Economics 590, fall 2007

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Homework 3

due 9/18/2007

Problem 1 (Limit points). *In the metric space* $(\mathbb{R}^2, |\cdot|)$*, where* $|\cdot|$ *is the Euclidean norm,*

a. is every point of every open set $O \subset \mathbb{R}^2$ *a limit point of* O*?*

b. is every point of every closed set $F \subset \mathbb{R}^2$ *a limit point of* F*?*

Problem 2 (Alternative definition of 'closed set'). *Let* $(A, d(\cdot))$ *be a metric space. Prove that* $F \subset A$ *is closed if and only if the complement of* F *is open (using the definitions of 'closed' and 'open' given in class).*

Problem 3 (Alternative definition of 'closed set' II). *Let* $(A, d(\cdot))$ *be a metric space. Prove the following statement:*

$F \subset A$ *is closed if and only if for every sequence* $\{x_n\}$ *contained in* F*,*

$$\lim_{n\to\infty} x_n = x \quad \Rightarrow \quad x \in F. \tag{1}$$

Again, use the definition of 'closed' given in class.

Problem 4 (Extreme values). *(Sundaram page 68, #16) Find the supremum, infimum, maximum, and minimum, if they exist, for the following sets:*

a. $A_1 = \{x \in [0, 1] : x \textit{ is irrational}\}$

b. $A_2 = \{x : x = \frac{1}{n}, \textit{ for } n = 1, 2, ...\}$

c. $A_3 = \{x : x = 1 - \frac{a}{n}, \textit{ for } n = 1, 2, ...\}$ *(note: take* a *to be some real number bigger than* 0*)*

d. $A_4 = \{x \in [0, \pi] : sin(x) > \frac{1}{2}\}$

e. $A_5 = \phi$*, the empty set*

Problem 5 (Convex sets). *A set* A *is convex if, for every* $x_1, x_2 \in A$*, and for every* $\alpha \in (0, 1)$*,*

$$\alpha x_1 + (1 - \alpha)x_2 \in A \tag{2}$$

(this is the usual definition from class).

If a set B *satisfies* $\frac{1}{2}x_1 + \frac{1}{2}x_2 \in B$ *for all* $x_1, x_2 \in B$*, does it follow that* B *is convex?*

Problem 6 (Metric spaces and open sets). *Any time we discuss open and closed sets, we carefully note the metric space we are working in. Give an example of a set which is open when seen as a subset of one metric space, and not open when seen as a subset of another. (hint: this is easy.)*

Problem 7 (Rational numbers). $\mathbb{Q} = \{x \in \mathbb{R} : x = \frac{a}{b}, \textit{ for integers } a, b\}$ *denotes the set of rational numbers. Is* $\mathbb{Q}$ *an open subset of the Euclidean space* $(\mathbb{R}, |\cdot|)$*, where* $|\cdot|$ *is the Euclidean metric? Is* $\mathbb{Q}$ *a closed subset of the same?*

Problem 8 (Intersections of open sets). *In* $\mathbb{R}^n$ *with the Euclidean metric, prove or counter each of the following:*

a. The intersection of any finite number of open sets is an open set.

b. The intersection of any infinite collection of open sets is an open set.