Problem set 1

answers

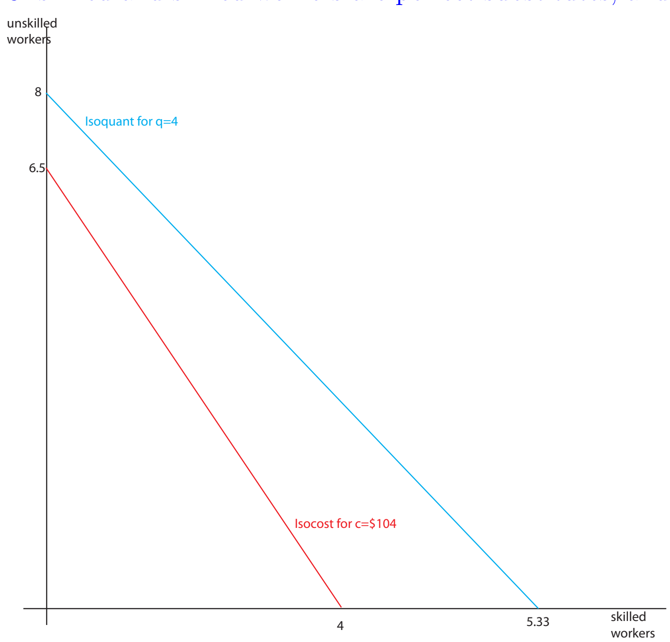
Problem 1 The Bouncing Ball Ping Pong Co. sells table tennis sets that consist of two paddles and one net. What is the firm's long-run expansion path if it incurs no costs other than what it pays for paddles and nets, which it buys at market prices? How does your answer depend on the relative prices of paddles and nets?

The expansion path simply measures the ratio in which the firm should use the two inputs to minimize costs. Here, the two inputs are perfect complements (the firm must always use 2 paddles and one net to make one set, a set with 3 or more paddles and only one net is no better than a set with only 2 paddles). Therefore, the firm buys inputs in the ratio of $2N = 1P$, where N is the umber of nets and P the number of paddles. $2N = P$ is the long-run expansion path. This is independent of the cost of the inputs. Note that the cost function would be $c(Q) = 2p_P Q + p_N Q$.

Problem 2 In February 2003, Circuit City Stores replaced skilled sales representatives who earn up to \$54,000/year with relatively unskilled workers who earned \$14 to \$18/hour. Suppose that sales representatives sell one particular Sony high definition TV. Let q represent the number of TV's sold per hour, s the number of skilled sales reps per hour, and u the number of unskilled reps per hour. Working eight hours/day, each skilled worker sells 6 TV's/day, while each unskilled worker sells 4. The wage rate of the skilled workers is $w_s = \$26/hour$, and the wage rate of the unskilled workers is $w_u = \$16/hour$.

a. Using a graph, show the isoquant for $q = 4$ with skilled sales representatives on the x-axis, and unskilled on the y-axis.

Unskilled and skilled workers are perfect substitutes, and hence the relevant isoquant is a straight line:



b. Draw a representative isocost line for $c = \$104$ per hour.

See the picture for part a.

c. Using an isocost-minimizing diagram, identify the cost-minimizing number of skilled and unskilled reps to sell $q = 4$ TV's per hour.

If you drew your graph to scale, you can tell graphically that the isocost is steeper than the isoquant, and so the cost-minimizing way to get to 4 output is to use only unskilled labor. If your graph is not to scale, you can also check this mathematically. The MPL per hour for an unskilled worker is $\frac{1}{2}$, while the cost of an unskilled worker is \$16. Therefore, for unskilled workers, $\frac{MPL}{w} = .03125$. For skilled workers, $\frac{MPL}{w} = .028846$. Therefore, unskilled workers are more cost-effective and the firm should hire only them, and no skilled workers. Therefore, to sell 4 TV's/hour, the firm should hire 8 unskilled workers and no skilled workers.

Problem 3 A bottling company uses two inputs to produce bottles of the soft drink Sludge: bottling machines (K) and workers (L). The isoquants have the usual smooth, curvy shape. The machines cost \$1,000 per day to run; the workers earn \$200/day. At the current level of production, the marginal product of the machines is an additional 200 bottles per day, while the marginal product of labor is 50 more bottles per day. Is this firm producing at minimum cost? If so, explain why. If not, explain how the firm should change the ratio of inputs it uses to lower its cost.

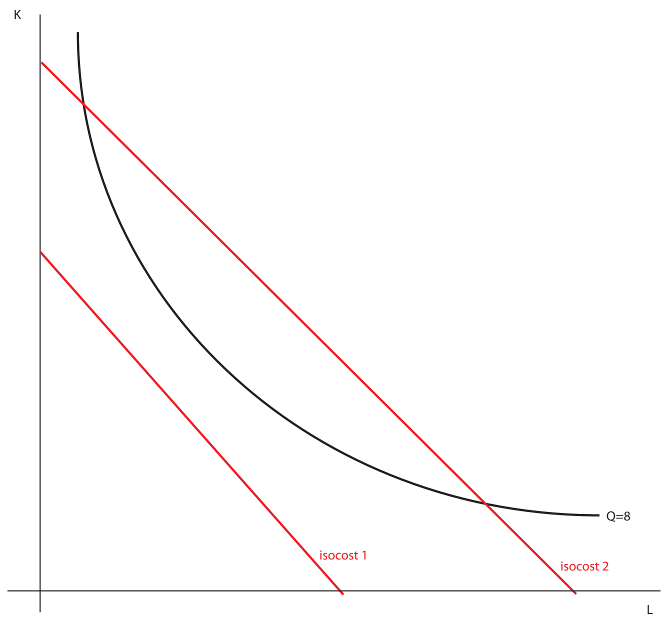
No, it is not minimizing costs. $\frac{MPL}{w} = \frac{50}{200} = \frac{1}{4}$, while $\frac{MPK}{r} = \frac{200}{1000} = \frac{1}{5}$, and so dollars are transformed into output more quickly using labor than using capital. The firm should therefore spend less on capital and more on labor.

Problem 4 Suppose Molly produces rocking chairs using both machines (K) and labor (L). Specifically, suppose that if Molly uses K machines and L labor, she produces $f(L, K) = \frac{1}{2}L^{\frac{1}{4}}K^{\frac{1}{2}}$ rocking chairs per hour. Suppose that the going rate for an hour of labor is \$20, while her rental rate of capital is \$10 per hour.

a. Using one of the methods outlined in class, sketch isoquants corresponding to outputs of $Q = 2$, $Q = 4$, and $Q = 8$.

To sketch the $Q = 2$ isoquant, set $2 = \frac{1}{2}L^{\frac{1}{4}}K^{\frac{1}{2}}$. Multiply both sides by 2 and take both sides to the 4th power to get $256 = LK^2$. Therefore, $K = \frac{16}{\sqrt{L}}$. Use a calculator to plot this, or just plot a few points and then connect the dots (e.g. $K = 16, L = 1$, $K = 4, L = 16$, $K = 1, L = 256$, $K = \frac{16}{5}, L = 25$, etc.

b. On a new graph, copy over the $Q = 8$ isoquant. Then, draw two isocost lines, one representing a cost that is *above* the minimum cost of producing $Q = 8$ output, and one representing a cost that is *below* the minimum cost. Be precise.



c. Using your graph from b, explain how you know that the minimum cost of producing 8 output is between the two cost levels identified in part b.

d. Go as far as you can, graphically or mathematically, in identifying the minimum cost of producing 8 units.

To get $Q = 8$ at a minimum cost requires $\frac{MPK}{r} = \frac{MPL}{w}$. Here, this means

$$\frac{\frac{1}{4}\frac{L^{\frac{1}{4}}}{K^{\frac{1}{2}}}}{10} = \frac{\frac{1}{8}\frac{K^{\frac{1}{2}}}{L^{\frac{3}{4}}}}{20}$$
$$\iff 4L = K$$

and so capital and labor must be used in the ratio $4L = K$. To get $Q = 8$, then, requires $8 = \frac{1}{2}L^{\frac{1}{4}}(4L)^{\frac{1}{2}}$, which yields $L = 16$ and $K = 64$.

e. (extra credit)[1] Identify Molly's cost function, $c(Q)$. Show that if the price of a rocking chair is set at \$320, Molly should produce 64 chairs/hour.

f. Fact: Molly maximizes her profits by producing 64 chairs, using 256 labor hours and 1,024 machine hours. Explain why MPL and MPK are not equal, despite Molly maximizing profits.

MPL is 2 times as much as MPL when the firm is profit-maximizing (try to verify this). While MPL is greater than MPK, meaning that labor is more productive than capital, it is also more expensive. When correcting for this by dividing by the costs of labor and capital, respectively, labor and capital have the same productivity per dollar.

Problem 5 Doug's production function for his economics midterm score is $\min\{O, 2I\}$, where I is time spent in class each week and O is time spent out of class studying each week. With any time not spent studying or in class, Doug can earn \$10 painting houses.

[1] in the sense that there may be a corresponding extra credit question on the quiz.

Describe Doug's "cost function" for achieving a midterm score of Q, that is the cost per week, measured in foregone wages, required for Doug to get a score of Q on the midterm.

Since time in class and time out of class are perfect complements, Doug should set $2I = O$. To get a score of Q on his midterm, then, requires $O = Q$ and $I = \frac{Q}{2}$. The cost of a score of Q is then $15Q$.