Economics 590, fall 2007
Instructor: Jeremy Sandford

Homework 1
due 9/4/2007

Problem 1 (Convex sets I). *Suppose A and B are convex sets in $\mathbb{R}^n$. Show that $A \cap B$ is also convex.*

Problem 2 (Convex sets II). *Let A and B be convex sets. Show (by counterexample) that $A \cup B$ need not be convex.*

Problem 3 (Mappings I). *Let $A = [0, 10]$ and $B = \mathbb{R}$.*

a. Give an example of a mapping from A to B which is neither injective nor surjective.

b. Give an example of a mapping from A to B which is injective but not surjective.

c. Give an example of a mapping from A to B which is bijective.

d. Give an example of a mapping from A to B which is surjective but not injective (note: for part d only, you may draw a picture for your answer).

Problem 4 (De Morgan's laws). *In class, we showed that for any sets $A_1, A_2, A_3, ..., A_n$,*

$$(\bigcup_{i=1}^{n} A_i)^c = \bigcap_{i=1}^{n} A_i^c$$

This is also know as the first of two De Morgan's laws, named for the mathematician and logician Augustus De Morgan (1806-1871). The second of De Morgan's laws says that

$$(\bigcap_{i=1}^{n} A_i)^c = \bigcup_{i=1}^{n} A_i^c$$

Prove the second of De Morgan's laws, (hint: imitate and/or use the proof of the first law from class).

Problem 5 (Mappings II). *Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by the rule $f(x) = x^3 - x$.*

Prove a-c to be true or false

a. f is injective

b. f is surjective

c. f is bijective

d. If you argued statement c to be false (hint: you should have), restrict the domain and/or range of f so that you get a new function $g : A \to B$ where $g(x) = x^3 - x$, $A \subset \mathbb{R}$, and $B \subset \mathbb{R}$ such that g is bijective. Graph g and g^{-1}. Note that there are many possible choices of g.

Problem 6 (Mappings III). *Consider sets A and B, each having a finite number of elements. That is, $A = \{a_1, a_2, ..., a_n\}$ and $B = \{b_1, b_2, ..., b_m\}$, for some integers m and n.*

Prove each of the following statements to be either true or false:

a. If $m < n$, there exists no function $g : A \to B$ that is injective.

b. If $m < n$, every function $g : A \to B$ is surjective.

c. If $m = n$, every function $g : A \to B$ is injective.