Familiarity and decision making: The unclear role of noise in accept/reject decisions

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Abstract

A decision maker observes a noisy signal of the quality of a project before deciding to accept or reject the project. We show 1- As the amount of noise increases, the minimum signal required for acceptance may either increase or decrease, and may be nonmonotonic. 2- Consequently, the average quality of accepted projects may either increase or decrease in the amount of noise. 3- The effect of increased noise on decisions depends in a straightforward way on which kind of mistake leaves the decision maker worse off, a rejection of a good project or an acceptance of a bad project.

Keywords: Asymmetric information, noise (JEL: C7, D8, L1)

1 Introduction

A decision maker decides whether to accept or reject a proposal based on noisy information about the proposal's merits. For example, a C.E.O. decides which capital budgeting proposals to fund, and which to reject. A consumer decides whether or not to agree to an expensive car repair recommended by his mechanic. A quality control manager decides whether or not the available evidence merits a recall. In each case, the decision maker has some, but not all, of the information relevant to his decision. This paper investigates the relationship between the amount of noise, or unknown information, and decision making. Specifically, we show that there is not necessarily *any* straightforward

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relation between noise and decision making. As noise increases, the decision maker may become more lenient, tougher, or may even vary his behavior non-monotonically. Consequently, our results predict that, for example, a C.E.O. may allocate either more or less capital towards his old division (whose proposals are observed with less noise). A quality control manager may become either more or less likely to issue a product recall as the quality of his information decreases. Either may behave non-monotonically as noise increases.

Anecdotal evidence indeed suggests a complicated relationship between noise and decision making. For example, on August 16, 2013, the Philadelphia Phillies fired Charlie Manuel, the winningest coach in team history. According to G.M. Ruben Amaro, the reason was that "this gives us a chance to see what we have in (new manager) Ryne Sandberg and see what he can do as manager of the Phillies."[1] In this case, more noise (the unproven Sandberg) was preferable to less (the veteran Manuel). Similarly, in 2011, J.C. Penney stockholders greeted the news of Apple's Ron Johnson's hiring as C.E.O. with enthusiasm, causing the stock's price to jump 17.5% the day he was hired, and another 24% upon his announcement of a transformative new retail strategy. That he was fired only 17 months later suggests the reaction to his hiring indicated a preference for noise rather than a belief he was supremely competent. Democratic congressman Gene Taylor, in explaining his vote for John McCain over Barack Obama in the 2008 election, offered "Better the devil you know" as an explanation,[2] apparently willing to choose a candidate farther from his own beliefs in return for lower noise. Our model directly addresses the inconsistent relationship between the amount of noise and decision making in these examples, by showing that more noise may be either an inducement or a deterrent in accept/reject decisions.

A simplified version of our model conveys the main idea of our paper. Suppose that a C.E.O. must accept or reject projects of two types, A and B. Suppose that any project will generate a profit drawn from a $U[-2/3, 1/3]$ distribution, regardless of its type. The C.E.O. wishes to accept profitable projects, and reject unprofitable projects. Finally, suppose that the C.E.O. is an expert on type A projects, and so can observes a signal which is strongly correlated with the true quality. Suppose further that he knows little about type B projects, and thus observes a signal which is weakly correlated with true quality. Now, consider the limiting cases: a type A projects signal is perfectly correlated with quality, while a type B project's signal has zero correlation. In this case, he will accept $1/3$ of type A projects (those with quality above 0), while rejecting all of the type B projects, regardless of signal. Therefore, even though his prior is that the two types of projects are of equal quality, he accepts more type A projects because he can observe their true qualities with greater accuracy. However, if the profit distribution were instead $U[-1/3, 2/3]$, then the C.E.O. would accept all type B projects,

[1] www.sportingnews.com/mlb/story/2013-08-16/phillies-manager-charlie-manuel-reported-to-be-fired

[2] thehill.com/blogs/ballot-box/house-races/125693-democratic-rep-taylor-says-he-voted-for-mccain-not-obama

while accepting only $2/3$ of type A projects. In this case, he rejects more type A projects precisely because he can observe their true qualities with greater accuracy.

The relationship between degree of information asymmetry and decision making has not gone unnoticed by the literature. Glaser et al. (2013) and Duchin and Sosyura (2013) empirically find that C.E.O.'s allocate capital disproportionately to managers with whom they have informal connections. Glaser et al. (2013) finds that this practice results in inefficient allocation of capital and therefore lower profits, whereas Duchin and Sosyura (2013) find a more mixed result. Specifically, the latter paper finds that the effect of this apparent favoritism is deleterious to profits in certain types of firms (those with weak governance, low manager ownership, and low institutional holdings), yet increases profits at other types of firms (firms with high variance in analysts' earnings forecasts, indicating high information asymmetry). The interpretation in Duchin and Sosyura (2013) of this dichotomy is that the former type of firm exhibit simple favoritism (for example, C.E.O.'s helping out their friends), while the latter type of firm benefit from what they term the "information hypothesis," the idea that informal connections between C.E.O.'s and managers lead to managers doing a better job of evaluating which projects are promising. We offer another, non-mutually exclusive explanation for the apparent variation in C.E.O. behavior, which is that there is no clear relationship between the amount of noise and the selectiveness of a decision maker, and therefore no clear relationship between noise of capital budgeting decisions and profits should be expected.

Both Duchin and Sosyura (2013) and Glaser et al. (2013) ably summarize a body of theoretical literature which considers settings in which information asymmetry within a firm introduces frictions in capital allocation (cf. Meyer et al. (1992), Bernardo et al. (2001), and Wulf (2009)). These papers generally predict that divisions with greater information asymmetries receive a lower amount of capital. Our contribution to the this literature is to point out that the amount of investment may increase or decrease in information accuracy, a result that has heretofore gone unnoticed in the literature.

Apart from capital budgeting, several empirical papers examine the effect of information asymmetry on decision making. French and Poterba (1991) find that investors prefer stocks from their home country to foreign countries. Levin (2001) finds that greater information asymmetries may increase or decrease the gains from trade. In scientific publishing, journal editors disproportionately publish papers authored by colleagues and former graduate students (Bardhan (2003)). Ang et al. (2013) find that C.E.O.'s favor segments they are more familiar with when making divestment decisions and offer evidence that the reason for this apparent favoritism is that C.E.O.'s are more informed about these segments. Our paper characterizes conditions under which noise negatively and positively influences decision making.

2 Model and results

A decision maker makes accept/reject decisions on projects. Project quality q is unobservable *ex ante*, but DM observes a noisy signal s which is correlated with q. Specifically, assume that $s \sim N(q, z\epsilon^2)$, where $z > 0$ is a measure of the noise of the signal. For example, a C.E.O. may have more accurate information about the true quality of proposed projects that come out of his old division than he does about those outside his area of expertise. Similarly, a referee of an academic journal observes quality with relatively lower noise the closer a paper is to his own research. A ISO certification body may observe the performance of companies with well-defined, structured tasks with less noise than those with many workers with non-specific responsibilities. A quality control manager samples output, deciding whether or not to recall an entire batch of product.

Suppose that, prior to observing the signal s, DM's belief is that that quality of any project is drawn from the following distribution:[3]

$$\text{DM's prior belief: } q \sim N(0, \epsilon^2) \tag{1}$$

Assume there is a threshold q_0, such that for $q \geq q_0$, DM prefers to accept projects, while for $q < q_0$, DM prefers rejection. Specifically, suppose that DM's objective is to minimize the expected value of the following loss function:

$$L(\text{accept}) = \begin{cases} 0 & \text{if} \quad q \geq q_0 \\ a & \text{if} \quad q < q_0 \end{cases} \qquad L(\text{reject}) = \begin{cases} 1 & \text{if} \quad q \geq q_0 \\ b & \text{if} \quad q < q_0 \end{cases} \tag{2}$$

where $a > b$ (if $b \geq a$, DM will always accept). The loss function in (2) assumes that DM does not care about the magnitude of $q - q_0$, the amount by which a given project exceeds or falls behind the threshold q_0 (this case is addressed in section 3). Within the class of loss functions with this property, (2) is without loss of generality.[4] In particular, parameters a and b measure the relative loss from to two types of error: accepting a bad project and rejecting a good project. Let $x = a - b$, or

$$x = L(\text{accept}|q < q_0) - L(\text{reject}|q < q_0) \tag{3}$$

If $x > 1$ ($x < 1$), this means that DM incurs a relatively greater (lesser) loss from accepting a bad project (rejecting a good project). As DM's decision is binary, his accept/reject decision depends

[3]Assuming a zero mean is without loss of generality. If the mean of the prior distribution is $\mu > 0$, then q_0 should be understood as $q_0 - \mu$.

[4]To normalize a loss function with four parameters, subtract $L(\text{accept}|q \geq q_0)$ from all values and divide all values by $L(\text{reject}|q \geq q_0) - L(\text{accept}|q \geq q_0)$. Define $a = \frac{L(\text{accept}|q<q_0) - L(\text{accept}|q\geq q_0)}{L(\text{reject}|q\geq q_0) - L(\text{accept}|q\geq q_0)}$ and $b = \frac{L(\text{reject}|q<q_0) - L(\text{accept}|q\geq q_0)}{L(\text{reject}|q\geq q_0) - L(\text{accept}|q\geq q_0)}$. Equation (2) results.

only on x, and not on a and b separately. Importantly, we will show that the monotonicity of DM's decisions with respect to signal noise depends on whether or not $x > 1$.

The case of a normally distributed signal centered around quality distributed with a normal prior is one of several conjugate distributions with a known posterior (see Casella and Berger (2002), page 326). Specifically, a Bayesian decision maker will have the following posterior belief of the quality of a project, denoted by F_s^z:

$$\text{posterior belief of } q \text{ conditional on } s\text{: } F_s^z = N\left(\frac{s}{1+z}, \frac{z\epsilon^2}{1+z}\right) \tag{4}$$

The mean of F_s^z is decreasing in signal noise z, while the variance is increasing. Clearly, it is optimal for DM to follow a threshold strategy, accepting projects if and only if $s \geq s_z^*$. We characterize the behavior of s_z^* as z increases, and show it may be increasing, decreasing, or even non-monotonic in z.

2.1 Solving the model

DM accepts a project if and only if $a\Pr(q < q_0) \leq \Pr(q > q_0) + bP(q < q_0)$, or if $x\Pr(q < q_0) \leq \Pr(q > q_0)$. To determine the limiting behavior of the model, it is useful to ask what decision DM would make based only on his prior, absent any signal. Given equation (1), we obtain the following result:

$$\text{DM rejects based on prior iff } q_0 \geq \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right) \tag{5}$$

where Φ is the c.d.f. of a standard normal distribution. If (5) holds, as z grows large DM rejects all projects ($\lim_{z\to\infty} s_z^* = \infty$), as his posterior belief will converge to his prior belief. Similarly, if (5) does not hold, DM accepts all projects with sufficiently noisy signals. However, DM's standard for acceptance, s_z^*, does not necessarily move monotonically in signal noise z.

Given a realized signal s, DM's expected loss from accepting a project is $aF_s^z(q_0)$, while the expected loss from rejecting is $(1 - F_s^z(q_0)) + bF_s^z(q_0)$. Therefore, the minimum value of s for which DM optimally accepts a project, s_z^*, is given by equation (6):

$$F_{s_z^*}^z(q_0) = \frac{1}{1+x} \tag{6}$$

Standardizing the normal distribution given in (4), we obtain a closed-form solution for s_z^*.[5] DM

[5] Specifically, equation (6) states that $\Pr(N\left(\frac{s_z^*}{1+z}, \frac{z\epsilon^2}{1+z}\right) \leq q_0) = \frac{1}{1+x}$, which implies that

$$\Pr\left(N(0,1) \leq \left(q_0 - \frac{s_z^*}{1+z}\right)\frac{\sqrt{1+z}}{\epsilon\sqrt{z}}\right) = \frac{1}{1+x}$$
$$\Rightarrow \left(q_0 - \frac{s_z^*}{1+z}\right)\frac{\sqrt{1+z}}{\epsilon\sqrt{z}} = \Phi^{-1}\left(\frac{1}{1+x}\right)$$

Solving for s_z^* yields equation (7).

minimizes his loss by following the rule outlined in (7):

$$\begin{array}{ll}\text{Accept if} & s > s_z^* \\ \text{Reject if} & s < s_z^*\end{array}, \text{ where } s_z^* = q_0(1+z) - \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)\sqrt{z(1+z)} \tag{7}$$

The derivative of s_z^* with respect to z depends on the sign of $\Phi^{-1}\left(\frac{1}{1+x}\right)$, which is negative if $x > 1$, positive if $x < 1$, and 0 if $x = 1$. If $x < 1$, (2) implies that mistakenly accepting an inferior project is less costly than mistakenly rejecting a superior project. From (6), an optimizing DM will set s_z^* so the probability that a project with a marginal signal in the neighborhood of s_z^* will be a failure is about $\frac{1}{1+x}$. Therefore, if $x < 1$, marginal projects are more likely to be failures than successes. This is entirely due to the preferences of the decision maker; as he values avoiding errors of one type (rejecting good projects) more than he does errors of another type (accepting bad projects), he shades in the direction of a more lenient threshold, so as to make the first type of error relatively less likely. It turns out that whether the probability of failure of a marginal product is above or below $\frac{1}{2}$ determines whether an increase in the variance of the posterior $F^z_{s_z^*}$ increases or decreases the threshold s_z^*. A decrease in the mean of $F^z_{s_z^*}$ caused by an increase in z always increases s_z^*.

If $x < 1$, so that the probability a marginal project fails is greater than $\frac{1}{2}$, an increase in the variance of the posterior distribution caused by an increase in signal noise z will increase the size of the right tail of the posterior distribution, meaning that for an unchanged cutoff s_z^*, a marginal project is relatively more likely to succeed. A DM with $x < 1$, however, is particularly worried about rejecting a good project than accepting a bad one. Hence, he optimally responds to the increase in the variance by making it easier for a project to be accepted, and increases threshold s_z^*. On the other hand, if $x > 1$, and the probability of failure for a marginal project is optimally set to be below $\frac{1}{2}$, an increase in the variance of the posterior distribution increases the left tail of the distribution and thus *increases* the probability a marginal project is a failure. Such a DM, relatively more concerned with accepting a bad project, responds by decreasing the threshold s_z^*. Finally, in the special case of $x = 1$, DM is equally concerned with both types of errors, and therefore sets the probability of failure of a marginal project to be exactly $\frac{1}{2}$. Hence, an increase in the variance of the posterior neither increases nor decreases the probability of failure, so no adjustment to the threshold s_z^* is necessary. In this case, $s_z^* = q_0(1+z)$, and s_z^* is clearly monotonically increasing in z.

2.2 Characterizing DM's acceptance threshold: non-monotonicity

Under equation (5), s_z^* approaches ∞ as $z \to \infty$; as the signal contains less and less information, DM is less and less likely to deviate from his prior belief. For symmetrical reasons, if (5) does not hold, $s_z^* \to -\infty$. The non-limiting behavior of s_z^* depends both on (5) and on the relationship between two quantities, $x = L(\text{accept}|q < q_0) - L(\text{reject}|q < q_0)$ and $1 = L(\text{reject}|q \geq q_0) - L(\text{accept}|q > q_0)$. Therefore, we separate the problem into four cases:

Case 1: Equation (5) holds and $x \geq 1$: DM rejects based on prior, and it is relatively more costly to accept a bad project than to reject a worthy project.

Case 2: Equation (5) holds and $x < 1$: DM rejects based on prior, and it is relatively more costly to reject a worthy project than to accept a bad project.

Case 3: DM accepts based on prior, and $x \geq 1$

Case 4: DM accepts based on prior, and $x < 1$

We find that only in cases 1 and 4 is DM's optimal threshold s_z^* monotonic in z. In cases 2 and 3, an increase in noise may prompt DM to either increase, or decrease his standard s_z^*. The result follows directly from inspection of equations (5) and (7), and the fact that the sign of $\Phi^{-1}\left(\frac{1}{1+x}\right)$ depends on whether or not x is larger than 1. Proposition 1 formalizes.

Proposition 1. *DM optimally accepts projects if and only if $s \geq s_z^*$. s_z^* varies in z as follows:*

- *Case 1 (DM rejects based on prior and $x \geq 1$): s_z^* increases monotonically to ∞.*
- *Case 2 (DM rejects based on prior and $x < 1$), s_z^* first decreases, then increases to ∞.*
- *Case 3 (DM accepts based on prior and $x \geq 1$), s_z^* first increases, then decreases to $-\infty$.*
- *Case 4 (DM accepts based on prior and $x < 1$), s_z^* decreases monotonically to $-\infty$.*

Proof: From (7), the derivative of s_z^* is given by:

$$\frac{\partial}{\partial z} s_z^* = q_0 - \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)\frac{1+2z}{2\sqrt{z(1+z)}} \tag{8}$$

Note that $\frac{1+2z}{2\sqrt{z(1+z)}} > 1$ for all $z > 0$, and that $\lim_{z\to\infty} \frac{1+2z}{2\sqrt{z(1+z)}} = 1$.

Consider case 1 first. Case 1 has two subcases. In subcase 1.A, $q_0 > 0 > \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)$, and in subcase 1.B, $0 > q_0 > \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)$. In 1.A, both terms of (8) are positive, and so $\frac{\partial}{\partial z} s_z^* > 0$ for all z. In 1.B, given that $\frac{1+2z}{2\sqrt{z(1+z)}} > 1$, we have that $\epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)\frac{1+2z}{2\sqrt{z(1+z)}} > q_0$, and so, from (8), $\frac{\partial}{\partial z} s_z^* > 0$ for all z.

In case 2, $q_0 > \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right) > 0$. The fact that $\lim_{z\to 0} \frac{1+2z}{2\sqrt{z(1+z)}} = \infty$ means that (8) is negative for sufficiently low z, while $q_0 > \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)$ and $\lim_{z\to\infty} \frac{1+2z}{2\sqrt{z(1+z)}} = 1$ imply that (8) is positive for sufficiently large z.

In case 3, $q_0 < \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right) < 0$. Case 3 is a mirror image of case 2, and so identical reasoning yields that (8) is positive for low z but negative for high z.

In case 4, there are again two subcases. In subcase 4.A, $q_0 < 0 < \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)$, and in subcase 4.B, $0 < q_0 < \epsilon\Phi^{-1}\left(\frac{1}{1+x}\right)$. Both are mirror images of subcases 1.A and 1.B, respectively, and so identical reasoning gives us that in each case, (8) is negative for all z. ■

We now turn to two numerical examples to illustrate intuition behind the possible non-monotonicity of s_z^*, illustrated in figures 1 and 2. Suppose that DM rejects based on his prior (i.e. (5) holds). An increase in z has two effects on DM's posterior distribution, F_s^z. First, it decreases the mean, and second, it increases the variance. The first effect causes s^* to increase regardless of the value of x (figures 1 and 2 both illustrate this effect, the former for $x = 2$ and the latter for $x = \frac{1}{3}$). However, an increase in the variance may spur either an increase or a decrease in s^*, depending on whether or not $x \geq 1$. The reason is that for $x \geq 1$, the cutoff s_z^* is set so that the probability that a marginal project will be a failure is about $\frac{1}{1+x} < \frac{1}{2}$, owing to DM being relatively more concerned with accepting a bad project than rejecting a worthy one. As F_s^z is a normal distribution, an increase in its variance increases the size of its left tail, and therefore increases $F_s^z(q_0)$, meaning that s_z^* must increase in order to compensate. Figure 1 demonstrates this observation. In subfigures 1d and 1e, an increase in the variance of F_s^z, holding the mean constant, increases $F_s^z(q_0)$ and therefore an increase in s^* is necessary for $F_s^z(q_0) = \frac{1}{1+x}$ to hold.

Now suppose that $x < 1$, and continue to assume that DM rejects based on his prior. Figure 2 demonstrates an example of this case. Here, an increase in the variance of F_s^z caused by an increase in z again causes both tails of F_s^z to increase. As q_0 is above the mean of F_s^z (because DM is relatively more concerned with rejecting worthy projects than accepting bad ones), this in turn *increases* the magnitude $F_2^z(q_0)$, and so a *decrease* of s_z^* is necessary so that, at the margin, DM is indifferent between acceptance and rejection ($\frac{1}{1+x} = f_s^z(q_0)$). Note that in the knife-edge case of $x = 1$, increasing the variance has no effect on $s_z^* = q_0(1 + z)$, as the probability that a marginal project is unworthy is exactly $1/2$.

Given an increase in z, whether the variance effect or the mean effect dominates determines the direction in which s_z^* moves. When z is large enough, s_z^* increases in z if (5) holds and decreases if (5) does not hold. However, s_z^* can move in either direction as z varies at intermediate values. Which competing effect dominates depends on the value of z, leading to the non-monotonicity described in Proposition 1.

Characterizing the fraction of projects accepted

Given an array of projects, some good and some bad, whose quality distribution is described by

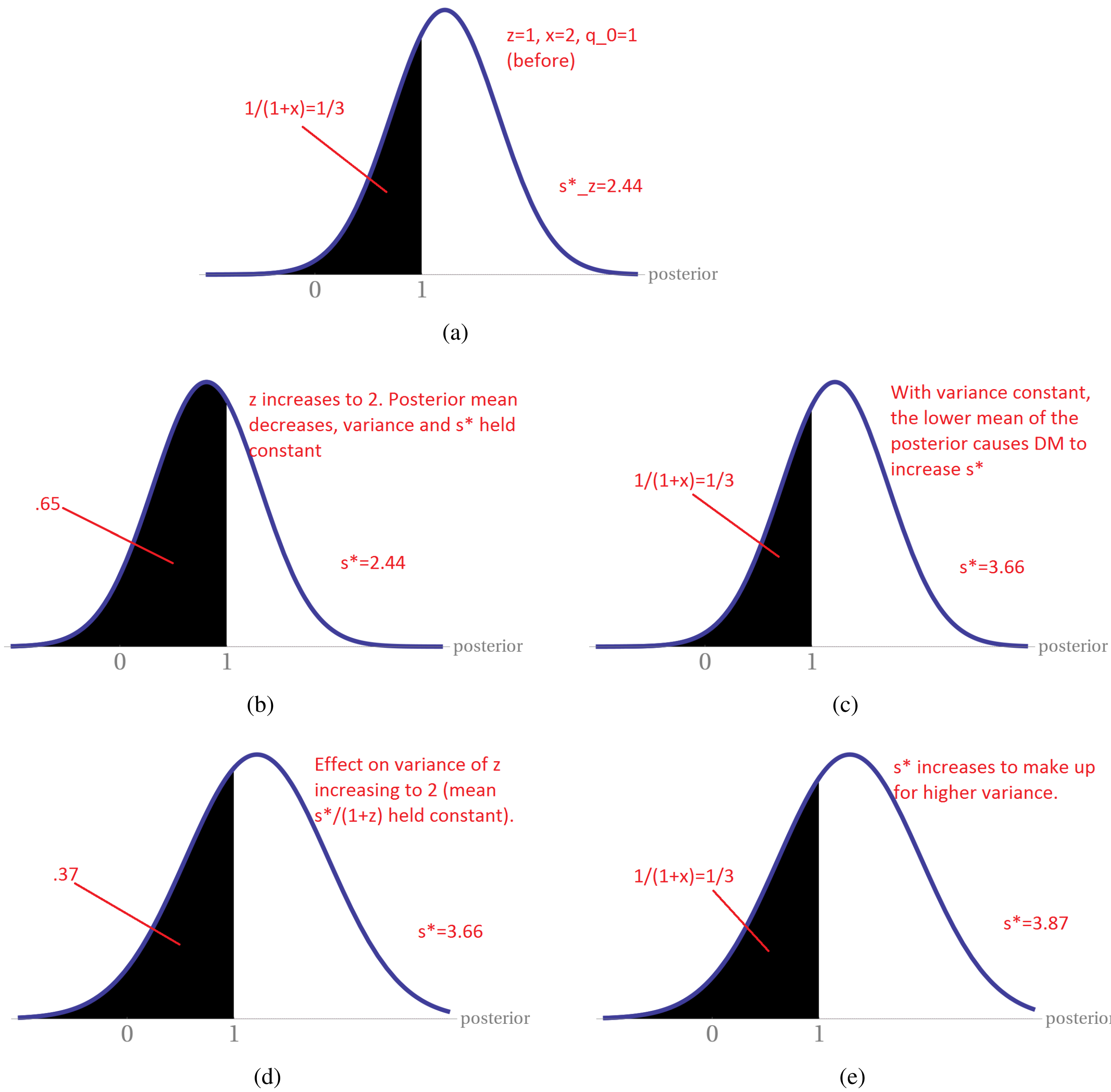

Figure 1: An illustration of case 1 of proposition 1. Starting from $z = 1$, $x = 2$, $q_0 = 1$, and DM rejects based on prior, the probability that a marginal project will be a failure is $\frac{1}{1+x} = \frac{1}{3}$. An increase in z has two effects. One, the mean of the posterior distribution F_s^z decreases, meaning that s^* must increase to balance equation (6), shown in figures 1b and 1c. Two, the variance of F_s^z increases, enlarging the tails of the distribution and increasing the area to the left of $q_0 = 1$, meaning that s^* must increase to balance (6). The second effect is show in figures 1d and 1e. Contrast the example of figure 1 with that of figure 2.

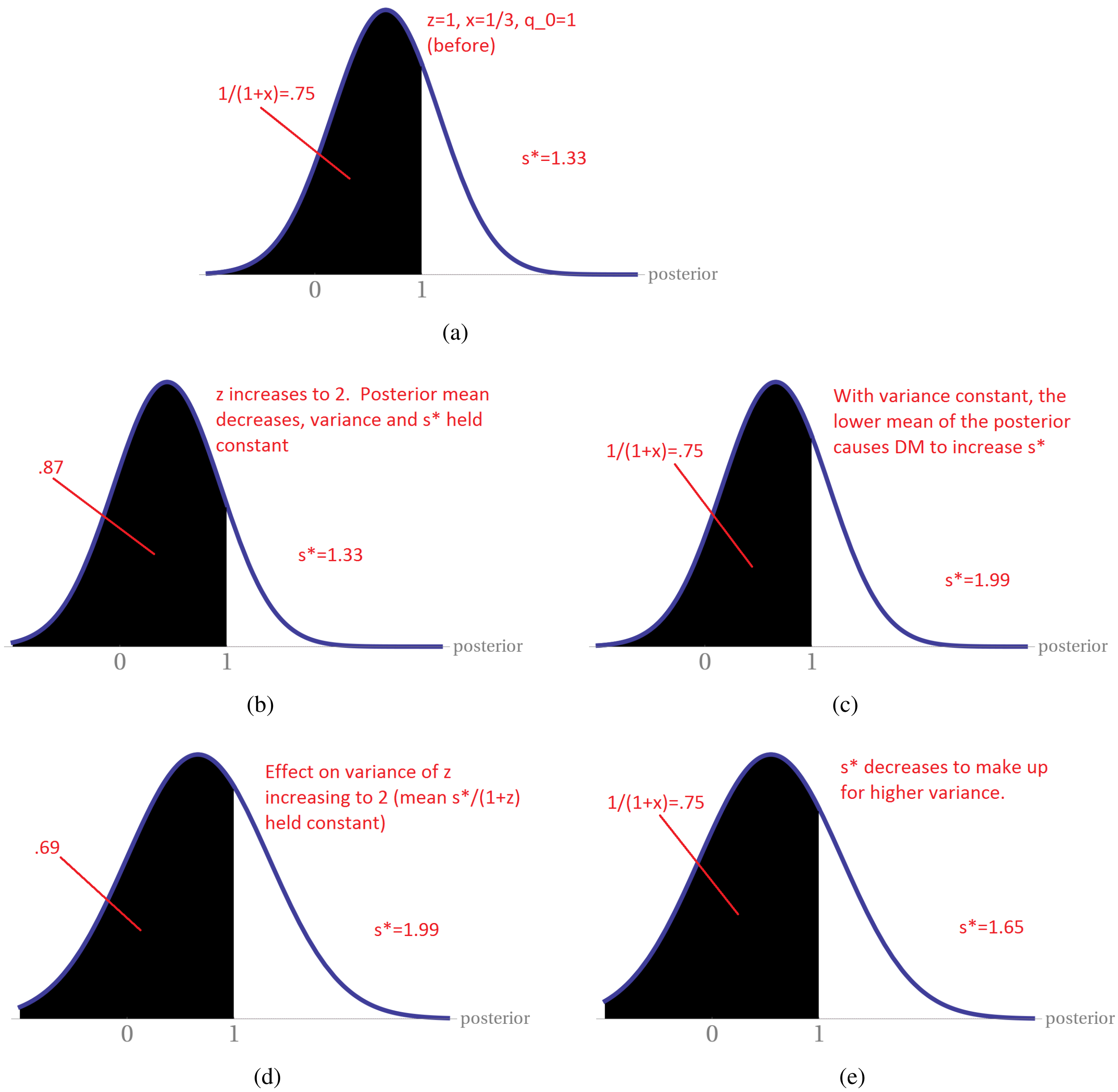

Figure 2: An illustration of case 2 of proposition 1. Starting from $z = 1$, $x = \frac{1}{3}$, $q_0 = 1$, and DM accepts based on prior, the probability that a marginal project will be a failure is $\frac{1}{1+x} = \frac{3}{4}$. An increase in z has two effects. One, the mean of the posterior distribution F_s^z decreases, meaning that s^* must increase to balance equation (6), shown in figures 2b and 2c. Two, the variance of F_s^z increases, enlarging the tails of the distribution and decreasing the area to the left of $q_0 = 1$, meaning that s^* must decrease to balance (6). The second effect is show in figures 2d and 2e. Contrast the example of figure 2 with that of figure 1.

$q \sim N(0, \epsilon^2)$ (Equation (1)), what fraction of projects does DM accept, as a function of signal noise z? Given that DM's acceptance threshold s_z^* does not necessarily move monotonically in z, DM may accept either a greater fraction or a lower fraction of projects as z increases, again depending on x. Indeed, the fraction of projects accepted by DM need not be monotonic in z.

Let $\Psi(z)$ denote the overall fraction of projects accepted by DM. In the case of a journal referee, given a fixed number of submissions, $\Psi(z)$ would describe the number of accepted papers. In the case of a C.E.O. making decisions on budgeting priorities, $\Psi(z)$ describes the number of new projects approved for funding. Proposition 2 formally characterizes $\Psi(z)$.

Proposition 2. *Letting $\Psi(z)$ equal the overall fraction of projects accepted, $\Psi(z)$ varies in z as follows:*

1. *If (5) holds and $x \geq 1$ (case 1), $\Psi(z)$ decreases to 0 as $z \to \infty$.*
2. *If (5) holds and $x < 1$ (case 2), $\Psi(z)$ decreases to 0 as $z \to \infty$, but may vary non-monotonically in z.*
3. *If (5) does not hold, and $x \geq 1$ (case 3), $\Psi(z)$ increases to 1 as $z \to \infty$, but may vary non-monotonically in z.*
4. *If (5) does not hold, and $x < 1$ (case 4), $\Psi(z)$ increase to 1 as $z \to \infty$.*

Proof: Suppose first that (5) holds (cases 1 and 2 in the statement of the proposition). Integrate $\Pr(s > s_z^*)(q)$ over q to get the *ex ante* acceptance probability:

$$\Pr(s > s_z^*) = 1 - \frac{1}{\sqrt{2\pi\epsilon^2}} \int_{-\infty}^{\infty} e^{-\frac{q^2}{2\epsilon^2}} \Phi\left(\frac{q_0 - q}{\epsilon\sqrt{z}} + \frac{q_0\sqrt{z}}{\epsilon} - \Phi^{-1}\left(\frac{1}{1+x}\right)\sqrt{1+z}\right) dq \qquad (9)$$

Under (5), the right-hand side of (9) approaches 0 as $z \to \infty$, monotonically if $x \geq 1$. The possibility of non-monotonicity follows from the numerical example in Figure 3.

A symmetrical proof follows in the case that (5) does not hold (cases 3 and 4 in the statement of the proposition). ■

Figure 3 illustrates cases 1 and 2 of Proposition 2. In Figure 3a, $\epsilon = 1$, $q_0 = 0$, $x = 1.7$, and DM prefers to reject projects based on his prior alone. For $z = 0$, DM perfectly observes quality and so accepts half of all projects, given $q_0 = 0$. As z increases, given that $x > 1$ (case 1), s_z^* increases monotonically, and so $\Psi(z)$ decreases monotonically to 0. In Figure 3b, $\epsilon = 1$, $q_0 = 1$, and $x = .3$, DM prefers to reject based on his prior (case 2). In this case, s_z^* first decreases, then increases in z and so $\Psi(z)$ first increases, then decreases to 0 as z increases to ∞. An increase in signal noise z from .5

to 1.5 (point A to point B) increases the fraction of projects accepted by DM, while a further increase from $z = 1.5$ to $z = 4$ (point B to point C) decreases the fraction of projects accepted. A similar figure illustrating cases 3 and 4 of Proposition 2 is omitted.

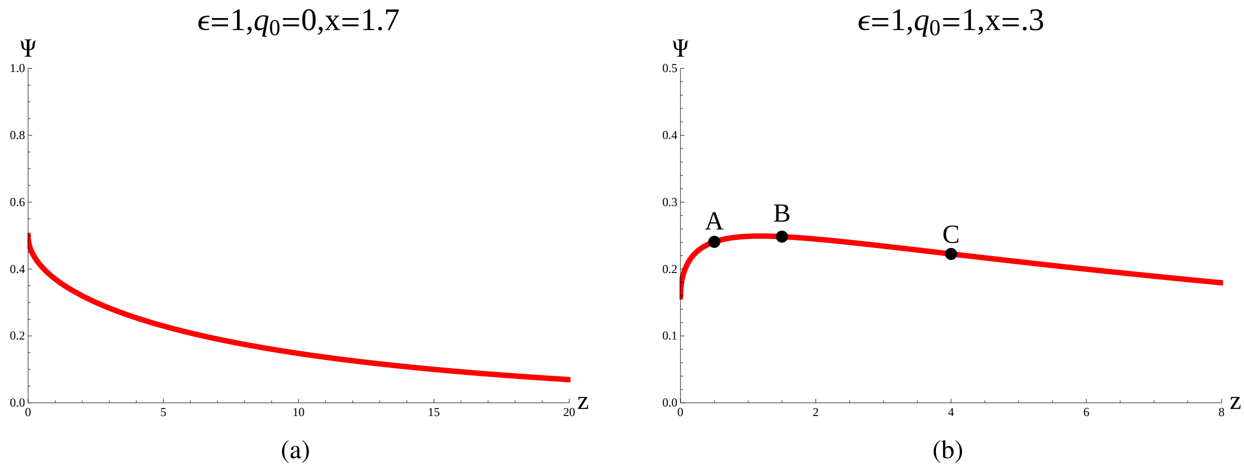

Figure 3: Illustration of cases 1 and 2 of proposition 2. In figure 3a (case 1), the overall fraction of projects accepted, $\Psi(z)$, decreases monotonically to 0 as z increases — increasing noise always means that DM accepts fewer projects. In figure 3b (case 2), $\Psi(z)$ first increases, then decreases to 0 as z increases — an increase in signal noise from point A to point B causes DM to increase the fraction of projects accepted, while an increase from point B to point C causes DM to decrease the fraction accepted.

It follows from proposition 2 that the average quality of accepted projects is also not necessarily monotonic in signal noise z. This means, for example, that the profitability of capital projects approved by a C.E.O. may increase or decrease in noise. This provides another explanation for the divergent empirical results discussed in section 1.

3 Discussion

One implication of our non-monotonicity result is that no particular empirical relationship between noise and fraction or quality of projects accepted. This is because the monotonicity of any outcome with respect to noise depends on x, an preference parameter. As x is unobservable, either a monotonic or a non-monotonic relationship between signal noise and acceptance standard can be expected empirically. Even in cases where it is clear that (5) holds or does not hold (for example, an academic journal which rejects well over half of its submissions could be fairly said to reject papers based on its prior belief alone), an empirical prediction can only be made for the limiting case as

$z \to \infty$.

A second implication is that there is no clear relationship between the amount of noise and the likelihood of acceptance for a project of any given quality level q. Hence, a strategic submitter should not necessarily favor a decision maker who can judge the quality of his project with more or less noise. This is true regardless of whether the submitter believes that the true quality is above or below q_0.

A reader might wonder about the role of risk aversion in explaining our results. The loss function (2) parameterizes two types of risk in relative terms: the risk of accepting a bad project, and the risk of accepting a good project. This results in an endogenous aversion to risk, as described by equation (7). Were DM to additionally have preferences over the absolute level of risk (suppose noise z entered as an argument in DM's utility function, with utility decreasing in z), the result would be to decrease threshold s_z^* for any value of z. Importantly, risk-aversion alone is therefore not capable of explaining the non-monotonicity we study in our model. To make this point especially clear, we assume a decision maker minimizing his expected loss throughout.

Finally, our model relies on the case of a normally distributed signal centered on quality, which has a normally distributed prior. What about other distributions? The tractability of our model comes from the case of a normal prior and signal having a known posterior distribution (also normal), and from the relative tractability of normal random variables. For arbitrary prior/signal distributions, the Bayesian posterior is generally unknown, meaning that no relationship between decision making and signal noise can be ruled out. Other conjugate distributions (see Casella and Burger (2002), page 326) generally produce an intractable posterior. One partial exception to this is the case of a Pareto prior and uniform signal. Readers interested in this case should refer to the working paper version of this paper.

Within the normal-normal model considered in his paper, two extensions present themselves. First, what happens when the loss function (2) is generalized so that DM cares about the amount by which q exceeds or falls short of q_0? In the case of a linear loss function, in which DM incurs a fixed cost from rejecting a paper, but a loss $q_0 - q$ from acceptance, DM's acceptance standard s_z^* moves monotonically in z. As discussed in section 2, the non-monotonicity observed in the main model is an artifact of a DM optimally setting the probability of failure of a marginal project above or below $\frac{1}{2}$, depending on his preference parameters; this does not happen in the linear loss case.[6]

Second, it is worth considering the discrete version of our model, in which true quality is binary (w.l.o.g., say quality is equal to -1 $w.p.\frac{1}{2}$ and 1 $w.p.\frac{1}{2}$, and that $q_0 = 0$), and the observed signal s

[6]For a full treatment of the linear loss case, see the working paper version of this paper.

follows an $N(q, z)$ distribution. Here,

$$F_s^z(q_0) = P(q = -1|s) = \frac{f(s; \mu = -1, \sigma^2 = z)}{f(s; \mu = -1, \sigma^2 = z) + f(s; \mu = 1, \sigma^2 = z)} = \frac{1}{1 + Exp\left(\frac{2s}{z}\right)} \tag{10}$$

where $f(s) = \frac{1}{\sigma\sqrt{2\pi}} Exp\left(-\frac{(s-\mu)^2}{2\sigma^2}\right)$ is the normal p.d.f. Recall from equation (6) that DM sets the marginal proposal s_z^* so that:

$$F_{s_z^*}^z(q_0) = \frac{1}{1 + Exp(\frac{2s_z^*}{z})} = \frac{1}{1+x}$$

$$\iff s_z^* = \frac{z}{2}\ln(x) \tag{11}$$

From equation (11), if $x = 1$ (DM has an equal aversion to both accepting bad projects and rejecting good ones), $s_z^* = 0$ for all z, and the standard is invariant in z. If $x > 1$ (accepting bad projects relatively more costly) s_z^* is increasing in z, meaning that rejection is more likely as z increases, while if $x < 1$, s_z^* is decreasing in z, making acceptance more likely. In each case, the movement is monotonic.

4 Conclusion

In this note, we show that there is no general relationship between noise and decision making. In particular, we investigate the behavior of the threshold value to induce an accept decision as the precision of the signal increases. Surprisingly, we find that the the relationship between noise and decision making is not even monotonic, a result that is new to the literature.

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