Problem set 4

answers

Problem 1 Lori employs Max. She wants him to work hard rather than to load. She considers offering him a bonus or not giving him one. All else the same, Max prefers to loaf.

		Max	
		Work	Loaf
Lori	Bonus	1,2	-1,3
	No bonus	3,-1	0,0

If Max and Lori choose actions simultaneously, what is the Nash equilibrium of this game?

The Nash equilibrium is 'No bonus, Loaf'. Note that this game is a prisoners dilemma.

Problem 2 Two firms are planning to sell either 10 or 20 units of their goods and face the following payoff matrix:

		Firm 2	
		10	20
Firm 1	10	30,30	50,35
	20	40,60	20,20

a.What is the Nash equilibrium if both firms make their decisions simultaneously? There are two Nash equilibria: '10, 20' and '20, 10'.

b. Suppose that firm 1 can decide first. What is the outcome?

1 produces 10, 2 produces 20 if 1 produces 10 and 10 if 1 produces 20

c. Now suppose that firm 2 can decide first. What is the outcome?

2 produces 10, 1 produces 20 if 2 produces 10, 1 produces 10 if 2 produces 20.

Problem 3 Suppose that Toyota and GM are considering entering a new market for electric automobiles and that their profits (in millions of dollars) from entering or staying out of the market are:

		GM	
		Enter	Don't enter
Toyota	Enter	10,-40	250,0
	Don't enter	0,200	0,0

a. If the firms make their decisions simultaneously, do either or both firms enter?

Firm 1 will enter, Firm 2 will not enter, in a Nash equilibrium.

b. How would your answer change if the US government committed to paying GM a lump-sum subsidy of $50 million on the condition that it would produce this new type of car?

In this case, they both will enter.

Problem 4 A thug wants the contents of a safe and is threatening the owner, the only person who knows the code, to open the safe. "I will kill you if you don't open the safe, and let you live if you do." Should the information holder believe the threat and open the safe? The table below shows the value that each person places on the various possible outcomes:

	Thug	**Safe's owner**
Open the safe, thug does not kill	4	3
Open the safe, thug kills	2	1
Do not open, thug kills	1	2
Do not open, thug does not kill	3	4

Such a game appears in many movies, including *Die Hard*, *Crimson Tide*, and *The Maltese Falcon*.

a. Draw the game tree. Who moves first?

The safe's owner would move first (obviously, should he be shot, he cannot open the safe). He decides to open or not, and the the thug decides to kill him or not.

b. What is the equilibrium?

The subgame perfect Nash equilibrium is for the owner to not open the safe, and the thug to not kill the owner.

c. Is the thug's threat credible?

No. Should the owner refuse to open the safe, the thug is better off not killing him.

d. Does the safe owner open the safe in a subgame perfect Nash equilibrium?

No.

Problem 5 Consider the following game between the IRS and a professor. Suppose that the professor can choose between cheating and not cheating when filing his tax return and IRS can choose between audit and no audit. Assume that an audit costs the IRS 10 and that it increases the revenues by 20 if the professor is cheating (it gives no additional revenues if the professor is not cheating). Furthermore, if the professor doesn't cheat, he pays a tax of 30 (independently of whether there is an audit or not), while if he does cheat he pays 20 in case he is not caught. However, if there is an audit and the professor is caught (auditing is perfect in the sense that cheaters are detected with probability 1) then the professor has to pay a tax of 30 and a fine of 10 (total=40).

a. Write down the payoff matrix of the game described above.

The matric below captures the story above:

		IRS: audit	IRS: no audit
Professor	cheat	-40,30	-20,20
	don't cheat	-30,20	-30,30

b. Derive all Nash equilibria of the game, mixed as well as pure.

There are no pure-strategy Nash equilibria. There is one mixed equilibrium, in which the professor cheats with probability $\frac{1}{2}$, and the IRS audits with probability $\frac{1}{2}$.

Problem 6 Consider the game below:

		Stringer: L	Stringer: R
Avon	T	4,8	0,0
	B	8,20	X,Y

a. If (B, R) is the Nash equilibrium of this game, what must be true of X and Y? Your answer should be two inequalities, one for X and one for Y.

It must be that $X \geq 0$ and $Y \geq 20$.

b. If this game is played sequentially, with Avon moving first and (B, R) is the subgame perfect equilibrium outcome, what must be true of X and Y? Your answer should be two inequalities, one for X and one for Y.

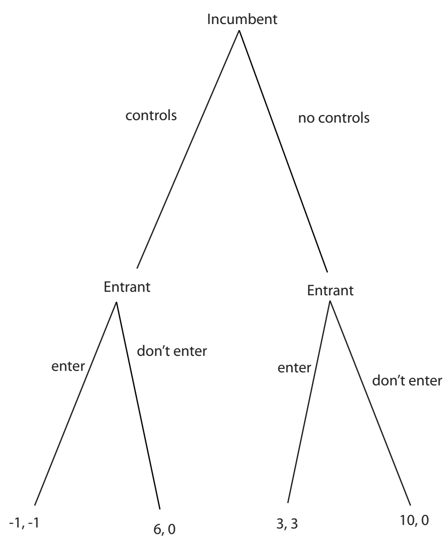
It must be that $X \geq 4$ while $Y \geq 20$.

Problem 7 An incumbent monopoly is currently earning a profit of \$10M. A second firm is considering entering the market; if it does so, both firms will earn profit of \$3M. The incumbent firm is considering urging the government to require all firms in the industry to install pollution control devices, which will lower profit by \$4M.

a. Suppose the incumbent firm chooses between 'pollution controls' and 'no controls' and the entrant chooses between 'enter' and 'don't enter', and that their decisions are made simultaneously. What are the Nash equilibria of this game?

The above game has one Nash equilibrium, at 'no controls, enter'.

b. Now suppose the incumbent firm moves first, and then, upon observing whether pollution controls are in place or not, the entrant chooses between 'enter' and 'don't enter'. What is the subgame perfect Nash equilibrium of this game?



Problem 8 A town has 100 voters: 51 conservatives and 49 liberals. A conservative and a liberal candidate are running for mayor. Voting is by simple majority, and in the case of a tie assume the liberal candidate wins. A conservative voter gets a payoff of 10 if the conservative candidate is elected and -10 if the liberal is elected; vice versa for a liberal voter. It costs a citizen 1 to vote.

a. Explain why it is not a Nash equilibrium for everybody to vote.Because if everyone votes, the conservative wins 51-49, and any one voter could have stayed home, without affecting the outcome of the election, saving the cost of voting.