Economics 590, fall 2007

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Homework 7

due 11/15/2007

Problem 1 (Minimum distances). *Answer each of the following:*

a. Find the maximum and minimum distance from the origin to the ellipse $x^2 + xy + y^2 = 3$.

b. Find the point on the parabola $y = x^2$ *which is closest to the point* $(2, 1)$.

c. Find the point closest to the origin in $\mathbb{R}^3$ *which is on both the plane* $3x + y + z = 5$ *and* $x + y + z = 1$.

Use second order sufficient conditions to verify that the points you solved for in a-c are, in fact, local minima and maxima, as appropriate.

Problem 2 (Second order conditions). *Return to Homework 6 and check second order conditions for the constrained maximization/minimization of the functions in problem 3, if you did not do so already.*

Problem 3 (Constrained optimization). *(Jevons, 1871) Your ship is overdue in port, and the beer is running out. The remaining supplies are divided up and you get 22.5 fluid ounces. The ship will not reach port before tomorrow morning, and there is a 60% chance that it will arrive then. You can't take beer with you when you leave the ship, so you could drink it all today, to make sure it isn't wasted. On the other hand, there is a 40% chance that you will still be afloat all day tomorrow, and a 10% chance that you will be afloat the day after that. You could save some beer in case you need it for the second day, or the third. It is certain that you will reach port before the fourth day.*

You are an expected utility maximizer,[1] *and your utility function is* $U(B) = 6000B - 250B^2$, *where* B *is daily beer consumption. How much beer should you drink today, and how much should you save for day 2? For day 3?*

Problem 4 (Two-part tariffs). *A profit-maximizing monopoly firm sells to many identical buyers. The marginal cost of production is a constant,* c, *with* $c < .5$. *Each buyer's demand function is* $p = 1 - q$. *The firm decides to charge a two-part tariff, so that a consumer who buys a quantity* q *must pay the amount* $y = a + bq$. *How would the firm choose* a *and* b*?*

Problem 5 (Constrained optimization II). *Find local and global maxima and minima of the function* $f(x, y) = \frac{1}{3}x^3 - \frac{3}{2}y^2 + 2x$ *subject to the constraint* $g(x, y) = x - y = 0$.

Problem 6 (Social welfare maximization). *You are a social planner distributing* a *units of apples and* b *units of bananas to two agents, One and Two. One has utility function* $u_1(a_1, b_1) = a_1b_1$ *and Two has* $u_2(a_2, b_2) = a_2b_2$. *You don't necessarily value One's welfare equivalently with Two's welfare; indeed, you are interested in maximizing* $\alpha u_1(a_1, b_1) + (1 - \alpha)u_2(a_2, b_2)$, *where* $\alpha \in (0, 1)$. *There is no borrowing or lending of apples or bananas, so your constraints are* $a_1 + a_2 = a$ *and* $b_1 + b_2 = b$. *Naturally, it must be the case that* $a_1 \geq 0$, $a_2 \geq 0$, $b_1 \geq 0$, *and* $b_2 \geq 0$.

[1] This means that if you were to consume B_1 beer on day 1, B_2 on day 2, and B_3 on day 3, your total utility would be $U(B_1) + .4U(B_2) + .4 * .1U(B_3)$

One frequently-employed tactic for dealing with nonnegativity constraints like these is to ignore them, solve the problem without them, and argue that the solution to the problem with no nonnegativity constraints is identical to that of the problem with nonnegativity constraints.

a. Solve the social welfare maximization problem ignoring the nonnegativity constraints.

b. Do you think your solution in a. also solves the problem with the nonnegativity constraints?