Final exam

5/6/11

Note: Throughout, points will be deducted mercilessly for insufficiently supported answers. When in doubt, err on the side of maximum verbosity.

Problem 1 (50 points) Consider a market with two or more firms and a continuum of workers. Each firm has two types of jobs, "old" jobs and "new" jobs. The profit to the firm and the payoff to the worker, when the worker is assigned to an old job, is 0. The payoff to a worker assigned to a new job is 1. The payoff to a firm when assigning the worker to the new job is 1 if the worker is skilled, and -1 if the worker is not skilled (all payoffs already include wages). A worker must pay a cost of c to acquire skills. The value of c differs across different workers, with c being uniformly distributed on $[0, 1]$.

a. Suppose that workers first decide whether to acquire skills and then are matched to firms, who assign them to jobs. Suppose that the firms *can* observe whether each worker has acquired skills. Find the pure-strategy equilibrium job-assignment and skill-acquisition decisions.

b. Now suppose that workers first decide whether to acquire skills and then are matched to firms, who assign them to jobs. Suppose that firms *cannot* observe whether a worker has acquired skills. Find the pure-strategy equilibrium and skill-acquisition decisions.

c. Now suppose that workers first decide whether to acquire skills, then take a test, and then are matched to firms, who assign them to jobs. Firms cannot observe whether a worker has acquired skills, but can observe the outcome of the test, which is either a pass (p) or fail (f).[1] A worker who has acquired skills passes the test with probability $\alpha > \frac{3}{4}$ and fails with probability $1 - \alpha$, while a worker who has not acquired skills passes with probability $1 - \alpha$ and fails with probability α. Find the equilibrium job-assignment and skill-acquisition decisions. (There are multiple such equilibria. Find all the pure strategy equilibria first. Consider mixed strategy equilibria if time permits.)

d. Now suppose that workers come in two varieties, red and green. The colors have no effect on the cost of acquiring skills, test outcomes, the value of acquiring skills, or anything else, but are observed by firms. Is there an equilibrium in which different colored workers behave differently?

Problem 2 (15 points) Consider a variation on the education signaling game in which, first, nature draws a worker's productivity from some continuous distribution on $[\underline{\theta}, \overline{\theta}]$. Once the worker observes her type, she can choose whether to submit to a costless test that perfectly reveals her ability. Finally, after observing whether the worker has taken the test and its outcome if she has, two firms bid for the worker's services. Argue that in any subgame perfect equilibrium of this model all worker types submit to the test, and firms offer a wage no greater than $\underline{\theta}$ to any worker not doing so. (hint: note that there are two major differences from the education model studied in class. 1-There is no education, 2-there is a continuum of worker types.)

[1] This is a similar setup to a model studied in class, but note that here the test has only two possible outcomes, whereas in class, the test score was continuously measured.

Problem 3 (35 points) Consider an oligopoly with K firms. Each firm can produce costlessly, and market demand is given by $P = 1 - \sum_{i=1}^{K} q_i$.

a. Suppose $K = 3$, and the three firms are Cournot competitors. Solve for the Nash equilibrium profits of firms 1, 2, and 3 in this game.

b. Suppose $K = 3$. Suppose that firm 1 publicly commits to a quantity. After observing firm 1's choice of quantity, firms 2 and 3 simultaneously choose quantities. Solve for the subgame perfect equilibrium profits of firms 1, 2, and 3 in this game.

For parts c-e: Suppose that K firms choose quantity sequentially (that is, firm 1 publicly announces a quantity. After observing q_1, firm 2 publicly announces a quantity. After observing q_1 and q_2, firm 3 publicly announces a quantity, and so on).

c. Suppose $K = 3$. Solve for each firm's profit in the subgame perfect equilibrium of this game. You are not allowed to directly reference the result of parts d and e.

d. Now suppose K is a large number. Solve for the SPE of the game via guess and verify (I guess, you verify).

$$\textbf{Guess: } q_k = \frac{1 - \sum_{i=1}^{k-1} q_i}{2} \tag{1}$$

Verify this guess[2] by showing that firm 1's best response to firms $2, 3, ..., K$ playing this strategy is $q_1 = \frac{1}{2}$. Do this in two steps:[3]

1. Show that if firms $2, 3, ..., K$ produce according to (1), then $q_k(q_1) = \frac{1}{2^{k-1}} - \frac{1}{2^{k-1}} q_1$.
2. Show that 1 implies $q_1^* = \frac{1}{2}$.

e. Calculate firm k's SPE profits in the game described in part d. (Whether or not you were able to solve part d, you may take its result as given).

[2]Think about what this guess is saying. Each firm produces enough quantity to satisfy half the remaining demand on its turn.

[3]There may be a third step required to verify the guess. Do not worry about this. If you cannot do step 1, you may take its result as given and show step 2 alone for partial credit.