \mu f(e_B)), \\ p'_{AB} &= p_A \delta(1 - f(e)), \\ p'_{BAB} &= p_{BA} \delta(1 - f(e)), \\ p'_{ABA} &= p_{AB} \delta(1 - \mu f(e_B)), \\ p'_{BABA} &= p_{BAB} \delta \frac{\mu - \mu f(e_B)}{1 - \mu f(e_B)}(1 - f(e_B)), \\ m' &= f(e)\delta(p_A + p_{BA} + p_{ABA} + p_{BABA}). \end{aligned}$$

Assigning Lagrange multipliers λ, η, and θ for the fourth, fifth, and eighth constraints, respectively (the other five constraints are irrelevant to the firm's problem), we get the following first-order conditions:

$$\text{(A2)} \qquad \begin{aligned} e : c'(e) &= \frac{1}{T}\delta f'(e)(\theta(p_A + p_{BA} + p_{ABA} + p_{BABA}) - \lambda p_A - \eta p_{BA}), \\ m' : \beta V_m[p'_{2.5}, p'_2, p'_{1.5}, p'_1, m'] &= \theta, \\ p'_{AB} : \beta V_{p_{AB}}[\cdot'] &= \lambda, \\ p'_{BAB} : \beta V_{p_{BAB}}[\cdot'] &= \eta. \end{aligned}$$

Repeated substitution gives us the values for V_m, $V_{p_{AB}}$, and $V_{p_{BAB}}$ under the assumption of steady-state values of effort of e_A^* (own) and e_B^* (rival):

$$\begin{aligned}
V_m &= \frac{\pi - c(e_A^*)}{1-\beta\delta} = \frac{\theta}{\beta}, \\
V_{p_{AB}} &= \beta\delta(1-\mu f(e_B^*))\left(\pi - c(e_A^*) + \beta\delta f(e_A^*)\frac{\pi - c(e_A^*)}{1-\beta\delta}\right) = \frac{\lambda}{\beta}, \\
V_{p_{BAB}} &= \beta\delta\frac{1-2\mu f(e_B^*) + \mu f(e_B^*)^2}{1-\mu f(e_B^*)}\left(\pi - c(e_A^*) + \beta\delta f(e_A^*)\frac{\pi - c(e_A^*)}{1-\beta\delta}\right) = \frac{\eta}{\beta}.
\end{aligned}$$

Reasoning similar to that of section 3.2.1 gives us steady-state values of T, p_A, p_{BA}, p_{ABA}, and p_{BABA}, under the assumption that $e_A^* = e_B^* = e^*$:

$$\begin{aligned}
p_A^* = p_B^* &= \frac{\mu}{2}, \\
p_{BA}^* = p_{AB}^* &= \frac{\mu}{2}\delta(1-\mu f(e^*)), \\
p_{ABA}^* = p_{BAB}^* &= \frac{\mu}{2}\delta^2(1-\mu f(e^*))(1-f(e^*)), \\
p_{BABA}^* = p_{ABAB}^* &= \frac{\mu}{2}\delta^3(1-f(e^*))(1-2\mu f(e^*) + \mu f(e^*)^2), \\
T^* &= \frac{1}{2(1-\delta)}(1-\delta^4(1-2\mu f(e^*) - \mu f(e^*)^2)^2).
\end{aligned}$$

Substituting the previous eight equations into (A2) gives a necessary condition for equilibrium effort. Under the parameterization in section 2.2.1, it is direct to calculate that $e^* = 5$ is an equilibrium.

A.3 Three Firms

Now suppose that new customers choose one of three firms at random, and a customer who never gets a good outcome cycles through the remaining two firms, returns to her original firm, and then cycles through the two remaining firms again, and so on. In the example of section 2.2.1, customers still have patience 2. Calling the remaining two firms B and C, firm A's problem is

$$\begin{aligned}
V[\cdot] &= \max_{e, p'_{AB}, p'_{BAB}, m'} T(\pi - c(e)) + \beta V[\cdot'] \\
\text{subject to} \quad p'_{AB} &= p_A \frac{1}{2}\delta(1-f(e)), \\
p'_{AC} &= p_A \frac{1}{2}\delta(1-f(e)), \\
p'_{CAB} &= p_{CA}\delta(1-f(e)), \\
p'_{BAC} &= p_{BA}\delta(1-f(e)), \\
m' &= f(e)\delta(p_A + p_{BA} + p_{CA} + p_{BCA} + p_{CBA} + p_{ABCA} \\
&\quad + p_{ACBA} + p_{BACBA} + p_{CABCA} + p_{BCABCA} + p_{CBACBA}).
\end{aligned}$$

Let λ_i be the Lagrange multiplier associated with the ith constraint, $i = 1, 2, 3, 4$, and let θ be the multiplier associated with the fifth constraint. Constraints that are orthogonal to the firm's problem are omitted. We get the following first-order conditions:

$$\text{(A3)} \qquad \begin{aligned} e : c'(e) = \frac{1}{T}\delta f'(e)\Big(&\theta(p_A + p_{BA} + p_{CA} + p_{BCA} + p_{CBA} + p_{ABCA} + p_{ACBA} \\ &+ p_{BACBA} + p_{CABCA} + p_{BCABCA} + p_{CBACBA}) \\ &- \lambda_1 \frac{p_A}{2} - \lambda_2 \frac{p_A}{2} - \lambda_3 p_{CA} - \lambda_4 p_{BA}\Big), \end{aligned}$$

$$\begin{aligned} m' &: \beta V_m[\cdot'] = \theta, \\ p'_{AB} &: \beta V_{p_{AB}}[\cdot'] = \lambda_1, \\ p'_{AC} &: \beta V_{p_{AC}}[\cdot'] = \lambda_2, \\ p'_{CAB} &: \beta V_{p_{CAB}}[\cdot'] = \lambda_3, \\ p'_{BAC} &: \beta V_{p_{BAC}}[\cdot'] = \lambda_4. \end{aligned}$$

In a symmetric steady state ($e^*_A = e^*_B = e^*_C = e^*$), we have the following values of state variables and Lagrange multipliers:

$$\begin{aligned} V_m &= \frac{\pi - c(e^*_A)}{1 - \beta\delta} = \frac{\theta}{\beta}, \\ V_{p_{AB}} &= \beta^2\delta^2(1 - \mu f(e^*))^2 \left(\pi - c(e^*_A) + \beta\delta f(e^*) \frac{\pi - c(e^*)}{1 - \beta\delta}\right) = \frac{\lambda_1}{\beta}, \\ V_{p_{AC}} &= \beta^2\delta^2(1 - \mu f(e^*))^2 \left(\pi - c(e^*) + \beta\delta f(e^*) \frac{\pi - c(e^*)}{1 - \beta\delta}\right) = \frac{\lambda_2}{\beta}, \\ V_{p_{CAB}} &= \beta^2\delta^2(1 - 2\mu f(e^*_C) + \mu f(e^*_C)^2) \left(\pi - c(e^*_A) + \beta\delta f(e^*_A) \frac{\pi - c(e^*_A)}{1 - \beta\delta}\right) = \frac{\lambda_3}{\beta}, \\ V_{p_{BAC}} &= \beta^2\delta^2(1 - 2\mu f(e^*_C) + \mu f(e^*_C)^2) \left(\pi - c(e^*_A) + \beta\delta f(e^*_A) \frac{\pi - c(e^*_A)}{1 - \beta\delta}\right) = \frac{\lambda_4}{\beta}, \\ p^*_A &= \frac{\mu}{3}, \\ p^*_{BA} &= \frac{\mu}{3}\delta(1 - \mu f(e^*)), \\ p^*_{CA} &= \frac{\mu}{3}\delta(1 - \mu f(e^*)), \\ p^*_{BCA} &= \frac{\mu}{3}\delta^2(1 - \mu f(e^*))^2, \\ p^*_{CBA} &= \frac{\mu}{3}\delta^2(1 - \mu f(e^*))^2, \\ p^*_{ABCA} &= \frac{\mu}{3}\delta^3(1 - \mu f(e^*))^2(1 - f(e^*)), \\ p^*_{ACBA} &= \frac{\mu}{3}\delta^3(1 - \mu f(e^*))^2(1 - f(e^*)), \\ p^*_{BACBA} &= \frac{\mu}{3}\delta^4(1 - \mu f(e^*))(1 - 2\mu f(e^*) + \mu f(e^*)^2)(1 - f(e^*)), \end{aligned}$$

$$p^*_{CABCA} = \frac{\mu}{3}\delta^4(1-\mu f(e^*))(1-2\mu f(e^*)+\mu f(e^*)^2)(1-f(e^*)),$$
$$p^*_{BCABCA} = \frac{\mu}{3}\delta^5(1-2\mu f(e^*_B)+\mu f(e^*_B)^2)^2(1-f(e^*)),$$
$$p^*_{CBACBA} = \frac{\mu}{3}\delta^5(1-2\mu f(e^*_B)+\mu f(e^*_B)^2)^2(1-f(e^*)),$$
$$T^* = \frac{1}{3(1-\delta)}(1-\delta^6(1-2\mu f(e)-\mu f(e)^2)^3).$$

Substituting the previous 17 equalities into equation (A3) yields a necessary condition for equilibrium effort. Once again, given the parameterization in section 2.2.1, it is direct to calculate that $e^* = 6.55$ is an equilibrium effort level. Finally, we confirm that with $e = 6.55$, all customers continue to have patience 2.

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Jeremy Sandford
Gatton College of
Business and Economics
University of Kentucky
Lexington, KY 40506
U.S.A.
jeremy.sandford@uky.edu