

Homework 1

Due Friday, 9/7/12 by 5pm

Instructions: Complete all problems and turn in a set of answers either to me or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write “According to Excel, ...” in your answer.

Problem 1 Imagine that a randomly selected mutual fund has a fifty percent chance of beating the performance of the stock market in any given year. Furthermore, assume that a fund’s chances of outperforming the market are independent from year to year.

- a. What is the probability that a randomly selected mutual fund will outperform the market for five consecutive years?
- b. What is the probability that a randomly selected mutual fund will outperform the market for ten consecutive years?
- c. Out of 10,000 mutual funds, how many would you estimate will outperform the market for ten consecutive years?
- d. Does the existence of a handful of market funds that outperform the market year after year demonstrate “proof” that certain fund managers are amazingly skillful in selecting stocks, or is there an alternative way to explain their success?

Problem 2 Testing for Groat’s Disease is very expensive, and positive tests are quite rare. To cut costs, a hospital considers the following procedure: they pool blood samples from twenty patients (while preserving the individual samples), and test the pooled blood. If the test on the pooled blood sample is negative, all 20 patients test negative. If the pooled test is positive, the samples are then tested individually to determine where the positive blood came from. Suppose that the probability any one individual’s blood is positive is .018, and that the samples are independent across patients.

- a. What is the probability that a pool of 20 individual samples will test negative?
- b. What is the probability that a pool of 20 individual samples will test positive?
- c. Let N represent the number of tests required to determine positive/negative status of a group of 20 patients, when following the pooled sample procedure described above. What is the expected value of N ?
- d. Dr. House suggests that while the pooled procedure is superior to testing each sample individually, the hospital can save even more money by pooling samples from X individuals, where $X \neq 20$. Show that he is correct.

Problem 3 A supermarket's market research team determines that among the set of customers purchasing diapers, the number of bottles of whiskey purchased is a random variable, X , following the distribution below:

x	$p(x)$
0	.05
1	.1
2	.15
3	.4
4	.2
5	.1

- a. Determine the expected value of X .
- b. Determine the variance and standard deviation of X .
- c. What is the probability that at least 3 bottles of whiskey are purchased by a customer in this segment?

Problem 4 Sahara, Lexington's best restaurant, is considering an all-you-can-eat special on their famous spicy chicken entree. Based on a similar promotion a few years back, the owners have assembled a probability distribution for the random variable X , representing the number of servings of chicken a customer will eat:

x	$p(x)$
1	.210
2	.535
3	.199
4	.056

- a. Determine the expected value, variance, and standard deviation of X .
- b. Suppose that a serving of chicken costs the restaurant \$5. What is the minimum price for the all-you-can-eat option so that to break even in expectation?
- c. What is the minimum price for the all-you-can-eat option so that they break even or better with 94.4% probability?

Problem 5 The Bermuda department of tourism estimates that 80% of visitors who attend a conference on the island bring a guest or family member along. Suppose we would like to use a binomial random variable X to describe how many of the 200 attendees of the Architectural Coating Manufacturer's Conference will bring somebody with them.

- a. What are the mean and variance of X ?
- b. What is $P(X = 160)$?
- c. What is $P(X = 180)$?
- d. What is $P(X \geq 120)$?
- e. What is $P(X \leq 150)$?

Problem 6 The number of arrivals at a car wash is a Poisson random variable, with an expected value of 8 per hour.

- a. What is the probability that 10 cars will arrive in the next hour?
- b. What is the probability that more than 5 cars will arrive in the next hour?
- c. What is the probability that fewer than 10 cars will arrive in the next hour?

Problem 7 Shutouts in the National Hockey League occur randomly and independently at a rate of 1 every 20 games. Calculate the probability of the following events.

- a. 2 shutouts in the next 10 games.
- b. 25 shutouts in 400 games.
- c. a shutout in tonight's game.

Problem 8 In a recent election, the mayor received 60% of the vote. Last week, a survey asked 100 people whether they would vote for the mayor's reelection or not. Assuming that his popularity has not changed since the election, what is the probability that more than 50 people in the sample say they would vote for the mayor's reelection?

Problem 9 During Eco 391's exams, you will be given a photocopy of certain probability tables from the back of your textbook. To practice using these tables, calculate the following probabilities first in Excel, and then using tables 1 and 2 in Appendix B. Note that the probability tables give cumulative probabilities, that is $P(X \leq x)$, so that if you want to calculate $P(X = 7)$, for example, you must compute $P(X \leq 7) - P(X \leq 6)$. Make sure you get the same answer with each method:

- a. X is binomial with $n = 10$ and $p = .4$. What is $P(X \leq 5)$? What is $P(X \geq 2)$? What is $P(X = 7)$?
- b. X is binomial with $n = 25$ and $p = .75$. What is $P(X = 16)$? $P(X \leq 10)$? $P(X \geq 20)$?
- c. X is poisson with mean 3. What is $P(X = 2)$? $P(X = 4)$? $P(X \geq 0)$?