

## Homework 3

### answers

**Problem 1** Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by  $C = 200 + 2q^2$ , where  $q$  is the level of output and  $C$  is total cost. (The marginal cost of production is  $4q$ ; the fixed cost is \$200.)

a. If the price of watches is \$100, how many watches should you produce to maximize profit? Set  $p = MC$ , or  $100 = 4q$  to get  $q = 25$ .

b. What will the profit level be?

Revenue is  $\$100 * 25 = 2500$ . Costs are  $200 + 2 * 25^2 = 1450$ . Profit is therefore \$1,050.

c. What is the smallest the price can be for the firm to produce in the short-run? In the long-run?

In the short-run, the firm will produce so long as  $MC \geq AVC$ , or  $p \geq 0$ . In the long-run, set  $mc = AC$  to get  $q = 10$  and  $p = \$40$ . Firms will stay active in the long-run so long as price is at least \$40.

**Problem 2** Each of 100 firms in a competitive market has a cost function of  $c(Q) = 72 + 2Q^2$ , meaning each firm has a marginal cost of  $MC = 4Q$ . The market demand curve is  $Q^D = 600 - 5p$ .

a. Solve for the short-run equilibrium price and quantity (hint: first, solve for an individual firm's supply curve, and then multiply the quantity by 100 to get the market supply curve).

The equilibrium price in the short-run is \$20. The total quantity is 500.

b. What profit level will each firm earn in the short-run?

Each firm produces a quantity of 5, meaning each firm earns a profit of  $\$20 * 5 - 72 - 2 * 5^2 = -\$22$ .

c. In the long-run, will there be entry or exit from the market? Explain. What number of firms will be active in the long run?

Since firms are earning a negative profit in the short-run, there will be exit in the long run. This will raise the price. In the long-run, each firm will produce at the minimum of its ATC curve, or where  $\frac{72}{q} + 2q = 4q$ , i.e.  $q = 6$ . This means the long-run price is \$24, and the market quantity is 480, so there are 80 firms active in the long-run equilibrium.

**Problem 3** Magee's Bakery, in downtown Lexington, estimates that its demand for transparent pies has a price elasticity of  $-1.5$ .

a. Suppose Magee's were to increase its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation. Given that demand is elastic, a price increase would decrease revenue, decrease costs, and its unclear in which direction profits would move.

b. Suppose Magee's were to decrease its price. In which direction would each of the following move: revenue, total costs, profit? For each, answer "increase", "decrease", or "uncertain", along with a brief explanation.

Revenue would increase, costs would increase, and profits might increase or decrease.

c. Finally, suppose for part c only that you now have the additional information that the marginal cost to Magee's of making one transparent pie is \$2 (and is constant). Magee's currently charges \$5 for each transparent pie. Should they increase or decrease this price, or should they leave it at \$5?

Recall from class that the optimal markup over marginal cost is given by  $\frac{p-MC}{p} = -\frac{1}{\text{elasticity}}$ . Therefore, the profit-maximizing price is  $\frac{p-2}{p} = \frac{1}{1.5}$ , or  $p = 6$ . Magee's should raise its price.

**Problem 4** Answer the following two questions about perfectly competitive markets:

a. There are many taxi drivers in New York City, all of whom sell identical taxi rides. Taxi drivers in New York are required to hold a special license to operate, and the government only issues 5,000 licenses. Will these drivers necessarily earn zero profit in the long run, or is it possible for them to earn positive profit?

In a competitive market, firms enter or exit until there is zero profit. Suppose that in the market for NYC taxi licenses, this would happen when there are 7,500 tax cabs. That there are only 5,000 licenses means that entry is artificially halted before the long-run equilibrium number of taxis. While further entry would shift the market's supply curve to the right, lowering price and thus profit, since this entry is prohibited, price would stay about the zero-profit price and profits would stay positive. So, no, it is not necessarily the case that a competitive market without free entry will converge to zero economic profits.

b. Carlos owns a gas station in Lexington. Carlos estimates his total costs are given by  $TC = 400 + .01q^2$  and his marginal costs are given by  $MC = .02q$ . The price of gas is currently \$3.75/gallon. Should Carlos stay in business in the long run? In the short run?

The highest profit Carlos can obtain is  $-\$48/44$  (to find this, set the price of \$3.75 equal to his marginal cost to get Carlos' profit-maximizing quantity, and then calculate profit=revenue - cost from there). Since he is losing money, he clearly should shut down in the long run. However, net of his fixed costs of \$400, Carlos is actually profitable, and so there is no reason to shut down in the short run.

**Problem 5** John runs the only carwash in town; he is a monopolist. John estimates his daily demand for carwashes is given by the expression  $Q = 100 - 4P$ , where  $Q$  is the number of carwashes drivers will purchase at price  $P$ . It costs John \$5 in electricity, soap, etc to run the carwash once. Additionally, John has fixed costs of \$300/day.

a. What price should John set for a carwash? What will be his daily profit at this price?

Note that John's demand is  $p = 25 - \frac{1}{4}q$ , and so his marginal revenue is  $MR = 25 - \frac{1}{2}q$ . Setting this equal to marginal cost, we have  $q = 40$ , and  $p = \$15$ , for a daily profit of \$100.

b. If John were to lower his price by \$1, he would sell more carwashes, and still be able to charge a price above his marginal cost. Explain intuitively why it would not be profit-maximizing to do so.

If John lowered his price by a dollar, he could sell 4 more carwashes for a price greater than his marginal cost of \$5. However, John would have to lower his price for all his customers not just the marginal customers, and doing so would cause John to lose money overall. A profit-maximizing monopolist sells a lower quantity than would be efficient, in order to be able to charge a high price.

c. What is John's elasticity of demand at his profit-maximizing price? Is it elastic or inelastic? If it is elastic, why does he not lower his price, as this would surely bring in many more customers? If inelastic, why does he not raise his price?

At a price of \$15 and a quantity of 40, the elasticity of demand is  $-1.5$ . His demand is elastic. If he lowered his price, this would increase his revenues, but it would also increase his costs. At John's current price of \$15, the marginal increase in revenue from a price decrease exactly equals the marginal increase in costs from a price decrease. Were John to decrease his price below \$15, the second effect would dominate.

**Problem 6** Suppose a profit-maximizing monopolist is producing 800 units of output and is charging a price of \$40 per unit.

- a. If the elasticity of demand for the product is  $\epsilon = -2$ , find the marginal cost of the last unit produced.

\$20

- b. What is the firm's percentage markup of price over marginal cost?

a 50% markup over marginal cost

- c. Suppose that the average cost of the last unit produced is \$15 and the firm's fixed cost is \$2,000. Find the firm's profit.

If we take "average cost" to mean AVC, then profit is  $800 * (\$40 - \$15) - \$2000 = \$18,000$ .

**Problem 7** Skywalker Farms supplies water to the town of Mos Eisley (they are "the water company", i.e. a monopoly supplier). Given their infrastructure of pipes, treatment centers, etc, the marginal cost of supplying one gallon of water is only \$.50 (and is constant), though maintaining their infrastructure has a daily fixed cost of \$75,000. Mos Eisley's daily demand for gallons of water is  $P = 20 - \frac{1}{1000}Q$ .

- a. Solve for Skywalker Farm's profit-maximizing price. What daily profit do they earn when they charge this price?

Skywalker Farms will charge a price of \$10.25 (selling a quantity of 9,750 gallons of water), and earns a profit of \$20,062.50.

- b. Emperor Palpatine, in a rare burst of populism, suggests regulating Skywalker Farms by capping the price they are allowed to charge at \$9/gallon of water. What profit will Skywalker Farms earn at this price?

We saw in part a that Skywalker Farms would like to charge a price of \$10. Here, they are prohibited from doing so by the government, and must charge a price of only \$9. At this price, the quantity demanded is 11,000, and so their profits are \$18,500.

- c. You are hired as a consultant to advise the Galactic Empire on this matter. Explain in words why such a price ceiling might be a good idea (i.e. could such a price ceiling be welfare-improving?).

A monopolist has an incentive to decrease the amount of output sold relative to what would be efficient (where willingness to pay as measured by the demand curve equals marginal cost). This results in a deadweight loss. A price ceiling dampens this incentive, increasing the amount of output produced, and decreasing the amount of deadweight loss.