Homework 10

due 12/1/08

Problem 1 (Finite-horizon dynamic programming I) (Sundaram, pg 278)

Solve parts a, b, and c of problem 1 on page 278 of the Sundaram book.

Problem 1 (Finite-horizon dynamic programming II) (Sundaram pg 278)

Consider the problem of optimal harvesting of a natural resource. A firm (say, a fishery) begins with a given stock of $y > 0$ of a natural resource (fish). In each period $t = 1, 2, ..., T$ of a finite horizon, the firm must decide how much of the resource to sell on the market that period. If the firm decides to sell x units of the resource, it receives a profit of $\log(x)$. The amount $(y - x)$ of the resource left unharvested grows to an available amount of $(y - x)^\alpha$ at the beginning of the next period. The firm wishes to choose a strategy that will maximize the sum of its profits over the model's T-period horizon.

Solve this problem for the firm's optimal choices of x and y over periods $0, 1, ..., T$. Prove your answers.[1]

Problem 3 (Finite-horizon dynamic programming III) (Sundaram pg 279)

Let an $m \times n$ matrix A be given. Consider the problem of finding a path between entries a_{ij} in the matrix A which (i) starts at a_{11} and ends at a_{mn}, (ii) which moves only to the right or down, and (iii) which maximizes the sum of the entries a_{ij} encountered. Express this as a dynamic programming problem. Using backwards induction, solve the problem when the matrix A is given by

$$\begin{bmatrix} 4 & 9 & 3 & 6 & 3 \\ 5 & 6 & 6 & 4 & 4 \\ 6 & 7 & 1 & 1 & 0 \\ 4 & 3 & 5 & 1 & 9 \end{bmatrix}$$

[1] Much of this was answered in the 11/20 lecture. What remains is to rigorously demonstrate what x and y are in period t, for generic t.