Midterm Exam

3/24/2016

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup, an intuitive explanation, or the right approach demonstrated on a simplified version of the problem. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have 90 minutes to complete the exam. Good luck!

Problem 1 (10 points) Consider the following game:

		Player 2		
		L	C	R
	U	5,1	1,4	1,0
Player 1	M	3,2	0,0	3,5
	D	4,3	4,4	0,3

a. Demonstrate that L is a strictly dominated strategy for player 2.

b. Demonstrate that, after removing L, U is a strictly dominated strategy for player 1.

c. Solve for all Nash equilibria of this game, mixed as well as pure.

Problem 2 (15 points) Consider the infinitely-repeated version of the following game:

		Player 2		
		L	C	R
	T	6,6	-1,7	-2,8
Player 1	M	7,-1	4,4	-1,5
	B	8,-2	5,-1	0,0

a. For which values of discount factor δ can the players support the pair of actions (M, C) played in every period? Your answer should fully describe the strategy used.

b. For which values of discount factor δ can the players support the pair of actions (T, L) played in every period (again, fully describe the strategy used)? Why is your answer different than for part a.?

Problem 3 (20 points) 99 shepherds share a common field in which they graze their sheep. Each shepherd purchases as many sheep as he/she likes, at a cost of $c = \$300$/sheep. The value of one sheep is given by:

$$v(G) = 2000 - S$$

where S is the total number of sheep which graze in the field (more sheep mean less grass/sheep, more sheep fights, etc). The common field is the only suitable location for grazing, and sheep die without grazing, so you may assume that all purchased sheep are brought to graze in the field.

a. In a symmetric Nash equilibrium, how many sheep does each shepherd purchase? How much profit is earned by each shepherd?

b. What is the socially optimal number of sheep? If the resulting total profit is split evenly amongst all shepherds, what is the profit for each shepherd?

c. Suppose a government imposes a tax on sheep of \$T/head, but that the revenue collected from the tax is distributed evenly to each of the 99 shepherds, regardless of how many sheep the shepherd owns. Is such a tax always welfare-reducing? Why or why not?

Problem 4 (20 points) Consider a market with inverse market demand given by $P = 10 - \frac{1}{100}Q$. Firm A is a monopoly producer, with marginal cost equal to \$2.

a. Calculate the market price and Firm A's profit as a monopolist.

Now, suppose that Firm A has discovered a new technology that will allow it to produce at a marginal cost of \$0. Implementing the new technology will cost firm A to incur a fixed cost of \$1,000.

b. Is it profitable for Firm A to implement the new technology?

Now, suppose that Firm A learns that Firm B is considering entering the market to compete with Firm A. To enter, Firm B would have to construct a factory at a cost of \$500, and then Firm A and Firm B would compete in Cournot oligopoly.[1] If firm B entered, its marginal cost would also equal \$2.

c. Calculate the market price, firm A's profit, and firm B's profit under Cournot competition. Would firm B profitably enter the market? (Assume for part c. that Firm A has not implemented the new technology.)

d. Consider an extensive form game with two rounds. In round 1, Firm A decides whether or not to implement the new technology. In round 2, Firm B decides whether or not to enter the market. Using your answers above (and possibly new calculations), determine the subgame perfect equilibrium of this game.

e. Policymakers sometimes worry that monopolists are less likely to innovate than firms in a competitive market.[2] Do your answers above suggest any caveats to this view?

[1] So that inverse market demand is given by $P = 10 - \frac{1}{100}(q_1 + q_2)$, where q_i is firm i's quantity.

[2] See e.g. "Enhanced market power can also be manifested in... diminished innovation.", page 2 of *Horizonal Merger Guidelines*, 2010, U.S. Department of Justice and Federal Trade Commission.

Problem 5 (15 points) Consider the infinitely repeated version of the following stage game:

		Player 2	
		A	B
Player 1	A	2,2	-1,3
	B	3,-1	1,1

Suppose each player plays the following strategy:

(Phase I) Play A initially; remain in phase I so long as no deviation occurred in the previous period. If either player deviates, move to Phase II in the following period.

(Phase II) Play (B, B) for T periods. If either player deviates, restart Phase II. After T periods, return to Phase I.

Suppose that both players have a common discount factor, $\delta = .6$. Find the minimum value of T for which the above strategies comprise a subgame perfect equilibrium of the repeated game. Most of the points will be awarded for correctly describing the inequality which determines this δ.

Problem 6 (20 points) Two bidders are bidding on a bottle of Scotch in a first-price, sealed bid auction. Bidder 1 values the bottle at v_1, and bidder 2 values the bottle at v_2. Neither bidder knows the other's valuation, but each knows that $v_i \sim U[0, 1]$, and that v_1 and v_2 are independent (note that this setting is identical to the first example studied in class). Bidders simultaneously submit hidden bids; the highest bidder gets the bottle for the price he paid.

a. Show that there is a Nash equilibrium in bidding strategies in which player i bids $b_i = \frac{v_i}{2}$.

b. Suppose that bidder 2 is irrational, and will bid $b_2 = v_2$. Demonstrate that $b_1 = \frac{v_1}{2}$ remains the best response for player 1.

c. Suppose a third bidder arrives to bid on the bottle of Scotch. Like bidders 1 and 2, bidder 3's valuation is private information, but is distributed $v_3 \sim U[0, 1]$. v_1, v_2, and v_3 are independent. Show that there is a Nash equilibrium in which player i bids $b_i = \frac{2v_1}{3}$.

To answer part c., you will need to use the fact that if X_1 and X_2 are independent $U[0, 1]$ random variables, $P(\max\{X_1, X_2\} < x) = x^2$ for $x \in [0, 1]$.