Problem set 5

answers

Problem 1 What is the duopoly Cournot equilibrium if the market demand function is

$$Q = 1000 - 1000p \tag{1}$$

and each firm's marginal cost is \$.28 per unit? Note that $Q = q_1 + q_2$.

$q_1^C = 240 = q_2^c$, and the price is \$.52.

Problem 2 If the demand curve facing a duopoly is $P = 1 - Q$, and firm 1 has marginal cost of 0, while firm 2 has marginal cost of x, what are the Cournot equilibrium quantities the two firms produce? Which firm produces more and which has the higher profit?

Firm 1 produces $q_1^C = \frac{1}{3} + \frac{x}{3}$ and earns a profit of $\left(\frac{1}{3} + \frac{x}{3}\right)^2$ while firm 2 produces $q_2^C = \frac{1}{3} - \frac{2}{3}x$ and earns a profit of $\left(\frac{1}{3} - \frac{2}{3}x\right)\left(\frac{1}{3} + \frac{x}{3}\right)$. It is straightforward to show that firm 1's profit is higher, and that $q_1^C > q_2^C$. It is also possible to answer this question using graphs of best response curves and a bit of logic.

Problem 3 Lori employs Max. She wants him to work hard rather than to load. She considers offering him a bonus or not giving him one. All else the same, Max prefers to loaf.

		Max	
		Work	Loaf
Lori	Bonus	1,2	-1,3
	No bonus	3,-1	0,0

If Max and Lori choose actions simultaneously, what is the Nash equilibrium of this game?

The Nash equilibrium is 'No bonus, Loaf'. Note that this game is a prisoners dilemma.

Problem 4 Two firms are planning to sell either 10 or 20 units of their goods and face the following payoff matrix:

		Firm 2	
		10	20
Firm 1	10	30,30	50,35
	20	40,60	20,20

a.What is the Nash equilibrium if both firms make their decisions simultaneously? There are two Nash equilibria: '10, 20' and '20, 10'.

b. Suppose that firm 1 can decide first. What is the outcome?

1 produces 10, 2 produces 20 if 1 produces 10 and 10 if 1 produces 20

c. Now suppose that firm 2 can decide first. What is the outcome?

2 produces 10, 1 produces 20 if 2 produces 10, 1 produces 10 if 2 produces 20.

Problem 5 Suppose that Toyota and GM are considering entering a new market for electric automobiles and that their profits (in millions of dollars) from entering or staying out of the market are:

		GM: Enter	GM: Don't enter
Toyota	Enter	10,-40	250,0
	Don't enter	0,200	0,0

a. If the firms make their decisions simultaneously, do either or both firms enter?

Firm 1 will enter, Firm 2 will not enter, in a Nash equilibrium.

b. How would your answer change if the US government committed to paying GM a lump-sum subsidy of $50 million on the condition that it would produce this new type of car?

In this case, they both will enter.

Problem 6 Arnold, Bob and Chester are neighbors. Each can paint his house white, brown or green. Each would prefer that all three houses be painted the same color, no matter what color it is. However, IF the houses are not the same color, Arnold most prefers his house to be white, then brown, then green. Similarly, if the houses are not painted the same color, Bobs choice for his house is green, then brown then white. Chester likes his house to be green and brown equally well, and likes white least of all.

Suppose the three neighbors must simultaneously decide what color to paint their house (i.e. they all go to different paint stores at the same time, and so cannot consult).

a. Is it a Nash equilibrium for Arnold, Bob, and Chester to all paint their houses green? If not, explain who wants to deviate and how.

Yes. No one can do better by switching colors.

b. Is it a Nash equilibrium for Arnold to paint his house white and Bob and Chester to paint their houses green? If not, explain who wants to deviate and how.

No. Arnold would prefer to switch his house's color to green.

c. Is it a Nash equilibrium for Arnold to paint his house white, Bob to paint his house green and Chester to paint his house brown? If not, explain who wants to deviate and how.

Yes. No one can switch colors and get a more preferred outcome.

d. Is it a Nash equilibrium for Arnold to paint his house white, Bob to paint his house brown, and Chester to paint his house green? If not, explain who wants to deviate and how.

No. If Bob switches to green paint, he will be better off.

Problem 7 A thug wants the contents of a safe and is threatening the owner, the only person who knows the code, to open the safe. "I will kill you if you don't open the safe, and let you live if you do." Should the information holder believe the threat and open the safe? The table below shows the value that each person places on the various possible outcomes:

	Thug	**Safe's owner**
Open the safe, thug does not kill	4	3
Open the safe, thug kills	2	1
Do not open, thug kills	1	2
Do not open, thug does not kill	3	4

Such a game appears in many movies, including *Die Hard*, *Crimson Tide*, and *The Maltese Falcon*.

a. Draw the game tree. Who moves first?

The safe's owner would move first (obviously, should he be shot, he cannot open the safe). He decides to open or not, and the the thug decides to kill him or not.

b. What is the equilibrium?

The subgame perfect Nash equilibrium is for the owner to not open the safe, and the thug to not kill the owner.

c. Is the thug's threat credible?

No. Should the owner refuse to open the safe, the thug is better off not killing him.

d. Does the safe owner open the safe in a subgame perfect Nash equilibrium?

No.