Problem set 6

Problem 1 Suppose there are two kinds of workers, productive workers and lazy workers. Firms cannot tell workers apart *ex ante*. If hired, a productive worker increases revenue by 9, while a lazy worker increases revenue by 1. Productive workers can acquire e years of education for a cost of e^2, while lazy workers can get e years for $2e^2$. Education does not increase productivity, but is observable to employers.

Solve for e^*, the minimum number of years of education productive workers must acquire to separate themselves from lazy workers. Show that, given your e^*, productive workers prefer e^* to all other levels of education, while lazy types prefer 0 years of education to all other possible levels.

The equation determining e^* is give by $9 - 2(e^*)^2 = 1$. The left-hand side is a lazy type's utility from getting e^* years education, the right-hand side utility from 0 years od education. Solving this equation yields $e^* = 2$. Employers then pay wages as follows:

$$wage(e) = \begin{cases} 1, & \text{if } e < 2 \\ 9, & \text{if } e \geq 2 \end{cases}$$

Problem 2 Consider an economy where people have two choices: work in the private sector or work at home. People who would be more productive in the private sector are also more productive at home (i.e. "high ability" people can do both things better). Employers cannot observe a worker's productivity before the worker is hired. Jobs are permanent: once hired at a given wage, a worker cannot be fired and his wage cannot be changed. Explain how adverse selection could cause the private-sector labor market to break down in this economy so that we could potentially end up with no workers employed in the private sector.

The lecture of 4/13/10 addresses this. The basic story is that if all workers seem identical *ex ante*, employers are willing to pay only for an average worker, but then the very best workers will be unwilling to supply their services, and so average quality will lower, driving the top workers who remain out of the market, lowering the average quality further, and so on. In the extreme case, the market breaks down completely.

Problem 3 An owner hires a manager to run his bar, Suppose the manager can either put in high effort, at a personal cost of c, or low effort, which costs 0. The manager's effort is not observable to the owner. However, suppose the bar's profit is either \$500 or \$0; this money goes to the owner. Suppose that which outcome occurs depends partly on the manager's effort, and partly on chance. Specifically, suppose that if the manager puts in high effort, the bar has profits of \$500 with probability .9, while if the manager puts in low effort, the bar makes a \$500 profit with probability .1. If the manager quits and takes a different job, he can earn a wage of 20.

a. Suppose the owner wants to condition the wage paid to the manager in such a way to induce the manager to put forth high effort. He also wants to pay him the lowest wage that will do this. Write down the incentive constraint and the participation constraint the owner faces.

$$\text{Incentive constraint:} \quad .9w_p + .1w_u - c \geq .1w_p + .9w_u$$

$$\text{participation constraint:} \quad .9w_p + .1w_u - c \geq 20$$

b. Suppose that $c = 8$. Solve for the cheapest contract the owner can offer the manager that will induce him to put forth high effort.

Solving the incentive and participation constraints from part a gives $w_p = \$29$ and $w_u = \$19$.

c. Now suppose c is a variable. Solve for the optimal contract as a function of c.

$w_p = \$20 + \$\frac{9}{8}c$, $w_u = \$20 - \frac{c}{8}$

Problem 4 Traditionally, doctors have been paid on a fee-for-service basis. Now doctors are increasingly paid on a capitated basis (they get paid for treating a patient for a year, regardless of how much treatment is required), though a patient may still have to pay a small fee each visit. What are the implications of this change in compensation for moral hazard?

One worry here is that if doctors are paid for each service they provide, they will have a perverse incentive to overprovide services, i.e. by ordering too many tests, scheduling too many follow-up vists, etc. If they are paid per patient, and not per visit, this incentive goes away, as they internalize the additional cost of more treatment/tests. Of course, then they have a different perverse incentive, that of underproviding services, i.e. not ordering tests which may be helpful. On balance, some sharing of the costs between doctors and patients may be most appropriate.

Problem 5 Consider a first-price sealed-bid auction with two bidders, each with a private value v. Neither bidder knows the other's value, but thinks that any v between 0 and 1 is equally likely for her opponent.

a. Describe qualitatively the optimal bidding behavior (i.e. should each person bid v, less than v, more than v?).

In a first-price, sealed-bid auction, each bidder shades his bid downward, i.e. bidding less that his true valuation.

Computing the *degree* of shading under optimal bidding is generally a hard problem. Here, we can do it by guessing that optimal shading is for each bidder to bid $\frac{v}{2}$, and then showing that this is optimal for each bidder, given that the other bidder is doing the same.

b. Suppose you're bidding against one opponent whose valuation v is equally likely to be anywhere between 0 and 1, and you suspect that whatever her v is, she will bid $\frac{v}{2}$. What is the probability you will win the auction if you bid .1? .4? .6?

If you bid .1, you win only if your opponent's v is below .2, which happens 20% of the time. Similarly, if you bid .4, you win 80% of the time, and if .6, 100% of the time.

c. Put together the answers for part b,; what is your probability of winning if you bid b, for any $b \in [0, 1]$?

$$P(win) = \begin{cases} 2b, \text{ if } b \leq .5 \\ 1, \text{ if } b > .5 \end{cases}$$

d. If your valuation is v and you bid b and win, you profit in the amount $v - b$, while if you lose, you profit 0. Calculate your expected profit from bidding b, given that your opponent is bidding half her valuation.

If your bid b is less than .5, your expected profit is $2b * (v - b)$. If your bid b is above .5, your payoff is $(v - b)$.

e. Find the value of b which maximizes your expression from d. This should be a function of your valuation v.

It is clearly never a good idea to play $b > .5$ (it doesn't increase your probability of winning the auction, but it does cost you more). Choosing b optimally then amounts to making the quantity $2b * (v - b)$ as large as possible. High school level calculus shows this happens at $b = \frac{v}{2}$.

f. Use your results to argue that it is a Nash equilibrium for both bidders to bid half of their valuation.

Other concepts to study for the final Anything mentioned in class may appear on the final. You should be especially concerned with being able to give an intelligent discussion of the following topics/concpets:

1. Efficiency wages
2. Each of the four different types of auctions discussed in class, and optimal bidding behavior therein
3. The winner's curse in common values auctions
4. Moral hazard
5. Indifference curves and budget lines (being able to describe and interpret a picture with a budget line and some indifference curves will suffice; not much analysis is required here.