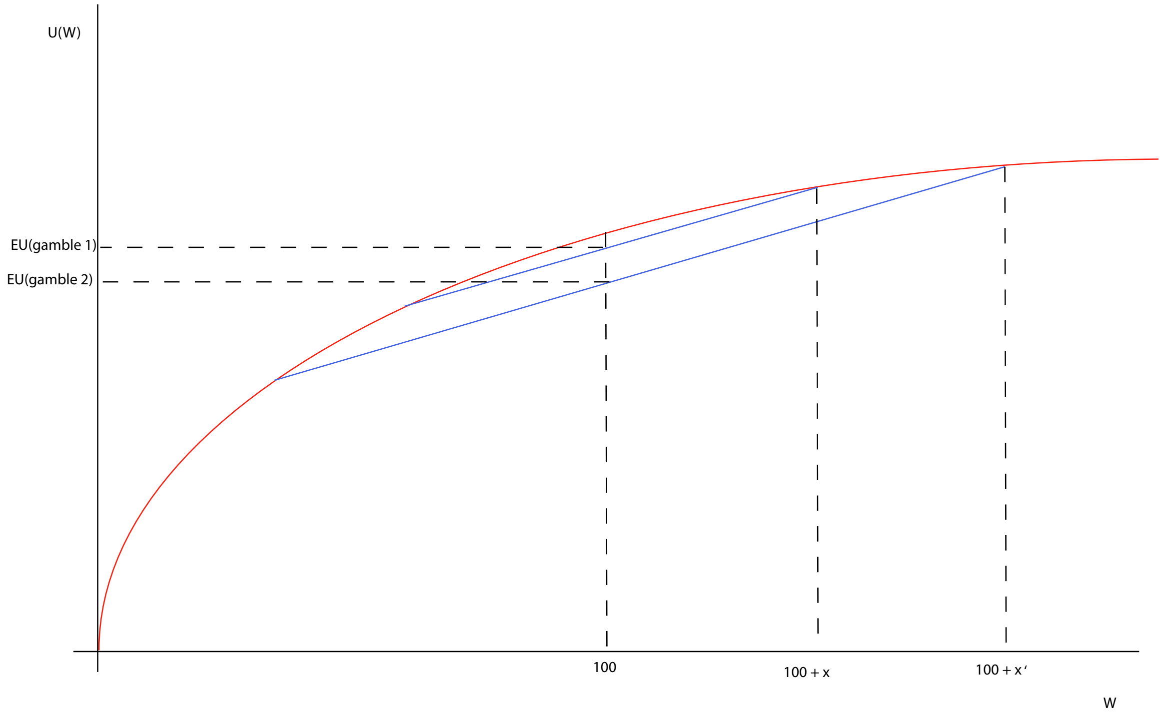
Homework 8

Problem 1 Suppose an individual is risk averse and has to choose between \$100 with certainty and a risky option with two equally likely outcomes: $\$100 - x$ and $\$100 + x$. Use either a graph or math to show that this person's risk premium is increasing in x.



See above figure. Let gamble 1 be a coin flip between $100 + x$ and $100 - x$ and gamble 2 between $100 + x'$ and $100 - x'$, with $x' > x$. Each gamble has an expected value of \$100. The individual's expected utility is lower with gamble 2, however. The amount he would accept with certainty as an alternative to the gamble is thus lower for gamble 2, and so the risk premium is greater for gamble 2.

Problem 2 By next year, the stock you own has a 25% probability of being worth \$400 and a 75% probability of being worth \$200. What are the expected value and the variance?

Expected value is $.25 * 400 + .75 * 200 = 250$. Variance is $.25(400 - 250)^2 + .75(200 - 250)^2 = 7500$.

Problem 3 Suppose that Gladys has a utility function of $u(w) = \sqrt{w}$ and an initial wealth of \$100.

a. How much of a risk premium would she want to participate in a gamble that has a 50% probability of raising her wealth to \$120 and a 50% probability of lowering her wealth to \$80?

This gamble has an expected value of \$100. Her expected utility is $.5\sqrt{120} + .5\sqrt{80} = 9.95$. The amount which would give her this utility if she got it with certainty is given by $\sqrt{w} = 9.95$, i.e. $w = \$98.99$. The difference between this amount and the gamble's expected value, \$1.01 is her risk premium.

b. Redo part a. if Gladys' utility function is instead given by $u(w) = \ln(w)$.

Now, her expected utility is $.5\ln(120) + .5\ln(80) = 4.58$, so the minimum amount she would take with certainty over the gamble solves $\ln(w) = 4.58$, or \$97.98, so her risk premium is \$2.02.

Problem 4 Lisa just inherited a vineyard from a distant relative. In good years (no rain or frost), she earns \$10,000 from the vineyard. In bad years, she earns only \$2,500. She estimates that the probability of a good year is 60%.

a. Calculate the expected value and variance of Lisa's income from the vineyard.

Expected value is $.6 * 10,000 + .4 * 2,500 = 7,000$. The variance is $.6 * (10,000 - 7,000)^2 + .4 * (2,500 - 7,000)^2 = 9,450,000$.

b. Suppose Lisa has utility function $u(w) = \sqrt{w}$, where w is her wealth. Assume she has 0 initial wealth. Ethan, a grape buyer, offers to lease the vineyard from Lisa for \$6,500 next year, so that Lisa would get \$6,500 regardless of whether it was a good year or a bad year. Will Lisa accept this offer?

Her expected utility from the gamble is $.6\sqrt{10,000} + .4\sqrt{2,500} = 80$. She will accept any amount with certainty that gives her at least 80 utility. Her utility from getting \$6,500 with certainty is $\sqrt{6,500} = 80.62$, so yes, she will accept Ethan's offer.

c. Why might Ethan make such an offer? Give three reasons, and explain each. One of these reasons should refer to his attitude toward risk.

It could be Ethan is naturally less risk averse than Lisa (for example, he may be risk neutral). It could be that he has a much greater wealth than Lisa, and so even if he has the same utility function as Lisa, the sums involved may be much smaller to him, making him less risk averse. Finally, it could be that he can diversify the risk of a bad grape harvest, perhaps by buying other vineyards in other locations or by buying stock in a beer company.

Problem 5 Suppose that two investments have the same three payoffs, but the probabilities associated with each payoff differs, as follows:

payoff	Probability (investment A)	Probability (investment B)
\$300	.10	.30
\$250	.80	.40
\$200	.10	.30

a. Find the expected return and standard deviation of each investment.

Investment A has expected value $300 * .1 + 250 * .8 + 200 * .1 = 250$, variance $.1 * (300 - 250)^2 + .8 * (250 - 250)^2 + .1 * (200 - 250)^2 = 500$, and standard deviation $\sqrt{500} = 22.36$. Investment B has expected value 250, variance 1500, and standard deviation 38.73.

b. Jill has the utility function $u(w) = 5w$, where w is the investment's payoff (assume she has initial wealth 0). Which investment does she prefer?

Jill's expected utility from both investment A and investment B is 1250. She is indifferent between the two gambles. We can tell this immediately by recognizing she is risk-neutral (linear utility function), meaning she chooses the gamble with the higher expected value.

c. Ken has the utility function $u(w) = 5\sqrt{w}$. Which investment will he choose?

EU(investment A) = 78.98, EU(investment B) = 78.82, so he will choose investment A. We can tell this immediately by recognizing that Ken is risk-averse (concave utility function), and so will choose the gamble with the lower standard deviation, since they have the same expected value.

d. Delores has the utility function $u(w) = 5w^2$. Which will she choose?

Without doing any calculation, she is risk loving (convex utility function) and so will choose the gamble with the higher standard deviation, as they both have the same expected value. thus she will choose investment B.

Problem 6 Larry owns a house worth \$100,000. There is a 10% chance it will burn down, in which case it will be worth \$20,000. There is a 90% chance it will not burn down and continue to be worth \$100,000. Larry's utility function is $u(w) = \sqrt{w}$, where w is how much his house is worth.

a. Suppose Eagle Insurance offers Larry \$1 worth of insurance for 10 cents. That is, Larry can transfer wealth to the state of the world in which the house burns down from the state in which it does not at the rate 10:1. How much insurance will Larry purchase?

Larry is risk averse, so if offered actuarially fair insurance (i.e. the insurance company will break even, on average, on this policy), he will fully insure, meaning he will buy \$72,727.27 worth of insurance at a cost of \$7,272.72. Showing this rigorously requires a bit of calculus, we can also just keep track of the fact that a risk averse person buys enough just enough actuarially-fair insurance to equalize his wealth across states of the world, which is true generally.

b. Is Eagle's price for \$1 of insurance likely to be higher or lower than that of part a? Why? Will Larry buy more or less insurance than in part a?

The insurance company will have to charge a higher than break even price to cover overhead, employees, etc. Thus, Larry will less than fully insure, i.e. he will purchase less than \$72,727.27 worth of insurance.