Problem set 3

"due" 3/4/2010

Problem 1 The Grand Theater is a movie house in a medium-sized college town. On any given night, if the theater is open, it must pay \$500 in fixed costs (paying electricity, ushers, etc) regardless of how many people come to the theater. If the theater is closed, its costs are 0. There are two groups of people who come to the Grand Theater, students and non-students. Students have demand function $q_s = 220 - 40p_s$ while non-students have demand function $q_n = 140 - 20p_n$.

a. Suppose that the theater cannot tell students apart from non-students. What price will it charge? How many students will come? How many non-students? What will the profits of the Grand Theater be?

In this case, the total demand will be $Q = 360 - 60p$, marginal revenue will be $6 - \frac{1}{30}Q$, and so profit-maximizing quantity will be 180 patrons, at a price of \$3. Profits are \$40 (\$3*180-\$500).

b. Now suppose that the cashier can accurately tell students from non-students by asking students to show their student IDs. Students cannot resell their tickets to non-students after purchase. Will the Grand charge students and non-students different prices? What will these prices be? What will be the Grand's profits?

Treating students and non-students as two separate groups, the theater will charge students \$2.75 and sell them 110 tickets, while charging non-students \$3.50 and selling them 70 tickets. Total profits are \$47.50.

c. Finally, suppose that the Grand Theater can only hold 150 people. If the theater is able to charge separate prices to students and non-students, what prices will it charge, and how many students and non-students will come?

We know that the number of students admitted plus the number of non-students admitted must equal 150. We also know that to be maximizing profits, it must be that the marginal revenue from the last student admitted must equal that from the last non-student admitted. Were this not so, the theater could admit one less from the low marginal revenue group and one more from the high marginal revenue group, and increase profits. Thus, the following two equations define the solution:

$$q_s + q_n = 150$$
$$5.5 - \frac{1}{20}q_s = 7 - \frac{1}{10}q_n$$

which has solution $q_n = 60$, $q_s = 90$, meaning $p_s = \$3.25$ and $p_n = \$4$.

Problem 2 Your firm produces 2 products, each at 0 marginal cost. You face four types of customers, each comprising 25% of your total customers (say you have N total customers). The groups have the following willingness to pay for your product:

customer	good 1	good 2
A	\$25	\$100
B	\$40	\$80
C	\$80	\$40
D	\$100	\$25

a. Compare selling these two products separately to bundling them and selling them together for one price. Which leads to a higher profit?

To sell the two goods bundled, you would charge a price of \$120 and make a profit of \$120N. To sell them separately, you would charge a price of \$80 for each good, and get profits of \$80N. Clearly, bundling them is superior to not.

b. Now consider the possibility that you sell these goods both bundled and unbundled (that is, you set three prices, one for good 1 alone, one for good 2 alone, and one for the bundle of good 1 and good 2). Would doing this improve upon the outcome of part a? Explain.

It does not seem that you could increase profits from the bundling outcome of part a by also selling the products separately.

c. Now suppose that the production of each good entails a marginal cost of \$30. How does this information change your answers to a and b above? Is it better to sell the goods unbundled, bundled, or both bundled and separately?

Here, A values good 1 and D values good 2 at less than their marginal costs. Consider a bundled price of \$120, a price of \$95 for good 1 and a price of \$95 for good 2. In this case, A buys only good 2, B and C buy the bundle, and D buys only good 1. Profit is \$62.5N. This is higher than could be achieved with only bundling or only separate prices.

Problem 3 Firm 1 and Firm 2, are Cournot competitors. The market demand curve is $p = 120 - q_1 - q_2$. Firm 1 has a constant marginal cost of \$20, while Firm 2's is \$10.

a.What are the Cournot equilibrium quantities? What is the equilibrium price?

The Cournot equilibrium is $q_1^C = 30$, $q_2^C = 40$. The equilibrium price is \$50.

b. How much profit does each firm make in the Cournot equilibrium?

Firm 1 makes a profit of \$900. Firm 2, \$1,600.

Problem 4 Consider a market with demand $Q = 20 - P$ supplied by two firms engaged in Bertrand price competition. That is, each firm simultaneously names a price, and then whichever firm names the lower price sells $20 - P$ units, and the firm naming the higher price sells 0. Assume both firms have a constant marginal cost of production equal to \$5.

a. Under price competition, what price does firm 1 set in an equilibrium? Firm 2? Under Bertrand price competition, each firm will set $p = \$5$.

b. How much profit does firm 1 earn? Firm 2? They will each earn 0 profit.

c. Suppose now that the two firms form a cartel and act as a monopolist. By how much would they be able to increase total profit? What would happen to price? If they collude and act as a monopolist, they produce 7.5 total units (3.75 each if they are splitting production), and charge a price of \$12.50, for a total profit of \$56.25 (\$28.12 apiece if they are splitting it).

Problem 5 Two Cournot quantity competitors face the following demand curve:

$$P = 9 - q_1 - q_2$$

to keep the problem simple, assume that both firms can produce at zero cost.

a. Solve for the Nash equilibrium. What are q_1 and q_2 in equilibrium, and how much profit does each firm earn? $q_1 = 3$, $q_2 = 3$, and each firm earns a profit of \$9.

b. Suppose firm 1 has the option of outsourcing 4.5 units of production at cost $\$K$, and producing nothing on its own (essentially, it is paying $\$K$ to commit to producing 4.5 units). Firm 2 is aware of firm 1's outsourcing decision prior to its determining how much to produce itself. What is the maximum value of K for which outsourcing is a good idea for firm 1? If firm 1 can commit to producing 4.5 units before firm 2 makes any production decisions, firm 2 then maximizes its profits by setting $q_2 = 2.25$, and so the price will be \$2.25, and firm 1 will earn a profit of \$10.12. Therefore, he is willing to pay up to \$1.12 in order to commit to producing 4.5 units of output.

Problem 6 Duopoly quantity-setting firms face the market demand

$$p = 150 - q_1 - q_2$$

Each firm has a marginal cost of \$60/unit.

a. What is the Cournot equilibrium? $q_1 = 30$, $q_2 = 30$, wtih $p = \$90$, and each firm earning a profit of \$900.

b. What is the Stackelberg equilibrium when firm 1 moves first? $q_1 = 45$, $q_2 = 22.5$, the price is $p = \$82.50$, and firm 1 earns a profit of \$1,012.50, while firm 2 earns \$506.25.

c. If firm 1 is currently a Cournot competitor, how much would it be willing to pay to become a Stackelberg leader? \$112.50