

## Final Exam

5/5/2016

**Instructions:** You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. The six questions are equally weighted.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 150 minutes to complete the exam. Good luck!

**Problem 1** Suppose that *qualified* and *unqualified* workers look alike, but perform differently on a pre-employment test. Specifically, suppose that an unqualified worker's test score is drawn from a  $U[0, \frac{3}{4}]$  distribution, while a qualified worker's score is drawn from a  $U[\frac{1}{2}, 1]$  distribution.

**a.** Suppose that prior to seeing a worker's test score, a firm believes a worker is qualified with probability  $\pi$ . What is the firm's posterior belief that a worker is qualified if his score is between 0 and  $\frac{1}{2}$ ? Between  $\frac{1}{2}$  and  $\frac{3}{4}$ ? Between  $\frac{3}{4}$  and 1?

**b.** Suppose that a firm can place a worker into a good job or a bad job. If a qualified worker is placed into a good job, the firm earns a profit of \$4,000, while if an unqualified worker is placed into a good job, the firm loses \$9,000. If a worker is placed into a bad job, the firm breaks even.

Consider a worker whose test score is between  $\frac{1}{2}$  and  $\frac{3}{4}$ . Characterize the firm's decision to place the worker into a good or a bad job as a function of  $\pi$ .

**c.** Suppose the firm places a worker into a good job if and only if the worker scores at least  $\frac{1}{2}$  on the test. Suppose further that the cost  $c$  of becoming qualified varies across workers, following  $c \sim U[0, 1]$ , and that the net benefit to a worker of being in a good job is  $\omega = 1$ . What fraction of workers will become qualified?

**d.** Now suppose the firm places a worker into a good job if and only if the worker scores at least  $\frac{3}{4}$  on the test. What fraction of workers become qualified?

**e.** Determine whether one or both of the scenarios of parts c-d constitute an equilibrium.

**Problem 2** Workers choose their education levels and then apply for jobs.  $\frac{1}{4}$  of workers are *high-ability*, while the remainder are *low-ability*. Firms cannot determine a given worker's ability level until after the worker is hired. Regardless of education level, high ability workers increase firm profits by \$100, while low ability workers increase firm profits by \$36. The cost of education is  $c_L(e) = e^2$  for low-ability workers, while it is  $c_H(e) = 2e$  for high ability workers. Assume that workers are paid their expected productivity.

For outcomes 1-4 below, determine if each outcome is an equilibrium of the game. If so, determine whether or not it satisfies the intuitive criterion, and prove your answer.

a. Outcome 1:

$$\begin{aligned} e_H &= 12 \\ e_L &= 2 \\ w(e) &= \begin{cases} \$100 & \text{if } e = 12 \\ \$36 & \text{if } e \neq 12 \end{cases} \end{aligned} \quad (1)$$

b. Outcome 2:

$$\begin{aligned} e_H &= 1 \\ e_L &= 1 \\ w(e) &= \begin{cases} \$52 & \text{if } e < 25 \\ \$100 & \text{if } e \geq 25 \end{cases} \end{aligned} \quad (2)$$

c. Outcome 3:

$$\begin{aligned} e_H &= 5 \\ e_L &= 5 \\ w(e) &= \begin{cases} \$36 & \text{if } e < 5 \\ \$52 & \text{if } e = 5 \\ \$(32 + 4e) & \text{if } e \in (5, 17) \\ \$100 & \text{if } e \geq 17 \end{cases} \end{aligned} \quad (3)$$

d. Outcome 4. For this part, your answer should depend on the variables  $X$  and  $Y$ .

$$\begin{aligned} e_H &= X \\ e_L &= 0 \\ w(e) &= \begin{cases} \$36 & \text{if } e < Y \\ \$100 & \text{if } e \geq Y \end{cases} \end{aligned} \quad (4)$$

**Problem 3** An incumbent firm is either a low-cost type ( $\theta_L$ ) or a high-cost type ( $\theta_H$ ), each with equal probability. In period 1, the incumbent is a monopolist and sets one of two prices,  $p_L$  or  $p_H$ , with its profits in period 1 given by the following table:

Type	Profit from $p_L$	Profit from $p_H$
$\theta_L$	6	8
$\theta_H$	1	5

After observing the period 1 price, a potential entrant (which does not know the incumbent's type) can choose to enter the market (E) or to stay out (O) in period 2. The payoffs of both players in period 2 are as follows:

Type	Entrant's choice	Incumbent's profit	Entrant's profit
$\theta_L$	E	0	-2
$\theta_L$	O	8	0
$\theta_H$	E	0	1
$\theta_H$	O	5	0

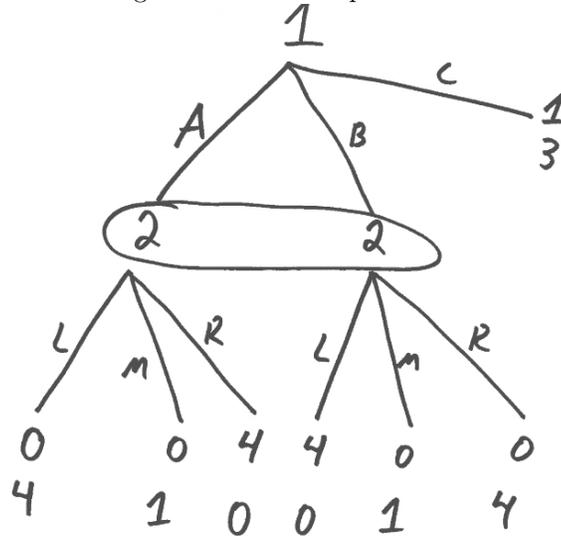
At the beginning of the game the incumbent discounts period 2 profits using discount factor  $\delta \leq 1$ .

- a. Draw the extensive form game. Make sure to include all payoffs, as a function of  $\delta$ .
- b. For the special case of  $\delta = 1$ , find a pooling perfect Bayesian equilibrium in which both incumbent types choose  $p_L$  in period 1. Does your equilibrium satisfy the intuitive criterion?
- c. Find the range of discount factors for which a separating equilibrium exists in which type  $\theta_L$  chooses  $p_L$  and type  $\theta_H$  chooses  $p_H$  in period 1.

**Problem 4** Consider the game in Figure 1 below.

- Draw the reduced normal form. Find all pure strategy Nash equilibria. There is a mixed Nash equilibrium in which 1 randomizes between A and B, and 2 randomizes between L and R. Find it.
- Find all of the game's perfect Bayesian equilibria (pure as well as mixed).
- Explain in intuitive terms any differences between your answers to part a and part b.

Figure 1: Game for question 4



**Problem 5** Consider two Cournot oligopolists, facing demand curve  $P = 1 - q_1 - q_2$ . Firm 1's marginal cost is as follows:

$$c_1 = 0 \text{ w.p. } (1 - \alpha) \text{ (Firm 1 is low cost)}$$

$$c_1 = X \text{ w.p. } \alpha \text{ (Firm 1 is high cost)}$$

Firm 1 knows its marginal cost, but Firm 2 knows only the distribution given above. Firm 2 has marginal cost equal to 0.

- a. Solve for Firm 1's best response functions. Note that since there are two types of Firm 1 (high and low cost), Firm 1 has two best response functions.
- b. Solve for Firm 2's best response function.
- c. In the oligopoly game's Bayesian Nash equilibrium, what quantity does Firm 2 produce? What quantity does Firm 1 produce if its costs are low? If its costs are high? What is the market price in each case?
- d. What is the derivative of  $q_1$  with respect to  $X$  in the case that Firm 1 is a low cost firm? In the case Firm 1 is high cost? Why does Firm 1's quantity depend on  $X$  even in the former case?

**Problem 6** Larry owns a gold mine, which contains  $X$  gold. Larry knows  $X$ , but the rest of the world knows only that  $X \sim U[0, 1]$ .

Larry can mine the gold himself, which would give him a payoff of  $3X$ . He could also sell to a mining company. Since the mining company is more efficient at extracting gold, the mining company's payoff would be  $4X - P$ , where  $P$  is the purchase price (assume the mining company is risk neutral). If Larry sells the mine, his payoff equals  $P$ .

**a.** Consider a given price  $P$ . For what values of  $X$  would Larry prefer to sell? For what values would he prefer to mine himself?

Now consider the following two-stage pricing game. In stage one, the mining company makes a take-it-or-leave-it offer  $P$ . In stage two, Larry accepts or rejects the offer, and payoffs are realized.

**b.** Solve for the Bayesian Nash equilibrium of this game. What is Larry's payoff, as a function of  $X$ ? What is the mining company's expected payoff (before it knows what  $X$  is)?

**c.** Suppose that a third-party mining expert can accurately determine  $X$  and report it to the mining company. The cost of this service is  $C$ . For what values of  $C$  would the mining company choose to hire the third-party expert?