Final Exam

answers

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

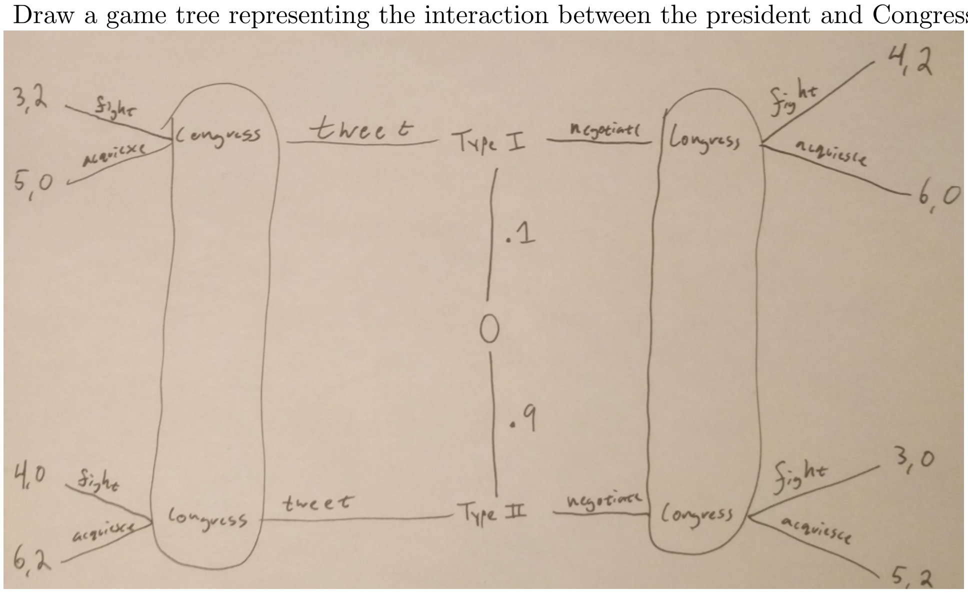
If you don't know how to answer a question, go as far as you can. Sometimes substantial points can be awarded for the right setup, an intuitive explanation, or the right approach demonstrated on a simplified version of the problem. Similarly, if a problem requires multiple steps, it is important that you clearly describe your progression through those steps, even if you know the correct numerical answer. You have until 8:45pm to complete the exam; no extra time will be given. Good luck!

Problem 1 (20 points) President Camacho is either a great strategic thinker (Type I) or a total moron (Type II), but no one is really sure which. Available evidence suggests a 10% chance the president is Type I, and a 90% chance he is Type II. During a tense standoff with Congress, the president may either negotiate with Congress or tweet childish insults about members of Congress. After observing the president's actions (but not his type), Congress chooses to either fight the president's agenda (risking a political battle which would damage both sides) or acquiesce to the president's preferred policies.

Payoffs are given in the following table. Each payoff lists the president's payoff first, followed by Congress'.

	payoffs if president is Type I	payoffs if president is Type II
(negotiate, acquiesce)	$(6, 0)$	$(5, 2)$
(negotiate, fight)	$(4, 2)$	$(3, 0)$
(tweet, acquiesce)	$(5, 0)$	$(6, 2)$
(tweet, fight)	$(3, 2)$	$(4, 0)$

a. Draw a game tree representing the interaction between the president and Congress.



b. Is there a separating equilibrium in which a Type I president plays "tweet", and a Type II president plays "negotiate"?

No. In any equilibrium in which Type I plays "tweet" and Type II "negotiate," Congress plays "fight" following "tweet" and "acquiesce" following "negotiate." But then a Type I president would prefer to deviate from "tweet" to "negotiate," as this would increase his payoff from 3 to 6. Note that this signalling game is equivalent to the beer-quiche game studied in class.

c. Is there a pooling equilibrium in which both types play "tweet"? If so, does it satisfy the intuitive criterion?

Yes, there is a pooling equilibrium in which both types play "tweet." If both types of president play "tweet" then Congress will acquiesce following a tweet, as it will believe there is a 90% chance a tweeting president is Type II. If Congress plays "fight" following a play of "negotiate" then neither type of president prefers to deviate. "Fight" is an optimal strategy for Congress if it believes there is a 100% chance that a president who plays "negotiate" is Type I.

The equilibrium *does* satisfy the intuitive criterion, as Type I could at least potentially be better off were he to switch from his equilibrium strategy (payoff 5) to "negotiate" (maximum payoff of 6.

Problem 2 (20 points) Consider an economy in which there are equal numbers of men and women, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$6,000, and if an unqualified worker is assigned to the good job the employer loses \$3,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than θ is θ^2. The probability that an unqualified worker will have a test score less than θ is θ. These probabilities are the same for men and women.

There is a fixed wage premium of \$16,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and \$12,000, for both men and women.

Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Can you find an equilibrium in which there are more men than women in the good jobs? You may make use of the following two hints:

Hint #1: You may make use of the WW and EE equations studied in class:

$$\frac{x_q}{x_u} = \frac{1-\pi}{\pi}\frac{f_u(\theta)}{f_q(\theta)} \quad \text{(EE)}$$

$$\pi = G(\omega\,(F_u(s) - F_q(s))) \quad \text{(WW)}$$

Hint #2: All equilibrium values of the test score threshold, s, are contained in the following set: $\{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{5}, \frac{2}{3}, \frac{3}{5}, \frac{3}{4}, \frac{4}{5}\}$.

Using the notation from class, we have:

$$x_q = 6000$$
$$x_u = 3000$$
$$F_q(\theta) = \theta^2$$
$$f_q(\theta) = 2\theta$$
$$F_u(\theta) = \theta$$
$$f_u(\theta) = 1$$
$$\omega = 16000$$
$$G(c) = \frac{c}{12000} \quad \text{for } c \in [0, 3000]$$

The EE and WW equations are then given by:

$$2 = \frac{1-\pi}{\pi}\frac{1}{2s} \quad \text{(EE)}$$
$$\pi = \frac{4}{3}s(1-s) \quad \text{(WW)}$$

Solving (EE) and (WW) for π and s yields two solutions in which both π and s are in $[0, 1]$:

$$\text{Solution 1: } s = \frac{3}{4}, \pi = \frac{1}{4}$$
$$\text{Solution 2: } s = \frac{1}{2}, \pi = \frac{1}{3}$$

If the $(s = \frac{3}{4}, \pi = 14)$ equilibrium is applied to B-workers, while the $(s = \frac{1}{2}, \pi = 13)$ equilibrium is applied to A-workers, then, despite A- and B-workers being ex ante identical, statistical discrimination against B-workers will persist in equilibrium.

Problem 3 (20 points) The best available test for Groat's disease is pretty accurate, but sometimes returns false positives and false negatives. Specifically, the test returns a positive result for 92 out of every 100 individuals with Groat's disease (and a negative test result for 8 out of 100 individuals who have Groat's). The test returns a negative result for 99 out of every 100 individuals who do not have Groat's disease (and a positive test result for 1 out of every 100 individuals who do not have Groat's). Only one in ten thousand people has Groat's disease.

a. Suppose that Alice receives a positive test result. What is the probability that Alice has Groat's? You may assume that prior to taking the test Alice's probability of having Groat's was one in ten thousand.

$$P(\text{Groat's}|\text{positive test}) = \frac{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's})}{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's}) + P(\text{no Groat's}) * P(\text{positive test}|\text{no Groat's})}$$
$$= \frac{\frac{1}{10000}\frac{92}{100}}{\frac{1}{10000}\frac{92}{100} + \frac{9999}{10000}\frac{1}{100}}$$
$$= .009117$$

So, there is less than a 1% chance that someone who tests positive actually has Groat's disease.

b. Suppose a new test is developed which has no false negatives; everyone with Groat's who takes the test receives a positive result. It is still the case that the test returns a negative result for 99 out of 100 people who do not have Groat's. Suppose that Bob takes this new test, and receives a positive result. What is the probability that Bob has Groat's? Again, you may assume that Bob's prior probability of having Groat's is one in ten thousand.

$$\begin{aligned} P(\text{Groat's}|\text{positive test}) &= \frac{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's})}{P(\text{Groat's}) * P(\text{positive test}|\text{Groat's}) + P(\text{no Groat's}) * P(\text{positive text}|\text{no Groat's})} \\ &= \frac{\frac{1}{10000}}{\frac{1}{10000} + \frac{9999}{10000}\frac{1}{100}} \\ &= .009902 \end{aligned} \tag{1}$$

c. What can you conclude about the effectiveness of preventative screening for rare diseases?

For any test of a rare disease with even a slight probability of a false positive, a positive result is much more likely to have come from an individual who does not have the disease than from one who does. Hence, most positive results will be false positives. If false positives are costly, e.g. because of unnecessary surgeries or patient stress, it is not clear that such preventative screening is a good idea.

Problem 4 (20 points) Suppose that normal workers increase a firm's revenue by \$6, while smart workers increase revenue by \$$A$, where $A > 6$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's educational level.

Any worker can acquire as much education as she wishes, but getting e years costs a normal worker $B * e$, where $B > 1$, while e years cost a smart worker only e. A worker's utility function equals her wage, minus the cost of education.

a. Define e^* as the minimum years of education that smart workers must get to differentiate themselves from normal workers in a separating equilibrium. Solve for e^*. Your answer will be a function of the variables A and B.

The normal workers must be indifferent between 0 education (and utility 6) and e^* education (and utility $A - Be^*$). This gives

$$e^* = \frac{A-6}{B}$$

b. Describe, verbally or using a picture, a wage function that supports a separating equilibrium in which smart types obtain e^* units of education. Does this equilibrium satisfy the intuitive criterion?

There is a range of acceptable answers. One easy answer is $w(e) = 6$ for $e < e_H$ and $w(e) = A$ for $e \geq e_h$.

c. As A increases, does e^* increase or decrease? Explain intuitively why this is the case.

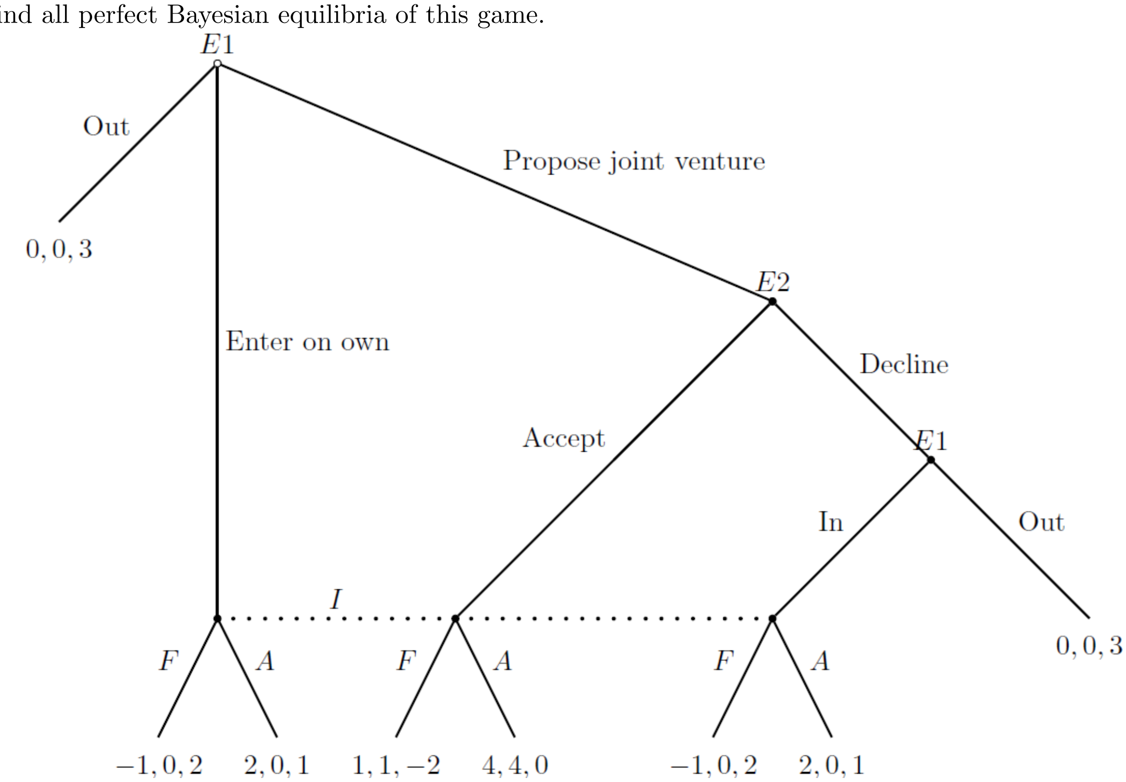
e^* is increasing in A. As smart workers become more productive, they are paid more in a competitive labor marker, and so there is more of an incentive for normal workers to try to pool with the smart workers by getting education. Therefore, education must be made costlier, by increasing the number of years required to be identified as a smart worker.

d. As B increases, does e^* increase or decrease? Explain intuitively why this is the case.

As B increases, e^* decreases. This is because as education becomes costlier to normal workers, less education is required for smart workers to differentiate themselves from normal workers.

Problem 5 (20 points) Consider the 3-player game between two entrants ($E1$ and $E2$) and an incumbent (I) in figure 2. Entrant $E1$ will either enter on his own or as part of a joint venture with entrant $E2$. If entry occurs, firm I chooses between fighting entry with a price war and acquiescing to normal competition. Firm I cannot observe whether or not entry was the result of a joint venture.

Find all perfect Bayesian equilibria of this game.



This is example 9.C.2 in MWG.