Homework #2

answers

Problem 1 Suppose that most people will not speed if the expected fine is greater than $200. Given current police practices, the probability of being caught speeding is 20%.

a. How high must the actual fine for speeding (what you have to pay if you get a ticket) be to deter most people from speeding?

The expected fine is equal to the probability of being caught times the actual fine, so we have $\$200 = .2*X$, or $X = \$1000$.

b. Suppose the Lexington mayor caps the amount police can fine speeding motorists at $500. The police vow to step up enforcement in order to cintinue to deter speeding. How high must the probability of being caught speeding be in order to deter most people from speeding?

Here, $\$200 = p * \$500$, or $p = .4$. Police must catch speeders 40% of the time to deter most speeders.

Problem 2 Answer the following questions about expected utility and risk aversion:

a. True/false/uncertain: Rex, who is risk averse, prefers a salaried job that will pay him $100,000 with certainty to a commission-based job that will pay him $20,000 with probability .5 and $200,000 with probability .5. (Explain why you think it is true, false, or uncertain.)

Uncertain. The expected salary from the commissioned job is $110,000, but it is risky. The safe job has a (certain) salary of $100,000. If Rex is only slightly risk-averse, the increase in expected salary will be enough to compensate him for the extra risk, but if he is very risk averse, he'll definitely prefer the safe job. If it helps, note that someone who is extremely (infinitely) risk averse makes decisions based only on the worst possible outcome, however unlikely that outcome may be, while someone on the other extreme (risk neutral) cares only about the average outcome.

b. Kyle is an English PhD student; there is some chance he will get a prestigious university job that will pay him $80,000/year (probability p). If he does not get this job, he will have to take a job at Starbuck's, paying $25,000/year (probability $(1-p)$). Kyle's utility function over salary is given by $\sqrt{w}$, where w is the amount of his salary. Before Kyle has a chance to finish his PhD, he is offered a job at Dunder Mifflin paying $60,000/year. How low does p have to be before Kyle is better off taking the Dunder Mifflin job? (hint: p is some number between 0 and 1. For example, $p = .25$ means there is a 25% chance of Kyle getting the university job and a 75% chance of his working at Starbuck's.)

Kyle's utility from the Dunder-Mifflin job is $\sqrt{60,000}$, while his utility from waiting to graduate is $p\sqrt{80,000} + (1-p)\sqrt{25,000}$. The former is larger if $p \leq .696$. Therefore, if Kyle thinks there is less than a 69% chance Kyle gets the unviersity job, he'll leave school for the Dunder-Mifflin job.

Problem 3 A driver faces a 5% probability that his car will be in an accident and will be worth nothing. Consider three drivers with cars that have value $30,000. Abdulla's utility function over the value of his car W is $u(W) = ln(1+W)$. Bedriya's utility function is $u(W) = 100 + 0.5W$.

a. What is Abdulla's risk premium?

Abdulla's expected utility is $EU = .05 * ln(1) + .95 * ln(30,001) = 9.79$. His certainty equivalent is $ln(CE + 1) = 9.79$, or $CE = \$17853.3$. As his expected wealth is $0 * .05 + \$30,000 * .95 = \$28,500$, his risk premium is \$10,646.7.

b. What is Bedriya's risk premium?

Bedriya is risk neutral, so her risk premium is 0 (try calculating it following the model in part a).

c. Which of these two people is less likely to take on risk? Which is more likely? How do you know?

Abdulla is less likely to take on risk. For one, he has a higher risk premium. This reflects the fact that he is risk averse, while Bedriya is risk neutral.

Problem 4 Answer the following two questions about insurance markets:

a. Give a concrete example of adverse selection occurring in an insurance market. Explain how your example could lower the profit of an insurer, and what steps might be taken to mitigate the problem.

I decide to offer health insurance, and being a inherently nice person, I decide not to decline insurance to anyone, nor to charge a higher rate to people who are more likely to need medical care. I end up with lots of sick, old people buying insurance from me, and few healthy, young people. As such, if I price the insurance based on the average health of everyone in the country, instead of the average health of sick, old people, I will lose a lot of money.

b. Give a concrete example of moral hazard in an insurance market. Explain how your example could lower the profit of an insurer, and what steps might be taken to mitigate the problem.

The government establishes a system of universal health care, partly by prohibiting insurance companies from discriminating based on pre-existing conditions. But now there may be no need to buy health insurance when healthy, since, for example, a heart attack victim could call for health insurance while riding in the ambulance to the hospital. Thus a system predicated on universal care may discourage healthy people from buying insurance, thus making it hard to profitably pool risk. (Note: the solution to this is obviously to require healthy individuals to purchase insurance).

Problem 5 The production function for a firm's product is give by $q = f(K, L) = 5 * K * L$. The price of capital is \$10 and the price of labor is \$15.

a. Suppose the firm wishes to produce output of 500. List 5 combinations of capital and labor that the firm can transform into 500 output.

$(K, L) = (100, 1), (20, 5), (10, 10), (4, 25), (2, 50)$. There are many more such combinations.

b. For each of your 5 combinations from part a, give the cost of using that combination of capital and labor. Which is the lowest?

For example, for $(25, 4)$, the cost would be $\$10 * 25 + \$15 * 4 = \$310$. Similar for other input combinations.

c. For your lowest cost combination from part b, calculate the marginal product of capital (MPK) and the marginal product of labor (MPL).

For example, for $(25, 4)$, calculate the MPK by calculating $f(26, 4) = 520$ and calculate MPL by calculating $f(25, 5) = 625$. Therefore, $MPK = 20$ and $MPL = 125$.

d. For your answer in parts b-c, is your marginal product per dollar equal across the two inputs? If not, should the firm use more labor-intensive production or more capital-intensive production?

For $(25, 4)$, $\frac{MPK}{r} = 2$, while $\frac{MPL}{w} = 8.3$, so no, they are not equal. The firm could save money and get the same output by using more labor and less capital.

Problem 6 A firm has production function $f(K, L) = \sqrt{KL}$. In the short run, the firm has capital $K = 400$; this cannot be changed in the near future. The cost of a unit of capital is \$20, while the cost of a unit of labor is \$30.

a. In the short-run, how much labor does the firm need to employ in order to produce $q = 300$ output? What is the cost of producing 300 output?

With 400 capital, the firms needs L labor to produce $q = 20\sqrt{L}$ output, so setting $q = 300$ yields $L = 225$. Therefore, the cost is $225 * \$30 + 400 * \$20 = \$14,750$.

b. Repeat part a for $q = 400$, $q = 500$ and $q = 600$.

The cost of $q = 400$ is \$20,000, the cost of $q = 500$ is \$26,750, the cost of $q = 600$ is \$35,000.

c. What is the cost of producing q output in the short-run?

Producing q output requires $L = \frac{q^2}{400}$ labor. Including the fixed cost of the 400 capital (\$8,000), the total cost is $C(q) = 8,000 + \frac{3}{40}q^2$.

d. What is the marginal cost of producing a 301^{st} unit? A 401^{st} unit? If you are comfortable doing so, you may answer this question by writing down the marginal cost function directly, rather than recalculating total cost for $q = 301$ and $q = 401$.

To calculate MC directly, plug 300 and 301 into the above cost function and take the difference. To calculate it using slopes, MC is the derivative of the cost function, of $MC = \frac{3}{20}q$.

e. Given your answer to part c, draw a graph with the firm's average total cost, average variable cost, and marginal cost (hint: $MC = \frac{3}{20}q$).

f. Suppose the firm operates in a competitive market, and the price of the output good is $p = \$15$. How much output will the firm supply. What will the firm's profit be at this price?

A competitive firm supplies output until $p = MC$. Here, this occurs ar $15 = \frac{3}{20}q$, or $q = 100$. Its profit is $100 * 15 - 8000 - \frac{3}{40}100^2 = -\$7,250$, so the firm is losing money, but will continue to operate in the short-run, as ignoring its fixed costs, it is profitable.

g. Now suppose that the competitive price changes, to \$p. How much output will the firm supply, as a function of p? What is the minimum price that the firm needs in order to be profitable in the short-run? In the long-run?

At any price p, a competitive firm chooses output such that $p = \frac{3}{20}q$, or $q = \frac{20}{3}p$. The minimum price necessary for short-run profit is 0 (this is where $AVC = MC$, and the minimum price necessary for zero profit in the long run is where $ATC = MC$, or where $\frac{8000}{q} + \frac{3}{40}q = \frac{3}{20}q$, or $q = 326.6$. To translate this into a price, plug $q = 326.6$ into the firm's supply equation, $p = \frac{3}{20}q$, to get $p = \$48.99$.

h. Finally, suppose now the firm's marginal cost is still $MC = \frac{3}{20}q$, but the firm is a monopoly with demand curve equal to $p = 24 - \frac{3}{40}q$. Solve for the firm's profit-maximizing price and quantity. What is the monopolist's profit?

A monopolist's marginal revenue curve is $24 - \frac{3}{20}q$. Equating this with marginal cost gives $q = 80$, meaning the monopolist would charge a price of \$18. Its profit is $80 * 18 - 8000 - \frac{3}{40} * 80^2 = -\$7,040$.

Problem 7 Magee's Bakery, in downtown Lexington, estimates that its demand for transparent pies has a price elasticity of -1.5.

a. Suppose Magee's were to increase its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation.

Given that demand is elastic, a price increase would decrease revenue, decrease costs, and its unclear in which direction profits would move.

b. Suppose Magee's were to decrease its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation.

Revenue would increase, costs would increase, and profits might increase or decrease.

c. Finally, suppose for part c only that you now have the additional information that the marginal cost to Magee's of making one transparent pie is \$2 (and is constant). Magee's currently charges \$5 for each transparent pie. Should they increase or decrease this price, or should they leave it at \$5?

Recall from class that the optimal markup over marginal cost is given by $\frac{p-MC}{p} = -\frac{1}{\text{elasticity}}$. Therefore, the profit-maximizing price is $\frac{p-2}{p} = \frac{1}{1.5}$, or $p = 6$. Magee's should raise its price.