Final Exam

answers

Instructions: You may use a calculator, scratch paper, and a one-sided handwritten "cheat sheet',' but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam.

Problem 1 (20 points) Following the Coate and Loury paper, suppose that whether a worker is qualified for a given job depends on whether or not that worker made an unobservable investment prior to applying for the job. The cost of the investment is c, which is distributed uniformly between 0 and 50 across all workers (so that $G(c) = \frac{c}{50}$). Workers place into either good jobs or bad jobs; the net benefit to a worker from the good job is 50.

Workers are tested before placement, but test scores are noisy. All test scores are between 0 and 100. Qualified workers (i.e., all those who made the unobservable investment) score at or below θ with probability $\frac{\theta^2}{10{,}000}$ while unqualified workers score at or below θ with probability $\frac{\theta}{100}$. Firms use the test score to decide whether to place a worker into a good job or a bad job. Thus,

$$F_u(\theta) = \frac{\theta}{100}, f_u(\theta) = \frac{1}{100}, F_q(\theta) = \frac{\theta^2}{10,000}, f_q(\theta) = \frac{1}{5000}\theta$$

Firms have a prior belief of π that any given worker is qualified before observing that worker's test score. Firms earn a profit of $x_q = 50$ from putting a qualified worker into the good job, while firms lose $x_u = 10$ from putting an unqualified worker into the good job. Firms break even on all workers in the bad job.

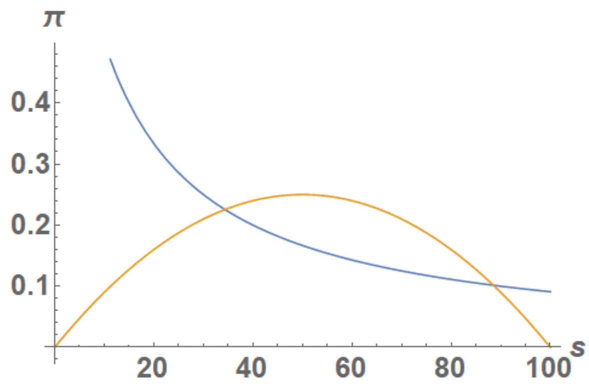
a. Suppose that $\pi = .1$. For what range of test scores would firms place a worker into the good job? The bad job? Firms would put any worker scoring at least 90 into the good job, and workers scoring below 90 into the bad job.

b. Suppose that $\pi = .2$. For what range of test scores would firms place a worker into the good job? The bad job? Based on your answers to a-b, what is the relationship between the prior π and how easy it is to place into the good job? Firms would put any worker scoring at least 40 into the good job, and workers scoring below 90 into the bad job. Evidently, the greater the prior, the lower the score needed to be placed into the good job.

c. Suppose that a worker is placed into the good job if he/she scores above 50. What fraction of workers will choose to become qualified? $\frac{1}{4}$

d. Does the fraction of workers choosing to become qualified increase or decrease if the test score cutoff is increased to 60? What if the cutoff is decreased to 40? Based on your answers, what is the relationship between test score cutoff and the fraction of qualified workers. If the cutoff is either increased to 60 or decreased to 40, 24% of workers will become qualified. Evidently, the fraction of qualified workers peaks at a test score cutoff of 50.

e. Letting s refer to the minimum test score for placement into the good job, draw a graph depicting 1- the relationship between π and s described in your answer to part b. and 2- the relationship between π and s described in your answer to part d. Beyond reflecting your answers to b and d, your picture need not be precise. Below is a precise picture I drew in Mathematica. It is substantively identical to pictures we discussed in class. Any qualitatively similar answers will receive full points.



Problem 2 (20 points) A recent study of discrimination in the German labor market[1] sent resumes to various employers. The resumes were identical except for the name and/or photo included.

The study found that 18.8% of the "Sandra Bauer" resumes produced an interview invitation. 13.5% of the "Meryem Öztürk" (no headscarf) resumes produced an interview, while only 4.2% of the "Meryem Öztürk" (with headscarf) resumes produced an interview invitation.

a. Do you think this study is consistent with statistical discrimination? Explain, using concepts from the Coate and Loury paper, how it can be that equally qualified applicants differing in only one characteristic (name or appearance) can have different success rates in the labor market. Yes, this is consistent with statistical discrimination. The key idea an answer should mention is that firms may have different priors about different groups of otherwise identical workers (e.g., men versus women, or workers with German-sounding names versus workers with Turkish sounding names). These different priors translate into the same observable information (in this case, the resume) being interpreted differently across groups. It is possible that German firms have a low prior for Turkish or Muslim workers, and thus a relatively stronger resume is needed for such a worker to land an interview, relative to a candidate with a German name and an identical resume.

b. Regardless or your answer to a., the study is clearly consistent with taste-based discrimination (i.e., employers simply dislike workers with Turkish names or who wear headscarves). Discuss, using concepts from the Coate and Loury paper, how statistical discrimination differs from taste-based discrimination in the long-run. Would we expect to see taste-based discrimination gradually disappear as competition drives employers to minimize costs? Would we expect statistical discrimination to gradually disappear for the same reason? We have informally discussed in class the argument that taste-based discrimination will gradually disappear

[1] Weichselbaumer, D. (2016). "Discrimination against female migrants wearing headscarves" IZA Discussion Paper No. 10217

as firms facing competitive pressure must reduce costs to survive; those that continue to discriminate will be driven out of a competitive market. This is not necessarily true with statistical discrimination, which can be self-perpetuating. Coate and Loury's model suggests that different priors can become entrenched as they affect the incentives of different groups of workers to develop skills necessary for success in the labor market. If one group (e.g., women) faces low priors, they will have to truly excel in order to be rewarded for their investments in e.g. education. Since these investments are costly, low priors diminish these workers' incentives to invest, which, in turn, confirms the low priors. If another group (e.g. men) has a higher prior, they will have a more reasonable standard, and will thus have greater incentive to acquire skills. Thus, discriminatory beliefs can be self-perpetuating, and not necessarily self-correcting, even over the very long run.

Problem 3 (20 points) Three players value an item at v_1, v_2, and v_3, respectively. They engage in a second-price private values auction to see who gets the item. Each player secretly writes down a bid on a sheet of paper, an auctioneer collects the papers, and then announces the winner (the highest bid). The winner pays an amount equal to the second-highest bid for the item. Each player knows his/her own valuation, but cannot observe any other player's valuation. Everyone knows that each player's valuation is drawn from a uniform distribution between 0 and $1,000.

a. Suppose that bidders 2 and 3 each bid half of his/her valuation (so that $b_2 = \frac{1}{2}v_2$, and $b_3 = \frac{1}{2}v_3$). Show that it is a best response for bidder 1 to bid $b_1 = v_1$. For a given valuation v_1, bidder 1 chooses his bid to maximize $Pr(\text{win auction}) * (v_1 - E(P))$, where $E(P)$ is the expected price paid. Clearly, $Pr(winauction)$ is increasing in $b_1 \leq v_1$, while $v_1 - E(P)$ is invariant in b_1, so $b_1 = v_1$ gives utility at least as high as any $b_1 < v_1$. Finally, $b_1 = v_1$ gives 1 at least as much utility as any $b_1 > v_1$, as while raising b_1 above v_1 increases $Pr(\text{win auction})$, the "extra" auctions won would all involve 1 outbidding another player with a bid greater than v_1, meaning that 1's utility net of price $v_1 - E(P)$ is negative.

b. Suppose that bidders 2 and 3 each bid exactly his/her valuation (so that $b_2 = v_2$, and $b_3 = v_3$). Show that bidder 1's best response is to bid $b_1 = v_1$. The argument in a. is not dependent on the other players' strategies.

c. This auction game has a unique Nash equilibrium in bidding strategies. What is it? Each player bids his/her valuation, $b_i = v_i$.

Problem 4 (20 points) Consider a signaling game that satisfies:

1. Two types of player 1, tough and weak. Player 1's type is unobservable to player 2, but known to player 1. Each type is equally likely *ex ante*, and is chosen by nature.

2. Player 1 encounters player 2 in a competition for resources.

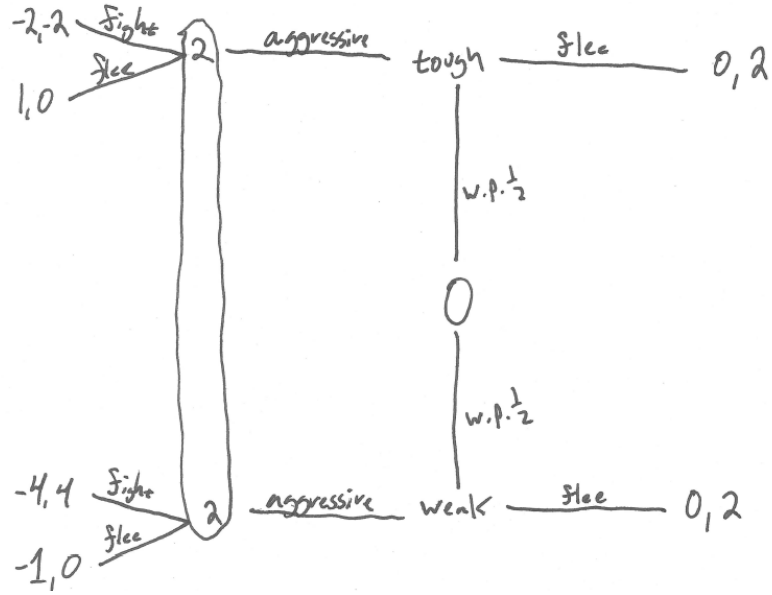
3. Either type of player 1 sends one of two signals. He can flee (game ends, player 1 receives 0, player 2 receives 2), or he can engage in costly aggressive behavior.

4. If player 1 behaves aggressively, player 2 can fight or flee. In this case, payoffs are as follows (the first

number is player 1's payoff):

	fight	flee
tough	$-2, -2$	$1, 0$
weak	$-4, 4$	$-1, 0$

a. Draw the extensive form of the game described above (hint: your picture will be similar, though not identical, to the signaling games we studied in class).



b. Is there a pooling equilibrium in which both types of player 1 behave aggressively? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion. No. Regardless of which action player 2 plays, the weak type would prefer to switch to "flee'.'

c. Is there a pooling equilibrium in which both types of player 1 flee? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion. Yes. If player 2 plays "fight," neither type of player 1 wish to deviate from "flee'.' For "fight" to be optimal for player 2, she must believe she is at the lower node with probability of at least $\frac{1}{3}$. This equilibirum does not satisfy the intuitive criterion, however, as player 2's belief places positive probability on player one being a weak type conditional on player 1 behaving aggressively. But the weak player 1's payoffs from aggressive behavior are dominated by his equilibrium payoff of 0. The tough player 1, on the other hand, could potentially increase his payoff by switching to aggressive behavior. Hence, the intuitive criterion requires that player 2 believe that if player 1 were to behave aggressively, then player 1 is tough with probability 1. Under this belief, player 2 would play "flee," which breaks the equilibrium as then the tough type of player 1 actually would prefer to switch.

d. Is there a separating equilibrium in which the tough type behaves aggressively and the weak type flees? If so, clearly state what the equilibrium is, and determine whether it satisfies the intuitive criterion. Yes. Player 2 plays "flee", and neither type of player 1 wishes to deviate. The intuitive criterion is trivially satisfied, as there are no unsent signals.

Problem 5 (20 points) Consider a version of the Cournot oligopoly game in which firm 2's costs are unknown to firm 1. Firm 2 knows its own cost, however. Specifically,

Inverse demand: $P = 1 - q_1 - q_2$

Firm 1's marginal cost: 0

Firm 2's marginal cost: $\begin{cases} c_L = .2 & \text{with probability } \frac{1}{4} \\ c_H = .4 & \text{with probability } \frac{3}{4} \end{cases}$

a. Suppose firm 2 is low cost (so that its marginal cost is $c_L = .2$). Solve for the value of q_2^L that maximizes firm 2's profits. (hint: your answer should be a function of q_1) If low cost, firm 2 solves:

$$\max_{q_2}(1 - q_1 - q_2)q_2 - .2q_2$$
$$\Rightarrow q_2^L = \frac{2}{5} - \frac{1}{2}q_1$$

b. Suppose firm 2 is high cost (so that its marginal cost is $c_L = .4$). Solve for the value of q_2^H that maximizes firm 2's profits. If high cost, firm 2 solves:

$$\max_{q_2}(1 - q_1 - q_2)q_2 - .4q_2$$
$$\Rightarrow q_2^H = \frac{3}{10} - \frac{1}{2}q_1$$

c. Solve for the value of q_1 that maximizes firm 1's profits for any values of q_2^L and q_2^H. Firm 1 solves:

$$\max_{q_1} \frac{1}{4}(1 - q_1 - q_2^L)q_1 + \frac{3}{4}(1 - q_1 - q_2^H)q_1$$
$$\Rightarrow q_1 = \frac{1}{2} - \frac{1}{8}q_2^L - \frac{3}{8}q_2^H$$

d. Clearly describe the oligopoly game's Bayesian Nash equilibrium. $q_1 = .45$, $q_2^H = .075$, $q_2^L = .175$