

Homeworks 3

due 2/15/2011

Problem 1 Mas-Colell, problem 9.B.7

Problem 2 Mas-Colell, problem 9.B.9

Problem 3 The “modified centipede” we considered in class (see figure 1) has a sequential equilibrium in which both players continue. In finding this equilibrium, we said that, from Bayes’ rule, $\mu_1(y) = \frac{19\sigma_2(c^1)}{19\sigma_2(c^1)+1}$ if $\sigma_1(C^1) > 0$. Since all information sets are reached with positive probability if $\sigma_1(C^1) > 0$, the belief $\mu_1(y)$ is trivially consistent, and so any equilibrium involving $\sigma_1(C^1) > 0$ will be a sequential equilibrium.

Now, argue that *even if* $\sigma_1(C^1) = 0$, the belief $\mu_1(y) = \frac{19\sigma_2(c^1)}{19\sigma_2(c^1)+1}$ is still the only consistent belief. Do this in two steps:

- a. Show that if $\sigma_1(C^1) = 0$ and $\sigma_2(c^1) > 0$, for any sequences $\epsilon^k \rightarrow 0$, $\alpha^k \rightarrow \sigma_2(c^1)$, the limit of the beliefs $\mu^k(y)$ implied by strategies $\sigma_1^k(C^1) = \epsilon^k$ and $\sigma_2^k(c^1) = \alpha^k$ is $\frac{19\sigma_2(c^1)}{19\sigma_2(c^1)+1}$
- b. Show that if $\sigma_1(C^1) = 0$ and $\sigma_2(c^1) = 0$, an implication of consistency implies that $\mu_1(y) = 0$.

Problem 4 It follows from problem 3 that the equilibrium in the “modified centipede” game we solved for in class in which both players continue is the unique sequential equilibrium. Does this game possess any other perfect Bayesian equilibrium profiles?

Problem 5 Ace-King-Queen poker is a two-card game that is played using a deck consisting of three cards: an ace (the high card), a king (the middle card), and a queen (the low card). Play proceeds as follows:

- Each player puts \$1 in a pot in the center of the table.
- The deck is shuffled, and each player is dealt one card. Each player sees only the card he is dealt.
- Player 1 chooses to raise (R) or fold (F). A choice of R means that player 1 puts an additional \$1 in the pot. Choosing F means that player 1 ends the game, allowing player 2 to have the money already in the pot.
- If player 1 raises, then player 2 chooses to call (c) or fold (f). A choice of c means that player 2 also puts an additional \$1 in the pot; in this case, the players reveal their cards and the player with the higher card wins the money in the pot.

a. Draw the extensive form of this game.

b. Find all sequential equilibria of this game.

c. If you could choose to be either player 1 or player 2 in this game, which player would you choose to be?

d. **(optional, do not turn in)** Suppose we modify the game as follows: instead of choosing between raise and fold, player 1 chooses between raise (R) and laydown (L). A choice of L means that the game ends, the players show their cards, and the player with the higher card wins the pot. Answer parts b and c for this modified game.

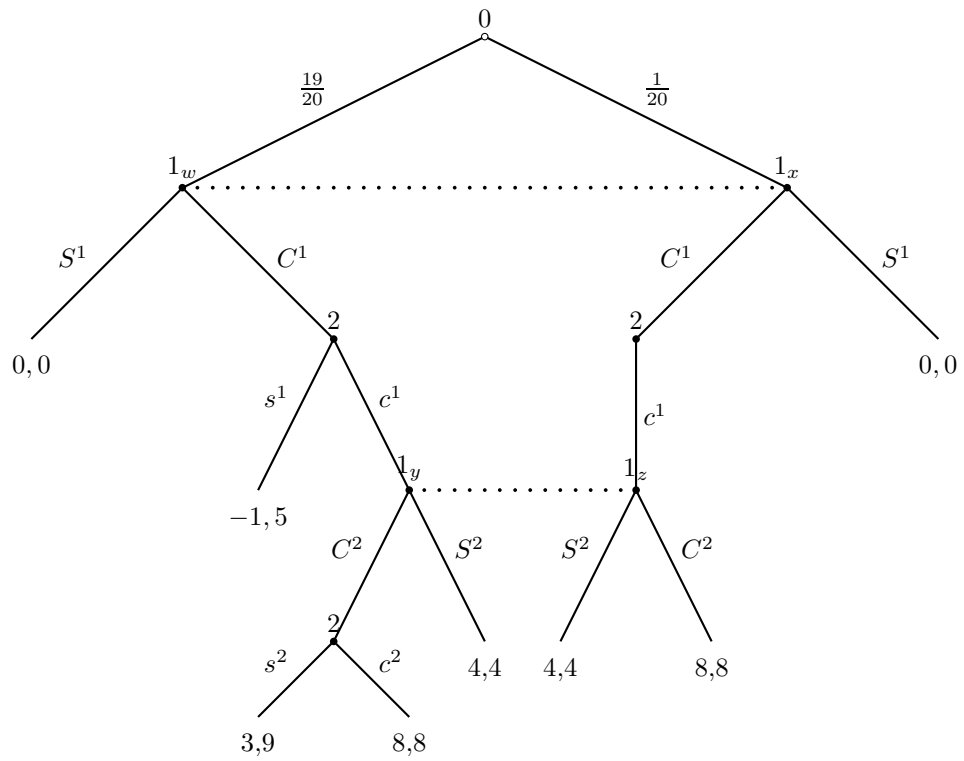


Figure 1: Modified centipede game