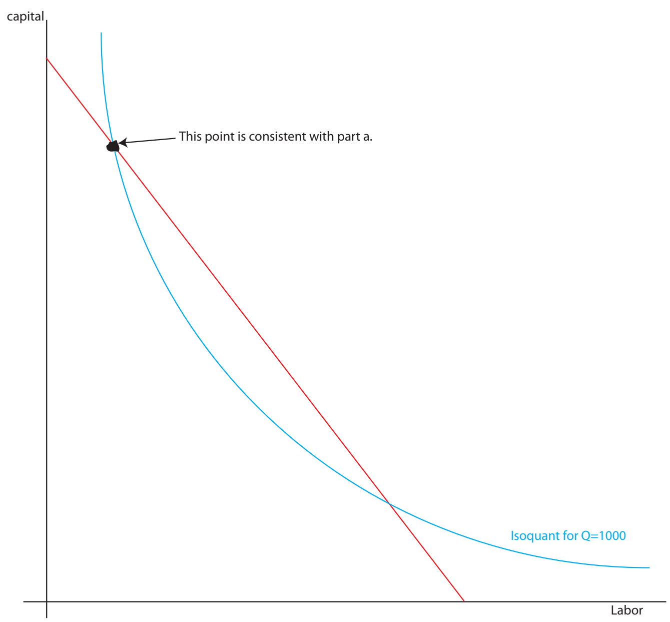
Quiz #1

Problem 1 Guatemalan Apparel uses labor and capital to produce hats. Suppose that at current input levels, the marginal product of capital is 44, while the marginal product of labor is 21. Suppose the hourly wage is \$7, while the hourly cost of capital is \$22.

a. Guatemalan Apparel hires you as a high-priced management consultant to determine if it is producing efficiently or not. Using the data above, what is your answer?

No, they are not, as $\frac{MPL}{w} = 3$ while $\frac{MPK}{r} = 2$. These are not equal. The firm could increase its output without increasing its cost by taking money away from capital and hiring more labor.

b. Suppose Guatemalan Apparel has a 'normal' production function, and is currently producing $Q = 1,000$ hats. On a graph with labor on the horizontal axis and capital on the vertical axis, sketch an isoquant for $Q = 1,000$ and the isocost line on which you believe Guatemalan Apparel is currently operating (your picture need be precise only so far as it supports your answer in part a).



Problem 2 Using labor L and capital K, Krustyburger produces Q meat-flavored sandwiches per hour, where $Q = \sqrt{LK}$. Krustyburger pays \$5 per hour of labor, and \$10/hour of capital.

a. Suppose in the short run, Krustyburger's capital is fixed at $\overline{K} = 16$. Write down Krustyburger's short-run cost of producing Q sandwiches, as a function of Q.

Krustyburger needs to hire enough labor so that $Q = \sqrt{16L}$, or $L = \frac{Q^2}{16}$. This costs $\$5 * \frac{Q^2}{16}$. Their cost of capital is $\$10 * 16 = \$160$, and so their total short-run cost is $c(Q) = \$160 + \$5\frac{Q^2}{16}$.

b. Fact: Given Krustyburger's production function, $\frac{MPK}{r} = \frac{MPL}{w}$ implies $2K = L$. Derive Krustyburger's long-run cost of producing Q sandwiches/hour ('long run' means capital is variable). Plus in $L = 2K$ into the production function $Q = \sqrt{LK}$ to get $Q = \sqrt{2K^2} = \sqrt{2}K$. Therefore, $K = \frac{Q}{\sqrt{2}}$ and $L = 2K = \sqrt{2}Q$, and so the long run cost function is $c(Q) = \$5 * \sqrt{2}Q + \$10 * \frac{Q}{\sqrt{2}}$ (sorry about the irrational numbers).

Problem 3 Mountain State University produces cutting edge research articles using professors and computers. Professors are completely useless when not in front of a computer, and so time spent without one is unproductive. Likewise, computers are not known to produce research on their own, so a computer without a professor in front of it is worthless. Suppose that it takes a professor sitting in front of his computer about 100 hours to write one research article. Suppose a professor's wage is \$50/hour, while the amortized cost of a computer is \$25/hour.

a. The university will hire L hours of professor time and K hours of computer time. Write down its production function which translates K and L into number of research papers Q (to keep things simple, assume it's possible to produce fractional research papers).

The text of the question strongly suggests that labor and capital are perfect compliments, with production function $Q = \min\{\frac{1}{100}L, \frac{1}{100}K\}$.

b. Write down the university's long-run cost function $c(Q)$.

To get Q papers, the university needs to hire $L = 100Q$ labor and $K = 100Q$ capital, at cost $\$50 * 100Q + \$25 * 100Q$, so $c(Q) = \$7,500Q$.

Problem 4 Suppose Scranton Vineyards have production function $Q = 2L^{\frac{1}{2}}K^{\frac{1}{2}}$, where Q is cases of wine and L and K are hours of labor and capital, respectively.

a. As precisely as you can, draw an isoquant corresponding to 2 cases of wine.

A good answer would either plot the line $K = \frac{1}{L}$ or it would sketch out a few points and then connect them.

b. The slope of the isoquant is sometimes called the *marginal rate of technical substitution*. Explain intuitively what it means that the marginal rate of technical substitution changes as Scranton Vineyards moves along the isoquant you drew.

The slope of an isoquant is simply the rate at which labor and capital can be substituted for each other. Specifically, if the slope is -7 at one point, it means that adding one worker decreases by 7 the number of machines needed to get the same output, while if it's $-\frac{1}{3}$ at another point says that here adding one more worker only decreases by $\frac{1}{3}$ the amount of machines needed for the same output. That the slope changes over an isoquant reflects that in a capital-intesive production process, adding another worker may allow you to eliminate many machines, while in a labor-intensive production process, adding another worker will have relatively little effect.

(extra credit) c. Go as far as you can in writing down Scranton Vineyard's cost function $c(Q)$, as a function of the wage rate w and the cost of capital r.

With this production function, $\frac{MPK}{r} = \frac{MPL}{w}$ implies $L = \frac{r}{w}K$. Therefore, $Q = 2(\frac{r}{w})^{\frac{1}{2}}K$, and so $K = \frac{1}{2}Q\frac{\sqrt{w}}{\sqrt{r}}$ and $L = \frac{1}{2}\frac{\sqrt{r}}{\sqrt{w}}Q$, so the cost of producing Q output is $c(Q) = \sqrt{rw}Q$.