

Homework #3

due Friday, October 18, by noon

Instructions: Complete all problems and turn in a set of answers either to me, in my mailbox, or under my door (office 335L) by the assigned due date. Do ask me questions via email or in my office hours. Do work together. Do not copy answers from another student or turn in answers that are substantively identical. To clarify, if you work with another student, I would expect that your numerical answers would be quite close, but that your verbal explanations would be similar, but not identical, reflecting that you wrote up your answers independently. Show your work, and write out explanations for your answers. If you use Excel or a similar tool, write “According to Excel, ...” in your answer.

Problem 1 A sample of $n = 64$ observations is drawn from a population with $\mu = 100$ and $\sigma = 16$. Find the following:

- $P(\bar{X} \geq 104)$
.02275
- $P(\bar{X} \leq 94)$
.001349898
- $P(\bar{X} \geq 110)$
 $2.87 * 10^{-7}$ (in other words, almost zero).

Problem 2 A statistics practitioner took a random sample of 50 observations from a population with a standard deviation of 25 and computed the sample mean to be 100.

- Use an interval estimate to estimate the population mean with 90% confidence.
[94.185, 105.815]
- Repeat part a. using a 95% confidence level.
[93.070, 106.930]
- Repeat part a. using a 99% confidence level.
[90.893, 109.107]
- Describe the effect on the confidence interval of increasing the confidence level.
A higher confidence level generates a longer interval.

Problem 3 The mean of a sample of 25 was calculated as $\bar{x} = 500$. The sample was randomly drawn from a population with a standard deviation of 15.

- Estimate the population mean with 99% confidence.
[492.273, 507.727]
- Repeat part a. changing the population standard deviation to 30.
[484.545, 515.455]
- Repeat part a. changing the population standard deviation to 60.
[469.090, 530.910]

d. Describe the effect on the confidence interval of increasing the population standard deviation.

As standard deviation increases, the width of the confidence interval increases.

Problem 4 A medical statistician wants to estimate the average weight loss of people who are on a new diet plan. In a preliminary study, he guesses that the standard deviation of the population of weight losses is about 10 pounds. How large a sample should he take to estimate the mean weight loss to within 2 pounds, with 90% confidence?

The margin of error in his estimate is $1.645 * \frac{10}{n}$, where n is the sample size. Setting this equal to 2 and solving for n gives us $n = 67.6$, so a sample of size 68 is required for the margin of error to be below 2 pounds.

Problem 5 The label on a three-quart container of orange juice claims that the orange juice contains an average of one gram of fat or less. Answer the following question for a hypothesis test that could be used to test the claim on the label.

a. Develop appropriate null and alternative hypotheses.

The most appropriate hypotheses to test are:

$$H_0 : \mu \leq 1$$

$$H_1 : \mu > 1$$

b. What is a Type I error in this situation? What are the consequences of making this error?

A type I error rejects H_0 when it is, in fact true. The consequences here are that one would incorrectly conclude that $\mu > 1$, when it is not. This could, for example, lead the manufacturer to drop an effective marketing line (“contains 1 gram of fat or less!”).

c. What is a Type II error in this situation? What are the consequences of making this error?

A type II error is failing to reject H_0 when it is false. A consequence of this could be the company putting out an incorrect marketing claim.

Problem 6 The manager of an automobile dealership is considering a new bonus plan designed to increase sales volume. Currently, the mean sales volume is 14 automobiles per month. The manager wants to conduct a research study to see whether the new bonus plan increases sales volume. To collect data on the plan, a sample of sales personnel will be allowed to sell under the new bonus plan for a one-month period.

a. Develop the null and alternative hypotheses that are most appropriate for this research situation (hint: be careful! Read pages 391-392. Is a one- or two-tailed test more appropriate?)

We are interested in whether or not sales have increases under the incentive plan, so we make the null the status quo (sales have not increased from 14 per month) and the alternative the event that sales have increased. A one-tailed test is more appropriate, since we don’t particularly care if sales have decreased under the plan:

$$H_0 : \mu = 14$$

$$H_1 : \mu > 14$$

b. Suppose a limited test is conducted among 25 randomly chosen salespeople, who sell an average of 14.7 cars under the bonus plan. Suppose the standard deviation for cars sold is 3. Test the hypotheses you identify in part a. with $\alpha = .1$, and interpret your conclusion.

We reject H_0 if and only if $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} > 1.28$. Given these data, the test statistic equals 1.167. Therefore, we fail to reject the null hypothesis. There is insufficient evidence to claim that the incentive program has increased sales.

Problem 7 Fightmaster and Associates Real Estate, Inc. advertises that the mean selling time of a residential home is 40 days or less. A sample of 50 recently sold residential homes shows a sample mean selling time of 34 days and a standard deviation of 20 days. Using $\alpha = .02$, test the validity of the company's claim.

Reject the null hypothesis that $\mu \geq 40$ if and only if the test statistic is less than -2.054 . Given these data, the test statistic comes out to -2.12 , so we reject H_0 .