

Homework 2

due Tuesday, September 18, in class

Instructions: Complete all problems and turn in a set of answers either to me or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write “According to Excel, ...” in your answer.

Problem 1 Let X be an exponential random variable, with $\lambda = .5$. Find the following probabilities.

- a. $P(X \geq 1)$
- b. $P(X \geq .4)$
- c. $P(X \leq .5)$
- d. $P(1 \leq X \leq 2)$

Problem 2 When trucks arrive at the Ambassador Bridge, each must be checked by customs agents. The time of an inspection is exponentially distributed, with a mean of 6 minutes. What is the probability that a truck requires more than 10 minutes to be checked?

Problem 3 Every day a bakery prepares its famous marble rye. A statistically savvy customer determines that daily demand is normally distributed with a mean of 850 loaves and a standard deviation of 90. How many loaves should the bakery make each day if:

- a. It wants the probability of running short on any day to be no more than 30%
- b. it wants the probability of running short on any day to be no more than 10%

Problem 4 Apple observes that the battery in the iPhone 5 lasts for an average of 26 hours under normal usage. Apple believes that battery life per charge is normally distributed, with a standard deviation of 5 hours.

- a. What is the probability that a battery lasts between 22 and 30 hours on one charge?
- b. What is the probability a battery lasts for less than 22 hours on one charge?
- c. What is the probability a battery lasts for more than 30 hours on one charge?

Problem 5 Suppose that the weight of airline passengers with luggage is normally distributed with a mean of 210 pounds and a standard deviation of 50 pounds.

- a. What is the probability that one randomly selected passenger (with luggage) weighs more than 280 pounds?
- b. What is the probability that a randomly selected sample of 10 passengers weigh more than 250 pounds, on average?

- c. Suppose safety regulations require that a plane carry less than 90,000 pounds. Suppose that the airline allows 400 people on the plane. What is the probability that the regulations are met?
- d. Bob Largebody weighs more than 99% of passengers. What is Bob's weight?

Problem 6 You collect a "sample" of 5 dice rolls. Let \bar{X} equal the mean outcome of the 5 rolls.

- a. Find $P(\bar{X} = 1)$
- b. Find $P(\bar{X} = 6)$
- c. Prove that $P(\bar{X} = 3) > P(\bar{X} = 1)$ (note: one way of doing this problem is to calculate $P(\bar{X} = 3)$, but there is a much simpler way to do it)

Problem 7 A normally distributed population has a mean of 40 and a standard deviation of 12. What does the Central Limit Theorem say about the sampling distribution of the mean if samples of size 100 are drawn from this population?

Problem 8 A sample of $n = 64$ observations is drawn from a population with $\mu = 100$ and $\sigma = 16$. Find the following:

- a. $P(\bar{X} \geq 104)$
- b. $P(\bar{X} \leq 94)$
- c. $P(\bar{X} \geq 110)$

Problem 9 The amount of time spent by American adults watching TV per day is normally distributed with a mean of 6 hours and a standard deviation of 1.5 hours.

- a. What is the probability that a randomly selected American adult watches television for more than 7 hours per day?
- b. What is the probability that the average time spent watching TV in a random sample of 5 American adults is more than 7 hours?
- c. What is the probability that the average time spent watching TV in a random sample of 30 American adults is more than 7 hours?

Problem 10 During Eco 391's exams, you will be given a photocopy of certain probability tables from the back of your textbook. To practice using these tables, calculate the following probabilities first in Excel, and then using table 3 in Appendix B. Note that the probability tables give cumulative probabilities, that is $P(X \leq x)$. Use two decimal places in all calculations.

- a. Suppose X is a normal random variable with mean -2 and standard deviation 10. Find $P(X < -5)$, $P(X \geq 0)$, and $P(X \geq 10)$.
- b. Suppose that X is a normal random variable with mean 100 and standard deviation 6. Find $P(X \leq 110)$, $P(X \geq 85)$, and $P(X \leq 100)$.
- c. Suppose that X is a normal random variable with mean 30 and standard deviation 2. Find $P(X \geq 28)$, $P(27 \leq X \leq 34)$, and $P(X \leq 35)$