Homework 1

due 1/25/2011

Problem 1 Suppose a player in an extensive form game has m information sets and that at his k^{th} information set, he can choose from among b_k actions.

i. How many pure strategies does this player have?

ii. What is the dimension of his set of mixed strategies? What is the dimension of his set of behavior strategies? (Hint: "dimension" means how many pieces of information you would need to completely understand his mixed or behavior strategy.)

Problem 2 Give an example of a game in which a player has a mixed strategy which does not admit an equivalent behavior strategy.

Problem 3 In the game Γ, player 1 moves first, choosing between actions A and B. If he chooses B, then player 2 chooses between actions C and D. If she chooses D, then player 1 moves again, choosing between actions E, F, and G. A choice of A or C ends the game. Payoffs are irrelevant for this question.

i. Find a behavior strategy which is equivalent to the following mixed strategy:

$$\sigma_1 = (\sigma_1(AE).\sigma_1(AF), \sigma_1(AG), \sigma_1(BE), \sigma_1(BF), \sigma_1(BG)) = (\frac{1}{2}, \frac{1}{3}, 0, 0, \frac{1}{12}, \frac{1}{12})$$

ii. Describe *all* mixed strategies which are equivalent to the following behavior strategy:

$$b_1 = ((b_1(A), b_1(B)), (b_1(E), b_1(F), b_1(G))) = ((\frac{1}{3}, \frac{2}{3}), (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}))$$

Problem 4 In the game Γ', player 1 moves first, choosing between actions L and R. Player 2 observes this choice. If 1 chooses L, then 2 chooses between actions A and B. If 1 chooses R, then 2 chooses between actions C and D. Let $b_2 = ((b_2(A), b_2(B)), (b_2(C), b_2(D))$ be a behavior strategy for player 2.

i. Describe the collection of mixed strategies which are equivalent to b_2. (Hint: you will need to write down equations describing the relationship between b_2 and $\sigma_2(AC), \sigma_2(AD)$, and so on.)

ii. Specify a single mixed strategy which is equivalent to b_2. (Hint: were you to have numbers for b_2, your answer should describe how to use those numbers to get a specific mixed strategy σ_2. There are many correct answers.)

Problem 5 Dave has preferences over lotteries which assign probabilities $p = (p_a, p_b, p_c)$ to three possible prizes: an apple, a banana, and a cherry. Suppose that Dave is indifferent between the lottery $p^1 = (1, 0, 0)$ and $p^2 = (0, \frac{1}{2}, \frac{1}{2})$, and that Dave strictly prefers the lottery $p^3 = (\frac{1}{2}, \frac{1}{2}, 0)$ to the lottery $p^4 = (0, \frac{3}{4}, \frac{1}{4})$. Are these preferences consistent with the von Neumann-Morgenstern axioms? (hint: when you are asked questions like this on HWs/exams, the answer is usually no, as it is much easier to disprove a statement like this than it is to prove it).

Problem 6 Find the reduced normal forms of the games in Figures 1 and 2. (Hint: for the 3 player game in figure 1, you will need to draw two payoff matrices, one for each of player 3's actions.)

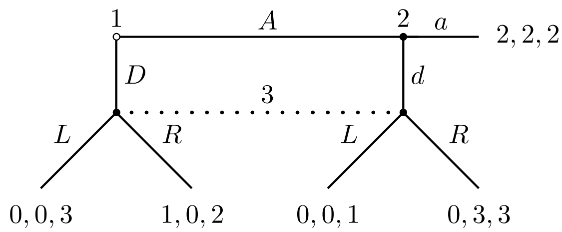

Figure 1: *Seltens horse*

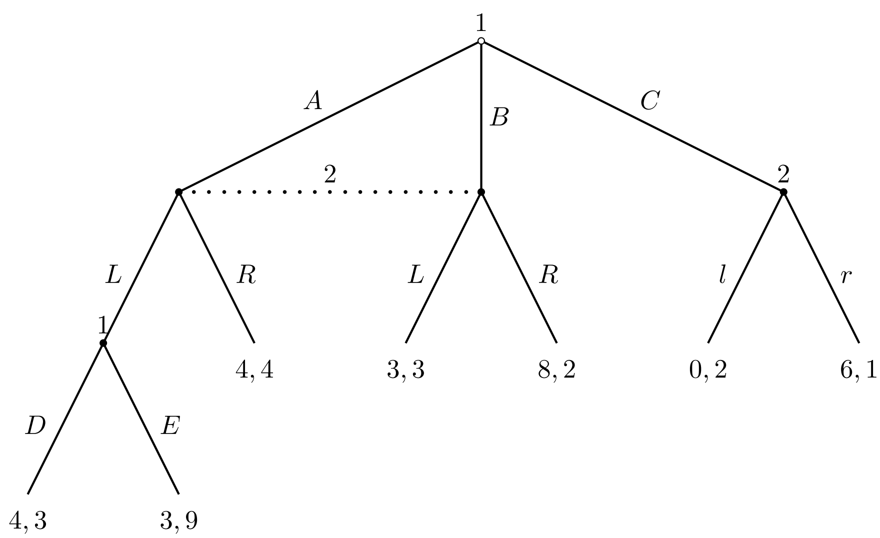

Figure 2: The unblinking eye