Homework 2

answers

Problem 1 Consider a market with demand curve $P = 1 - Q$, served by Cournot oligopolists with marginal cost c who compete by simultaneously setting quantity.

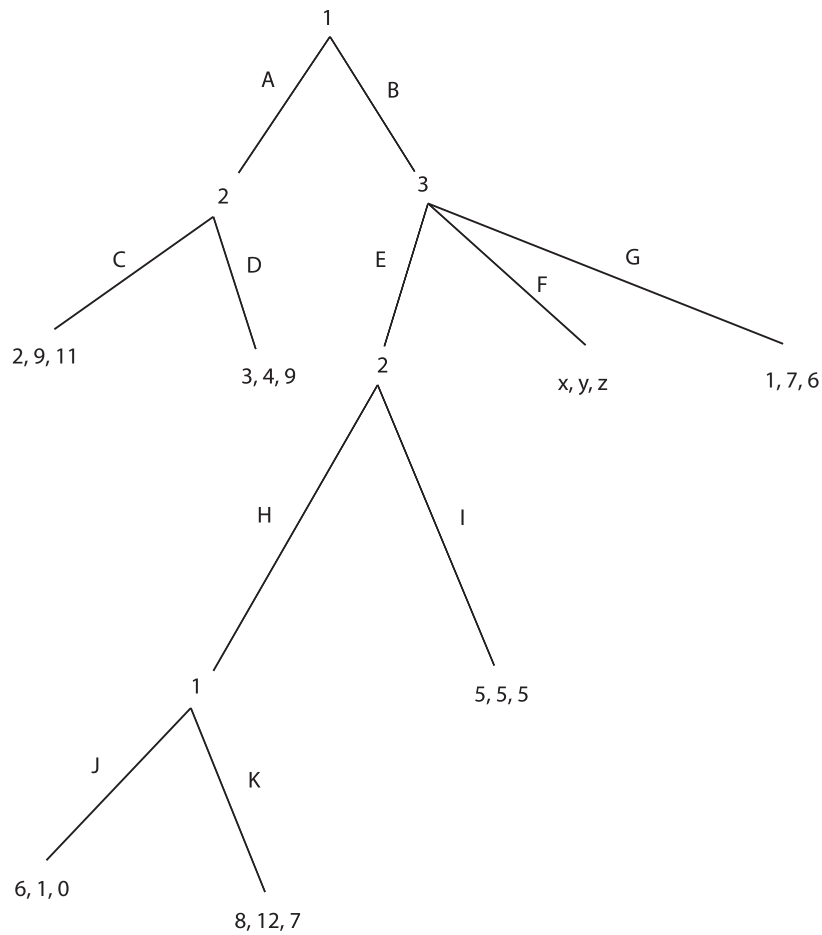
a. Suppose that there are 2 firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

In the Nash equilibrium, $q_1 = q_2 = \frac{1}{4}$. The price is $P = \frac{1}{2}$ and each firm earns a profit of $\frac{1}{16}$.

b. Now suppose that there are N firms, each with marginal cost $c = \frac{1}{4}$. Solve for the Nash equilibrium quantity, price, and profit for each firm.

In the Nash equilibrium, each firm produces $q = \frac{3}{4(N+1)}$. The market price is $\frac{4+N}{4(N+1)}$. Each firm earns a profit of $\frac{9}{16}\frac{1}{(N+1)^2}$.

Problem 2 Consider the sequential move game below. Each set of payoffs is ordered u_1, u_2, u_3, where u_i is player i's utility.



For all possible values of x, y, and z, What is the subgame perfect equilibrium of this game? Remember to describe players' choices at all nodes, including those that are unreached.

If $z < 7$, then 1 plays B and K, 2 plays C and H, and 3 plays E; payoffs are (8,12,7).

If $z > 7$ and $x > 2$, then 1 plays B and K, 2 plays C and H, and 3 plays F; payoffs are (x,y,z).

If $z > 7$ and $x < 2$, then 1 plays A and K, 2 plays C and H, and 3 plays F; payoffs are (2,9,11).

If $z = 7$, then any mixture of E and F is optimal for 3, including the pure strategies. If 3 plays E with probability p and F with probability $1 - p$, then player 1's payoff from B is $8p + x(1 - p)$, whereas if 1 plays A his payoff is 2. Hence, For all mixtures and values of x such that $2 > 8p + x(1 - p)$, there is a SPE in which 1 plays A and K, 2 plays C and H, and 3 plays $pE + (1 - p)F$. For all values of p and x for which $2 < 8p + (1 - p)x$, there is a SPE in which 1 plays B and K, 2 plays C and H, and 3 plays $pE + (1 - p)F$.

Finally, if $2 = 8p + (1-p)x$, then there is a SPE in which 1 mixes between A and B (any mixture is part of an equilibrium) and plays K, 2 plays C and H, and 3 plays $pE + (1-p)F$.[1]

Problem 3 Consider a market with inverse market demand given by $P = 10 - \frac{1}{100}Q$. Firm A is a monopoly producer, with marginal cost equal to \$2.

a. Calculate Firm A's optimal quantity, and its profit as a monopolist.

Firm A produces $Q = 400$, so that the market price is \$6, and Firm A's profit is \$1,600.

Now, suppose that Firm A has discovered a new technology that will allow it to produce at a marginal cost of \$0. Implementing the new technology will cost firm A to incur a fixed cost of \$1,000.

b. Is it profitable for Firm A to implement the new technology?

If Firm A has zero marginal cost, it will produce $Q = 500$, the market price is \$5, and Firm A's variable profit is $\$2,500$. Since this is only \$900 more than its profit in part a. with a higher marginal cost, Firm A will choose not to invest in the new technology.

Now, suppose that Firm A learns that Firm B is considering entering the market to compete with Firm A. To enter, Firm B would have to construct a factory at a cost of \$500, and then Firm A and Firm B would compete in Cournot oligopoly.[2] If firm B entered, its marginal cost would also equal \$2.

c. Calculate the market price, firm A's profit, and firm B's profit under Cournot competition. Would firm B profitably enter the market? (Assume for part c. that Firm A has not implemented the new technology.)

In Cournot equilibrium, Firms A and B each produce quantity $\frac{800}{3}$, the market price is \$4.67, and each firm earns a profit of \$711.11.

d. Consider an extensive form game with two rounds. In round 1, Firm A decides whether or not to implement the new technology. In round 2, Firm B decides whether or not to enter the market. Using your answers above (and possibly new calculations), determine the subgame perfect equilibrium of this game.

See the figure below. Note that the payoffs in the event Firm A invests and Firm B enter require an additional calculation. The SPE of this game is for Firm A to invest in the new technology, and for Firm B to not enter the market.

e. Policymakers sometimes worry that monopolists are less likely to innovate than firms in a competitive market.[3] Do your answers above suggest any caveats to this view?

Potential entrants may drive a monopolist to innovate to discourage entry.

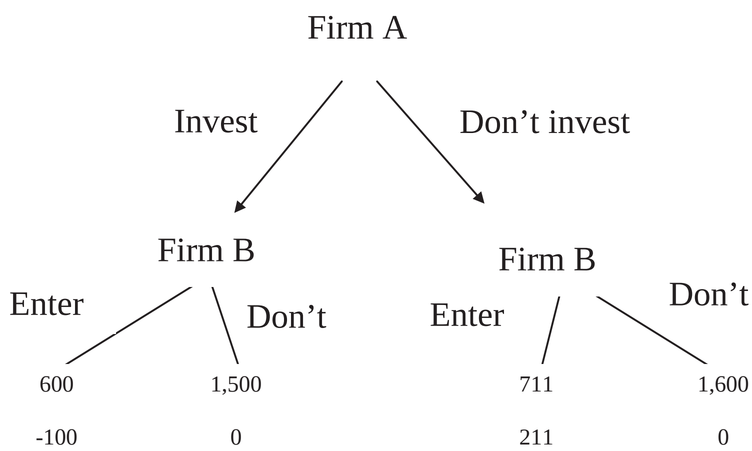
Problem 4 Consider a 2-player bargaining game, in which the players are choosing how to split a surplus of \$1. Each player discounts payoffs one period in the future by $\delta = \frac{1}{2}$. The rules are as follows:

[1] Most students ignored equilibria with mixed strategies in their answer, and my intention in asking this question was for you to determine how the pure strategies depend on the values of x, y, and z, so don't worry if this was not part of your answer.

[2] So that inverse market demand is given by $P = 10 - \frac{1}{100}(q_1 + q_2)$, where q_i is firm i's quantity.

[3] See e.g. "Enhanced market power can also be manifested in... diminished innovation.", page 2 of *Horizonal Merger Guidelines*, 2010, U.S. Department of Justice and Federal Trade Commission.

Figure 1: Relevant figure for question 3.d.



Period 0: Player 1 chooses a value of X between 0 and 1, at cost $c(X) = \frac{3}{8}X^2$.

Period 1: Player 2 makes an offer of $(y, 1-y)$, where y is between 0 and 1. Player 1 may either accept the offer, in which case the surplus is split accordingly and the game ends, or reject the offer, in which case the game moves on to period 2.

Period 2: Player 1 makes an offer to player 2. If the offer is accepted, the surplus is split accordingly and the game ends. If the offer is rejected, a mediator awards a payoff of X to player 1, and $1 - X$ to player 2 (nb. this is the same X chosen by player 1 in period 0; interpret the cost player 1 incurred in that period as that of hiring an attorney to investigate the most favorable mediation venue).

a. Solve for the subgame perfect equilibrium of this game. Is player 1 or player 2 better off in this setup? Solve the game using backward induction. In period 2, player 2's outside option is $\delta * (1 - X)$, so player 1's offer would be $(1 - \delta + \delta X, \delta - \delta X)$, and it player 2 would accetp the offer. In period 1, player 2 would offer $(\delta - \delta^2 + \delta^2 X, 1 - \delta + \delta^2 - \delta^2 X)$, and player 1 would accept the offer. In period 0, player 1 must equate the marginal cost of a higher X (which is $\frac{3}{4}X$ with the marginal benefit (which is δ^2). Given that $\delta = \frac{1}{2}$, then, we have that $X = \frac{1}{3}$. The subgame perfect equilibrium is thus player 1 chooses $X = \frac{1}{3}$ in period 0, and players 1 and 2 make offers as above in periods 1 and 2. Player 2's offer is accepted in period 1, and the game ends then. Player 1's payoff (from the perspective of period 1) is $\frac{1}{3}$, and player 2's payoff is $\frac{2}{3}$, so it is better to be player 2.

b. Intuitively, how would your answer change by an increase in δ? You do not need to work out the mathematical details.

For a constant $X > 0$, player 1's period 1 payoff of $\delta - \delta^2 + \delta^2 X$ is increasing in δ. Further, in period 0, the marginal benefit of choosing a higher X increases as player 1 becomes more patient. Hence player 1 will receive a greater share of the surplus. Acceptable anwers could also recalculate the equilibrium outcome for one or two higher values of δ to gain intuition.

Problem 5 Gibbons, problem 2.7

Begin in the second stage, where the n firms simultaneously choose L_i, $i = 1, 2, ..., n$, and wage w is taken as given. The firms are essentially playing a Cournot game. Solve for the symmetric Nash equilibrium using standard methods. Specifically, firm i solves:

$$\max_{l_i}(a - \sum_{i=1}^{n} L_i)L_i - wL_i$$

which has FOC $L_i = \frac{a-w-\sum_{j\neq i} L_j}{2}$. In a symmetric equilibrium, $L_1 = L_2 = ... = L_n$, so we have $L_i = \frac{a-w}{n+1}$ for all i.

Now, turning to stage 1, where the union chooses w, the union's utility-maximization problem is given by:

$$\max_{w}(w - w_a)\frac{n}{n+1}(a - w)$$

which has FOC $w = \frac{a+w_a}{2}$. The firm's maximized utility is $\frac{n}{n+1}\left(\frac{a-w_a}{2}\right)^2$, which is increasing in n. Evidently, the union's wage offer does not vary in n, as it is set to equate the marginal benefit of a higher wage (more money for union members) and the marginal cost (fewer employed workers in the union), which does not vary in the number of firms. However, the more firms there are, the higher total output is (as is standard in a Cournot model). Since more output requires more workers, the union prefers there to be as many firms as possible.

Problem 6 Alice and Bob compete in a race. At the start of the race, both players are 6 steps away from the finish line. Who gets the first turn is determined by a toss of a fair coin; the players then alternate turns, with the results of all previous turns being observed before the current turn occurs.

During a turn, a player chooses from these four options:

- Do nothing at cost 0;
- Advance 1 step at cost 2;
- Advance 2 steps at cost 7;
- Advance 3 steps of at cost 15.

The race ends when the first player crosses the finish line. The winner of the race receives a payoff of 20, while the loser gets nothing. Assume there is no discounting, but that all else equal each player prefers to finish the game more quickly.

Find the subgame perfect equilibria of this game.[4]

The table below describes how many steps a player will take on her turn, depending on the number of steps both she and her opponent have remaining. The table also describes a players continuation payoff, or the payoff she receives from that point forward in the game.

[4]Hint: In the SPE, a player's choice at a decision node only depends on the number of steps he has left and on the number of steps his opponent has left. Make a table with columns and rows numbered from 1-6, representing how many steps each player has left to finish. Solve for what one player will do at each possible state. Since the game is symmetric, solving for what one player will do at each point in your table is sufficient to solve the game.

The equilibrium outcome is that the first mover takes one step at a time until she crosses the finish line, while the second mover never takes a step. The first mover thus gets a payoff of 8.

Interestingly, if the cost of taking one step were decreased to 1, the first mover still wins the game, but her overall payoff is reduced to 2, despite steps being cheaper. This is because she now has to take 3 steps initially to deter player 2 from leapfrogging her.