Economics 590, fall 2007
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Noatation Appendix

We generally use upper-case letters to represent sets, and lower case letters to represent elements of sets. Common letters used for sets are A and B, while common letters used for elements of sets are a, b, x, and y. Additionally, O is often used to represent an open set and F is often used to represent a closed set.

To say "the element x is in the set A," we write $x \in A$. To say "the set A is a subset of the set B," we write $A \subset B$. $A \cup B$ means the set of all elements in either A or B, while $A \cap B$ means the set of all elements in both A and B. For $A \subset B$, we take $B \backslash A$ to be the set of all points in B but not A. If B is the largest set specified (for our purposes, virtually always $\mathbb{R}^n$), we write the complement of A as $A^c = B \backslash A$.

When we want to describe sets explicitly, we can either list all of the elements inside of curly brackets (i.e. $\{3, 4, 5\}$) or we can make use of the following well-known sets:

$$
\begin{aligned}
\mathbb{R} &= \text{the set of all real numbers} \\
\mathbb{Z} &= \text{the set of all integers} \\
\mathbb{Q} &= \text{the set of all rational numbers}
\end{aligned}
$$

Many, but not all, sets we're interested in discussing will be subsets of the above three, and so we can describe such sets in terms of the above three, as in $\{x \in \mathbb{R} : x > 3\}$ or $\{x \in \mathbb{Z} : x^2 > 7\}$, where the : sign means "such that". The following shorthand notation for intervals of real numbers is ubiquitous:

$$
\begin{aligned}
(a, b) &= \{x \in \mathbb{R} : a < x < b\} \\
[a, b] &= \{x \in \mathbb{R} : a \leq x \leq b\} \\
(a, b] &= \{x \in \mathbb{R} : a < x \leq b\} \\
[a, b) &= \{x \in \mathbb{R} : a \leq x < b\}
\end{aligned}
$$

Often we want to describe vectors of numbers, i.e. a list of n real numbers. The sets containing these vectors can be written in the form $A \times B$, where $A \times B$ is the set of all vectors of length 2 whose first element is in A and whose second element is in B. When $A = B$, the convention is to write $A^2 = A \times A$. Thus, for example, $\mathbb{R}^n$ is the set containing all n-length vectors of real numbers.

To describe vectors, we might write $(x_1, x_2, ..., x_n)$, i.e. simply list out each component, using either rounded or square brackets. Note that then the vector $(3, 7)$, i.e. an element of $\mathbb{R}^2$ whose first component is 3 and whose second component is 7, is written exactly the same as the set $(3, 7)$, the set of all real numbers between 3 and 7. It should be clear from context which is meant.

We might, for example, write $x \in \mathbb{R}^2$ to describe some vector of two numbers. With this notation, it is understood that $x = (x_1, x_2)$, i.e. that x actually consists of 2 components. To describe the inner product of two vectors, we write $x \cdot y$ or $x'y$ (nb. the former literally means "the inner product of x and y while the latter means "the transpose of the matrix x multiplied by the matrix y"). Take $|x|$ to be the norm of the vector x.