Homework 1 (hard version)

due Wednesday, September 14 by 9am.

Instructions: This version of homework 1 may replace the shorter and easier version that I expect most students will choose to complete. It is intended to offer students a more mathematically rigorous treatment of consumer theory than offered in lecture. To complete the assignment, you will need simple calculus, be comfortable manipulating and solving systems of equations, and be familiar with such concepts as logarithms and exponents. If you have never been trained in these topics, or have forgotten them, it is still possible to complete this assignment with extra work on the math (and you may ask for my help with the math as well).

To provide some incentive for students to attempt this version, the maximum possible score is 140 points, as opposed to the 100 point maximum for the regular version. If a student turns in both versions of the homework, I will only grade the easier version.

Complete all problems. Answers may be handwritten or typed. Students may work together, but must independently formulate their own answers. Failure to do so will result in a grade of zero.

Problem 1 Gabe allocates $500/month to consumption of his two favorite comestibles: caviar (C) and merlot (M). His preferences over these two goods are represented by the utility function $u(C, M) = C^{\alpha}M^{1-\alpha}$, where $\alpha \in (0, 1)$. The price of caviar is $20/unit, while the price of merlot is $30/unit.

a. Calculate Gabe's marginal utility of caviar by taking the partial derivative of his utility function ($MU_C = \frac{\partial}{\partial C}u(C, M)$). Do the same for merlot.

b. Recall that in order to maximize utility, Gabe must have $\frac{MU_C}{p_C} = \frac{MU_M}{p_M}$. Suppose $\alpha = \frac{2}{5}$. Determine the ratio of Gabe's consumption of caviar to his consumption of merlot by applying the equation given above.

c. Again suppose $\alpha = \frac{2}{5}$. Gabe's budget line is described by the equation $p_C * C + p_M * M = \$500$. Combine your answer to b, which describes Gabe's optimal ratio of caviar to merlot, with Gabe's budget line, which describes what Gabe can afford, to determine the exact number of units of caviar and merlot that Gabe should buy to maximize his utility.

d. Leave α as an unspecified parameter for this problem. Suppose that the prices of merlot and caviar are constantly changing. Leave them as parameters (price of caivar is p_C and price of merlot is p_M) and repeat steps a-c to determine the optimal consumption bundle as a function of p_C and p_M.

e. Is Gabe's demand for caviar increasind or decreasing in the price of merlot? Are the goods complements or substitutes (or neither)?

Problem 2 In class, I claimed that the utility function is not intended to measure *cardinal* happiness, only *ordinal* happiness. If this is true, then we should be able to reorder the utility numbers assigned to bundles arbitrarily without changing the preferences they represent, so long as we preserve the ranking of the bundles. That is, saying bundle A gives me utility 20 and bundle B utility 50 should be entirely equivalent to saying bundle A gives me utility of $-3,000,000$ while bundle B gives me utility of $10^{100^{10^{1000^{10}}}}$. Which of the following functions preserve order (that is, $x > y$ implies $f(x) > f(y)$)?

a. $f(x) = ln(x)$

b. $f(x) = x^2$

c. $f(x) = \sqrt{x}$ (note: restrict this function to only $[0, \infty)$

d. $f(x) = 5x$

e. $f(x) = -7x^2$

f. Verify that the function $ln(x)$ does not change the underlying preferences by redoing question 1e with the utility function changed to $ln(C^\alpha M^{1-\alpha}) = \alpha ln(C) + (1-\alpha) ln(M)$ and checking whether you get the same answer or not.

Problem 3 The local swimming pool charges nonmembers $10 per visit. If you join the pool, you can swim for $5 per visit, but you have to pay an annual fee of $F.

a. Suppose you have an income of $100. Draw an indifference curve diagram to graphically determine the value of F such that you are indifferent between joining and not joining. Put number of swimming pool visits on the horizontal axis and dollars spent on all other goods on the vertical axis.

b. Suppose the pool charged you exactly that F. Would you go to the pool more or fewer times as a member than as a nonmember?

c. Suppose your utility function is $u(P, X) = P * X$, where P is number of pool visits, and X is dollars spent on all other things. Solve for the value of F which would leave you indifferent between joining and not joining.

Problem 4 Erik's utility over xylophones (X) and yarn (Y) is $u(X, Y) = \sqrt{X} + \sqrt{Y}$. He spends $\$I$ on these two goods.

a. Solve for Erik's demand for xylophones as a function of his budget I, the price of a xylophone p_X and the price of yarn p_Y.

b. Are xylophones and yarn complements or substitutes for Erik?

Problem 5 Carlos has the following utility function: $u(X, Y) = \sqrt{X} + \frac{1}{10}Y$. Suppose that the price of both goods is $1/unit. Carlos has a budget of $I to spend on these two goods.

a. Calculate $\frac{MU_X}{p_X}$ and $\frac{MU_y}{p_Y}$. Under what condition is $\frac{MU_X}{p_X}$ larger?

b. Calculate Carlos' optimal consumption of X and Y, as a function of I. Be careful: this is a special case of preferences known as *quasilinear preferences*. Hint: as I increases by $1, Carlos allocates that extra dollar towards whichever good has the higher value of $\frac{MU}{P}$.

c. What types of goods might X and Y be, given the consumption pattern you solved for in part b.?