Problem set 4

"due" 11/4/2009

Problem 1 The demand curve a monopoly faces is $P = 100 - Q$. The firm's cost curve is $c(Q) = 10 + 5Q$ (so $mc = 5$). What is the firm's profit-maximizing quantity and price? What is the firm's profit? What is the value of consumer surplus and deadweight loss? What is this monopolist's Lerner index?

The monopolist maximizes profit by setting $Q = 47.5$ and $P = \$52.50$. Consumer surplus is 1128.125, producer surplus is 2256.25, profit is 2246.25. Deadweight loss is 1128.125. The Lerner index is .904.

Problem 2 The Albuquerque Isotopes, a minor league baseball team, have a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is \$5, yet this results in an average attendance of only 20,000 people.

a. Explain how it can be profitable to have 10,000 empty seats.

Were the Isotopes to fill the stadium, they would have to lower the ticket price from \$5. While this would generate additional revenue from the new spectators, it would lose revenue from those who came even when the price was \$5.

b. Next week the Isotopes play the Capital City Goofballs, who have offered to buy an unlimited number of tickets at \$4 each, to be resold only in Capital City. How many tickets should be sold to Capital City to maximize the Isotopes' profit? 10,000? More than 10,000? Explain.

Capital City's offer raises the marginal cost of selling a ticket to an Albuquerque fan by \$4; this is the opportunity cost of the seat given Capital City's offer. Increasing this marginal cost lowers the quantity which should be sold in Albuquerque (draw a picture to convince yourself of this).

c. Given your answer to b, what price should the Isotopes charge their own fans? \$4? \$5? More?

Given the answer to b., they should charge a price of more than \$5 to Albuquerque fans.

Problem 3 True/false: a monopolist will increase its output if the government institutes a binding price ceiling. Explain why. If the government wants to set a price ceiling which maximizes total surplus, what price should it choose? (Hint: use a graph to help answer this question).

True. A monopolist has an incentive to restrict output in order to inflate price. If the government mandates a price ceiling, this motive dissipates, and thus the monopolist will produce more.

Problem 6 There are 10 households in Lake Wobegon, Minnesota, each with a demand for electricity of $Q = 50 - P$. Lake Wobegon Electric's (LWE) cost of producing electricity is $c(Q) = 500 + Q$.

a. If the regulators of LWE want to make sure that there is no deadweight loss in this market, what price will they force LWE to charge? What will output be in this case? Calculate consumer surplus and LWE's profit with that price.

They will impose a price ceiling where marginal cost crosses the demand curve, at $P = \$1$. If LWE produces the demanded quantity of 49 per customer, their profit would be $10 * 1 * 49 - 500 - 49 * 10 = -500$. They would thus need a subsidy of at least \$500 were they to continue to operate at this price. Consumer surplus is $\frac{1}{2} * 49 * 49 = 1200.5$ per consumer, for a total of 12005.

b. If regulators want to ensure that LWE doesn't lose money, what is the lowest price they can impose? Calculate output, consumer surplus, and profit. Is there any deadweight loss?

They will impose a price ceiling where average cost crosses the demand curve. The total demand curve, summed across the 10 customers, is $Q = 500 - 10P$, or $P = 50 - .1Q$, so we need $50 - .1Q = \frac{500}{Q} + 1$, which holds at $Q = 479.57$, $P = \$2.04$. Profit would be 0. Consumer surplus is $\frac{1}{2} * 479.57 * 47.96 = 11500$. Deadweight loss is $\frac{1}{2} * 1.04 * 10.43 = 5.42$.

c. It is suggested that each household be required to pay a fixed amount just to receive any electricity at all, and then a per-unit charge for electricity. Then LWE can break even while charging the price calculated in a. What fixed amount would each household have to pay for the plan to work? Per the answer in a, if each household paid LWE a \$50 fee, LWE could break even charging a price of \$1, with no deadweight loss.

Problem 5 A monopolist faces the demand curve $p = 24 - Q$, and has constant marginal cost of \$4. If the firm runs an advertising campaign, its demand shifts out to $p = 32 - Q$. What is the largest amount the firm would be willing to pay for this campaign?

Profit is \$100 with no campaign and \$196 with the campaign, so the firm would be willing to pay any amount less than \$96.

Problem 6 The demand function for a monopolist's product is given by $p = 43 - 2q$. The monopolist has a constant marginal cost of \$3.

a. Find the monopolist's profit-maximizing price and quantity. What is his profit?

Price is \$23, quantity sold is 10, and profit is \$200.

b. Suppose the monopolist is able to perfectly price discriminate. What quantity will he sell, and what will his profit be?

He will sell quantity 20, at prices ranging from \$3 to \$43, for a profit of \$400.

Problem 7 The Grand Theater is a movie house in a medium-sized college town. On any given night, if the theater is open, it must pay \$500 in fixed costs (paying electricity, ushers, etc) regardless of how many people come to the theater. If the theater is closed, its costs are 0. There are two groups of people who come to the Grand Theater, students and non-students. Students have demand function $q_s = 220 - 40p_s$ while non-students have demand function $q_n = 140 - 20p_n$.

a. Suppose that the theater cannot tell students apart from non-students. What price will it charge? How many students will come? How many non-students? What will the profits of the Grand Theater be?

In this case, the total demand will be $Q = 360 - 60p$, marginal revenue will be $6 - \frac{1}{30}Q$, and so profit-maximizing quantity will be 180 patrons, at a price of \$3. Profits are \$40 (\$3*180-\$500).

b. Now suppose that the cashier can accurately tell students from non-students by asking students to show their student IDs. Students cannot resell their tickets to non-students after purchase. Will the Grand charge students and non-students different prices? What will these prices be? What will be the Grand's profits?

Treating students and non-students as two separate groups, the theater will charge students \$2.75 and sell them 110 tickets, while charging non-students \$3.50 and selling them 70 tickets. Total profits are \$47.50.

c. Finally, suppose that the Grand Theater can only hold 150 people. If the theater is able to charge separate prices to students and non-students, what prices will it charge, and how many students and non-students will come?

We know that the number of students admitted plus the number of non-students admitted must equal 150. We also know that to be maximizing profits, it must be that the marginal revenue from the last student

admitted must equal that from the last non-student admitted. Were this not so, the theater could admit one less from the low marginal revenue group and one more from the high marginal revenue group, and increase profits. Thus, the following two equations define the solution:

$$q_s + q_n = 150$$
$$5.5 - \frac{1}{20}q_s = 7 - \frac{1}{10}q_n$$

which has solution $q_n = 60$, $q_s = 90$, meaning $p_s = \$3.25$ and $p_n = \$4$.

Problem 8 Your firm produces 2 products, each at 0 marginal cost. You face four types of customers, each comprising 25% of your total customers (say you have N total customers). The groups have the following willingness to pay for your product:

customer	good 1	good 2
A	\$25	\$100
B	\$40	\$80
C	\$80	\$40
D	\$100	\$25

a. Compare selling these two products separately to bundling them and selling them together for one price. Which leads to a higher profit?

To sell the two goods bundled, you would charge a price of \$120 and make a profit of $\$120N$. To sell them separately, you would charge a price of \$80 for each good, and get profits of $\$80N$. Clearly, bundling them is superior to not.

b. Now consider the possibility that you sell these goods both bundled and unbundled (that is, you set three prices, one for good 1 alone, one for good 2 alone, and one for the bundle of good 1 and good 2). Would doing this improve upon the outcome of part a? Explain.

It does not seem that you could increase profits from the bundling outcome of part a by also selling the products separately.

c. Now suppose that the production of each good entails a marginal cost of \$30. How does this information change your answers to a and b above? Is it better to sell the goods unbundled, bundled, or both bundled and separately?

Here, A values good 1 and D values good 2 at less than their marginal costs. Consider a bundled price of \$120, a price of \$95 for good 1 and a price of \$95 for good 2. In this case, A buys only good 2, B and C buy the bundle, and D buys only good 1. Profit is $\$62.5N$. This is higher than could be achieved with only bundling or only separate prices.

Problem 9 An airline has two types of customers who fly a given route, tourists and business travelers. Say that tourists have demand function $p = 20 - q$, where q is the quality level for the seat in which a tourist sits, and business travelers have demand given by $p = 40 - q$. Suppose that there is a fixed number of customers who fly this route, $\frac{1}{8}$ of them business travelers, and $\frac{7}{8}$ of them tourists. For simplicity, assume that the cost of providing a given quality level on a flight is 0 for the airline, and that the cost of taking on an additional passenger is also 0.

a. Suppose the airline can only set one quality level on this route. What quality level should the airline set, and what price should it charge for a ticket?

The only two quality levels which make any sense are $q = 20$ and $q = 40$. At $q = 20$, they can charge a price of \$200, and sell to everyone, making a per customer profit of \$200. At a quality level of 40, they can charge a price of \$800, sell only to the business travelers, and earn a per customer profit of $\frac{1}{8} * \$800 = \$100$. Clearly, the former is better.

b. Now suppose it is possible for the airline to offer both a first class section and a coach section, with different quality levels. Suppose the quality and price of a coach seat is the same as the quality and price you solved for in b. What should the price of a first class ticket be to maximize profit?

If the price of a coach ticket is \$200, the price of a first class ticket needs to be \$400 to induce the business travelers to buy a first class ticket.

c. Now suppose the airline reduces the quality in coach by 2 units. What is the new price in coach? In first class? Show that this quality reduction increases the airline's profits.

Reducing coach quality to 18 lowers the price the airline can charge for a coach ticket to \$198, while raising the price they can charge for a first class ticket to \$440. This increases per customer profit to $\frac{7}{8}\$198 + \frac{1}{8}\$440 = \$228.25$. In part b, per customer profit was only $\$200 + \frac{1}{8}\$200 = \$225$.