Economics 590, fall 2007
Instructor: Jeremy Sandford

Midterm
10/23/2007

Directions: **Answer 4 of the 5 equally weighted questions.** Take great pains to support your answers as thoroughly as you are able. Unsupported arguments and unaccompanied pictures are unlikely to receive many points. Remember, the standard you aspire to is having your answers be clear and intelligible to an intelligent person who is not in our class.

You may use results from lectures or books, but be as specific as possible in citing them. You have until 1:55pm. Good luck!

Problem 1 (25 points). *Consider the production function* $f(k,l) = 12k^{\frac{1}{3}}l^{\frac{2}{3}}$. *That is, a firm employing* l *labor and* k *capital produces* $f(k,l)$ *units.*

a. From an initial position of (k,l), *give the ratio in which* k *and* l *should be increased to increase* $f(k,l)$ *most rapidly.*

b. From an initial point of $l = 216$ *and* $k = 27$, *calculate the derivative of* f *in the direction of maximal increase. Show that, from this initial point, increasing* k *and* l *together in this direction increases* f *more rapidly than does increasing* k *or* l *alone.*

c. From an initial position of $(k,l) = (27, 216)$, *give the direction* v *in which* $D_v f(k,l) = 0$, *that is, in which production does not change at all.*

d. Provide an economic interpretation of your answer in c.

Problem 2 (25 points). *In* $\mathbb{R}$ *with metric* $d(x,y)$, *a sequence* $\{x_n\}$ *is called a Sandford sequence if, for any* $\epsilon > 0$, *there exists a number* $N(\epsilon)$ *such that* $n, m > N(\epsilon)$ *implies that* $d(x_n, x_m) < \epsilon$.

Prove that any convergent sequence in $\mathbb{R}$ *is a Sandford sequence.*

Problem 3 (25 points). *If* A *is a non-empty closed subset of* $\mathbb{R}^2$, *and* $x \notin A$, *is there a point in* A *that is nearest to* x, *under the standard metric* $d(y,z) = \sqrt{y^2 + z^2}$*? Is it unique? Prove your claims.*

Problem 4 (25 points). *Let* $g : \mathbb{R} \to \mathbb{R}$ *be a function (not necessarily continuous) which has a maximum and minimum on* $\mathbb{R}$. *Let* $f : \mathbb{R} \to \mathbb{R}$ *be continuous. Does* $h(x) = f(g(x))$ *necessarily have a maximum on* $\mathbb{R}$*? Prove your answer, or provide a detailed counterexample.*

Problem 5 (25 points). *Prove whether or not each of the following functions is quasiconcave:*

a. $f : \mathbb{R}_+ \to \mathbb{R}$, $f(x) = -x^2$

b. $f : \mathbb{R}_+ \to \mathbb{R}$, $f(x) = x^2$

c. $f : [0, 2\pi] \to \mathbb{R}$, $f(x) = sin(x)$

d. $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x_1, x_2) = 4x_1 + 2x_2 - x_1^2 + x_1x_2 - x_2^2$