Midterm

answers

Instructions: You may use a calculator and scratch paper, but no other resources. In particular, you may not discuss the exam with anyone other than the instructor, and you may not access the Internet, your notes, or books during the exam. You have 150 minutes to complete the exam. Good luck!

Questions 1-2 pertain to the game in Figure 1:

		Player 2 L	Player 2 R
Player 1	U	10,10	4,50
Player 1	D	50,4	0,0

Figure 1: Problems 1 and 2 of this exam refer to this normal form game.

Problem 1 (20 points) Consider the game in Figure 1:

a. Find all Nash equilibrium strategies, pure as well as mixed.

There are three NE: (D, L), (U, R), and $(\frac{1}{11}U + \frac{10}{11}D, \frac{1}{11}L + \frac{10}{11}R)$.

b. For each of the Nash equilibria you solved for in a, state the payoff received by each player.

Payoffs are $(50, 4)$, $(4, 50)$, and $(\frac{50}{11}, \frac{50}{11})$, respectively.

c. Solve for each player's minmax payoff. Also state the strategy each player uses to minmax his/her opponent.

The minmax payoff for each player is 4. To minmax player 2, player 1 plays D, while to minmax player 1, player 2 plays R.

d. Now suppose the game is played sequentially, with player 1 choosing his strategy first, and player 2 observing 1's choice prior to choosing her strategy. Solve for all subgame perfect equilibria of this game.

In the unique SPE, 1 plays D, while 2 plays L if 1 plays D, and R if 1 plays U. SPE payoffs are $(50, 4)$.

Problem 2 (20 points) Now, suppose the game in Figure 1 is played repeatedly ad infinitum, with both players sharing a discount factor $\delta \in (0, 1)$.

a. What is the minimum value of δ for which (U, L) is sustainable in a SPE using grim trigger Nash reversion strategies?

The only NE that will work as a punishment is the symmetric mixed equilibrium, in which both players receive a payoff of $\frac{50}{11}$ in each period. For neither player to wish to deviate from (U, L), we must have:

$$\frac{10}{1-\delta} \geq 50 + \frac{\delta\frac{50}{11}}{1-\delta}$$
$$\iff \delta \geq \frac{22}{25} = .88$$

Now, consider a carrot and stick strategy, in which (U, L) is played initially, and players switch to (D, R) for T periods following any deviation from (U, L). If anything other than (D, R) is played during the punishment phase, the punishment restarts, meaning that (D, R) is played for an additional T periods. After T periods of (D, R), the game restarts at (U, L).

b. What condition on δ and T must hold for neither player to wish to deviate from (U, L) in phase 1 of the game?

$$\frac{10}{1-\delta} \geq 50 + \frac{10\delta^{T+1}}{1-\delta}$$
$$\iff \frac{1-\delta^{T+1}}{1-\delta} \geq 5$$

c. What condition on δ and T must hold for neither player to wish to deviate from (D, R) in phase 2 of the game?

$$\delta^T \geq .4$$

d. Suppose $\delta = .9$. Go as far as you can in determining for which values for T do the carrot and stick strategies comprise a subgame perfect equilibrium. T must be between 6 and 8, inclusive.

e. Describe in intuitive terms, why carrot and stick strategies fail SPE equilibrium conditions if T is very small or very large.

If T is large, following through with the punishment is unappealing. Thus, players are likely to deviate from the punishment path, and so the punishment is non-credible. But this makes players willing to deviate from phase 1 of the game. If T is too small, then the punishment is not severe enough to deter players from pursuing a short-run gain by deviating from phase 1 of the games.

Problem 3 (20 points) Two players must divide a surplus of one million dollars. Suppose they play the following bargaining game:

Period 1: Player 1 makes an offer to player 2 on how to divide the million dollars. If 2 accepts, the game ends. If 2 rejects the offer, the game moves to period 2.

Period 2: Player 2 makes an offer to player 1 on how to divide the million dollars. If 1 accepts, the game ends. If 1 rejects the offer, the game moves to period 3.

Period 3: Player 1 makes an offer to player 2 on how to divide the million dollars. If 2 accepts, the game ends. If 2 rejects the offer, both players immediately receive \$100,000, while the remaining \$800,000 is lost (e.g., to lawyers).

Players discount payoffs one period in the future by δ.

a. Solve for the game's subgame perfect equilibrium. Make sure to list what offer will be made in each period, in what period (if any) an offer will be accepted, and how the surplus is divided.

Solve by backwards induction. In period 3, player 1 will offer $(.9, .1)$ (numbers represent the fraction of the total surplus), and player 2 will accept. Then, in period 2, player 2 offers $(.9\delta, 1 - .9\delta)$, and 1 accepts. In period 1, player 1 offers $(1 - \delta(1 - .9\delta), \delta(1 - .9\delta))$, and 2 accepts.

b. Is player 2's equilibrium payoff (measured in period 1) increasing, decreasing, or nonmonotonic in the discount factor δ. Why is this?

2's payoff is $\delta(1 - .9\delta)$, which is nonmonotonic in δ, increasing for $\delta < \frac{5}{9}$, decreasing for $\delta > \frac{5}{9}$. Were δ very low, 2 would get a low payoff, since she would not want to wait for more favorable outcomes in future periods, so 1 would offer her a low amount in period 1. On the other hand, a higher δ *decreases* player 2's period 2 payoff, since 1 is more willing to wait for a favorable period 3 payoff. Thus, 2's payoff is maximized for an intermediate discount factor.

For parts c. and d., suppose that player 2 has the option to pay a bribe of $\$X$ at the beginning of period 1. Paying the bribe would increase the amount she would receive should the offer in period 3 be rejected to $200,000. No other aspect of the game is affected by the bribe.

c. Suppose the discount factor is $\delta = .9$. What is the maximum bribe she would be willing to pay?

Paying the bribe increases 2's share of the surplus from $\delta(1 - .9\delta)$ to $\delta(1 - .8\delta)$, or by $.1\delta^2$. At $\delta = .9$, the increase in her share of the surplus is thus $81,000, which is the maximum bribe she would pay.

d. If the discount factor were *smaller* than .9, would the maximum bribe player 2 would be willing to play increase or decrease? Why?

Paying the bribe increases her share of the surplus by $.1\delta^2$, which is increasing in δ. Thus, were her discount factor smaller, her willingness to pay the bribe would decrease. This is because the bribe works by increasing her outside option, but since this outside option is only attainable several periods in the future, it is less relevant the smaller the discount factor.

Problem 4 (20 points) Firm A is a monopoly seller of an intermediate good (e.g., glass for phone screens). For simplicity, suppose A's marginal cost of production is 0.

A sells its output to firm B for price c. Suppose firm B uses exactly one unit of the intermediate good to produce one unit of its final good (e.g., phones), and (for simplicity) that B has no costs other than the cost of the intermediate good purchased from A. Demand for B's final good is given by $Q = 1 - P$.

Suppose firms A and B interact as follows. First, A chooses a price for the intermediate good, c. Then, B observes c and chooses a price for its final good. Both A and B wish to maximize their profits.

a. Determine B's profit-maximizing price, as a function of c (hint: B takes c as fixed when choosing price).

Firm B sets price P to solve:

$$\max_P (1 - P)(P - c)$$

which has solution $P = \frac{1}{2} + \frac{c}{2}$.

b. Determine the quantity of the intermediate good B purchases, as a function of c (hint: recall that B needs exactly one unit of the intermediate good for each copy of the final good it sells).

At the price solved for in a., demand is $Q = \frac{1}{2} - \frac{c}{2}$.

c. Determine the value of c that maximizes firm A's profit. What quantity of the final good is produced?

Given the answer to b., firm A chooses c to solve:

$$\max_c c(\frac{1}{2} - \frac{c}{2})$$

which has solution $c = \frac{1}{2}$. At this value of c, quantity $\frac{1}{4}$ of the final good is sold.

Now, suppose that firm A and B merge. The merged firm maximizes the sum of A's profit and B's profit.

d. Solve for the profit maximizing values of c and P. What quantity of the final good is produced?

The merged firm chooses c and P to solve:

$$\max_{P,c} c * (\frac{1}{2} - \frac{c}{2}) + (1 - P)(P - c) \tag{1}$$

which has solution $c = 0$, $P = \frac{1}{2}$. Another way of thinking about this is that any positive value of c is a transfer from B to A (and thus irrelevant to the merged firm), while B's profit is clearly decreasing in c.

e. Combinations of producers of intermediate and final goods are often referred to as *vertical mergers.* Based on your answers to parts c. and d., do vertical mergers tend to increase or decrease prices of final goods?

The merged firm produces a greater quantity of the final good at a lower price, because firm B no longer has to pay a markup above marginal cost for the intermediate good. This effect is known as the *elimination of double marginalization*, and is commonly cited as a reason why vertical mergers are likely to be legal.[1]

Problem 5 (20 points) Consider a common good (e.g., clean air) that is depleted with use. N agents consume the good, and each gets utility both from its own consumption and from the remaining stock of the common good. There is no cost associated with consuming x_i. If agent i consumes x_i, agent i's utility is:

$$u_i(x_i, x_{-i}) = \ln(x_i) + \ln(6000 - 2\sum_{j=1}^{N} x_j)$$

a. Suppose $N = 2$. Show that the two agents' best response functions are given by:[2]

$$x_1(x_2) = 1500 - \frac{1}{2}x_2, \quad x_2(x_1) = 1500 - \frac{1}{2}x_1$$

Solve for the Nash equilibrium values of x_1 and x_2. Agent 1's maximization problem is:

$$\max_{x_1} \ln(x_1) + \ln(6000 - 2x_1 - 2x_2)$$

$$\text{First order condition: } \frac{1}{x_1} = \frac{2}{6000 - 2x_1 - 2x_2}$$

$$\Rightarrow x_1 = 1500 - \frac{1}{2}x_2$$

Agent 2's best response follows from symmetry. Plugging one best response into the other yields a Nash equilibrium of $x_1 = x_2 = 1,000$.

[1] See the Feburary 26, 2019 AT&T/Time Warner appellate decision at https://www.cadc.uscourts.gov/internet/opinions.nsf/390E66D6D58F426B852583AD00546ED6/$file/18-5214.pdf

[2] Recall that the derivative of $\ln(x)$ is $\frac{1}{x}$.

b. Now suppose that there are N agents. Solve for the symmetric Nash equilibrium, in which all agents choose the same value of x. Agent 1's maximization problem is:

$$\max_{x_1} \ln(x_1) + \ln(6000 - 2\sum_j x_j)$$

$$\text{First order condition: } \frac{1}{x_1} = \frac{2}{6000 - 2\sum_j x_j}$$

$$\Rightarrow x_1 = \frac{3000}{N+1} = x_j \text{ (from symmetry)}$$