Some notes on quantity discrimination (airline example)

1 Setup

- An airline sets one or more *quality levels* q throughout its plane. For example, 1^{st} class and coach could be two different quality levels.
- For simplicity, assume that the cost of producing quality level q is 0.
- The airline has two groups of customers, tourists, with demand $p_1 = a_1 - q_1$, and business travelers, with demand $p_2 = a_2 - q_2$, where $a_2 > a_1$. Let q_1 be the quality level in the section in which the tourist sits and q_2 the quality in the section in which business travelers sit.
- Think of the demand curves as measuring willingness to pay for different quality levels. For example, tourists have a willingness to pay for quality $q = 1$ of $a_1 - 1$. Perhaps $q = 1$ represents a heated plane. Tourists have an additional willingness to pay for quality level $q = 2$ of $a_1 - 2$. Perhaps quality $q = 2$ is a heated plane with seats. And so on. The total willingness to pay for a seat with quality $\tilde{q}$ is thus the area under the demand curve, from $q = 0$ to $q = \tilde{q}$.
- Suppose fraction t of all customers are business travelers, and fraction $1 - t$ are tourists.
- What tradeoffs does the airline set, what quality levels should it set, and what prices does it charge?

2 Case I: only one quality level is possible

- Area under a tourist's (business traveler's) demand curve represents the maximum price a tourist (business traveler) would be willing to pay for a ticket in a class with that quality.
- Suppose it is only possible for the airline to set one quality level (i.e. coach for everyone). I claim there are only two quality levels that make any sense: $q = a_1$, with price equal to \$A, or $q = a_2$, with price equal to \$A+\$B+\$C (see figure 2 below)
- If $q = a_1$, per-customer profits are \$A. If quality is $q = a_2$, per-customer profits are $t(\$A + \$B + \$C)$. Therefore, $q = a_1$ is optimal so long as t is not too big, and so long as a_2 is not too much bigger than a_1. Specifically, the airline sets $q = a_1$ if $A \geq \frac{t}{1-t}(B + C)$, or if $a_1^2 - 2a_2\frac{t}{1-t}(a_2 - a_1) \geq 0$.

3 Case II: airline can set different quality levels

- Good first guess is airline sets quality $q = a_1$ in coach at price $\$A$, and quality $q = a_2$ in 1^{st} class at price $\$A + \$B + \$C$. However, under this guess, business travelers prefer a coach ticket (extra surplus \$B) to a 1^{st} class ticket (extra surplus 0).

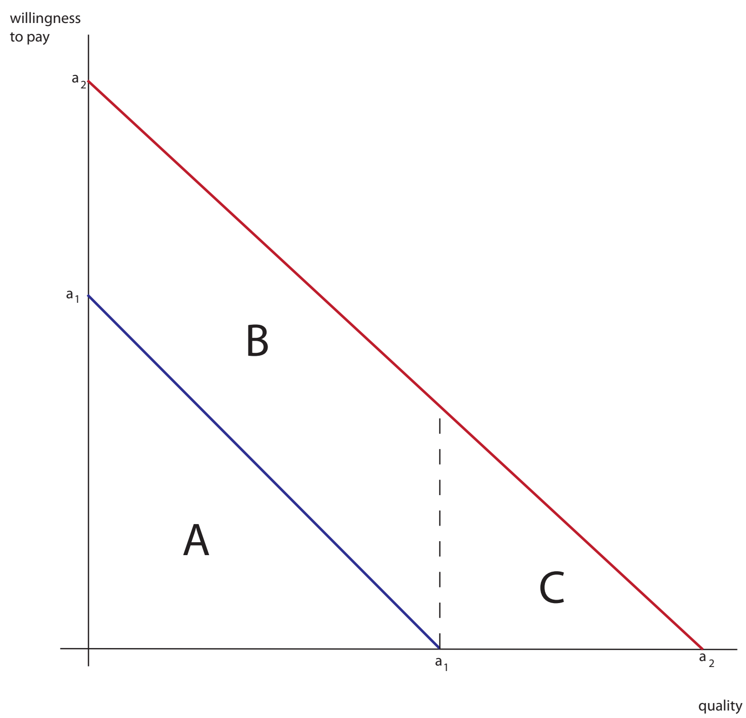

Figure 1: At quality level a_1, tourists are willing to pay a maximum of \$A, while business travelers are willing to pay a maximum of \$A+\$B. At quality level a_2, tourists are willing to pay a maximum of \$A, while business travelers are willing to pay a maximum of \$A+\$B+\$C.

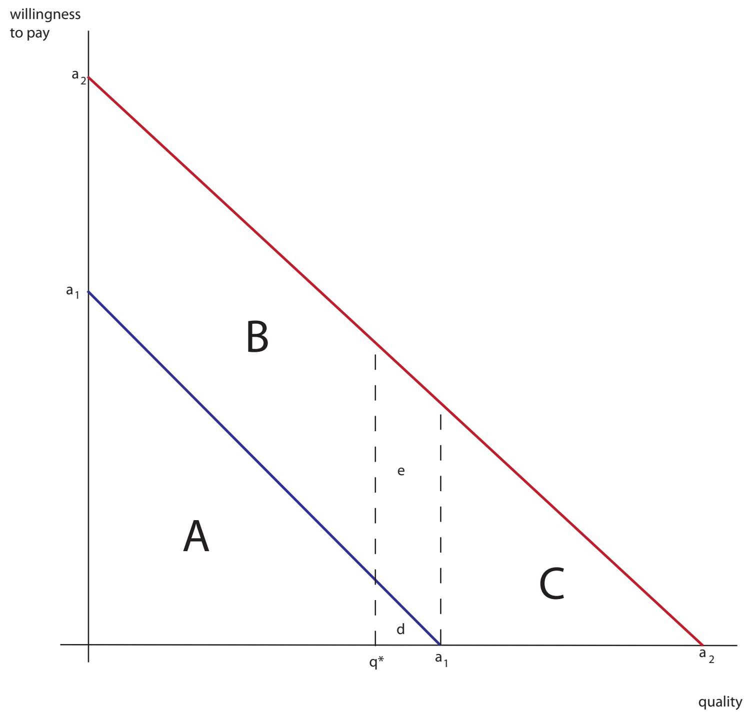

Figure 2: If the airline can set multiple quality levels throughout the plane (for example, first class and coach), they will lower the quality in coach in order to be able to increase the price of a first class ticket.

- Therefore, the airline lowers the price of a 1^{st} class ticket by $\$B$, in order to make business travelers willing to purchase a 1^{st} class ticket over a coach ticket. This increases their profits to $\$A + t\$C$. Note that this is a higher profit than can be earned when the airline is constrained to only be able to set one quality level.

- However, by lowering the quality in coach, the airline can increase the price it can charge for a 1^{st} class ticket despite lowering the price it can get for a coach ticket. The airline lowers quality in coach until its profits $\$A + t\$C$ are maximized. The more business travelers, the lower the quality in coach, and the greater the difference between a_2 and a_1, the lower the quality in coach.

- Consider, for example, quality level q^* in Figure 3. If the airline sets this quality level in coach, the price it can charge for a coach ticket ($\$A$) decreases by \$d, yet the price it can charge for a 1^{st} class ticket ($\$A + \$C + \$d + \$e$) increases (by \$d+\$e). The airline optimally sets q^* so that the decrease in price of a coach ticket, \$A, is exactly offset by the increase in a 1^{st} class ticket, weighted by the fraction of all customers who are business travelers. That is, the airline sets q^* such that $(1 - t)d = te$.

- The moral of this story is that coach is made to be unpleasant not to annoy the people who sit in coach (tourists), but to annoy the people who *don't* sit in coach (business travelers). The happier business travelers are in 1^{st} class relative to coach, the higher price they can be charged.