Homework 6

answers

Problem 1 In stage 1, each of a large pool of firms decides whether or not to enter a market, at startup cost K. In stage 2, the J firms that decided to enter in stage 1 compete in a Cournot oligopoly. The demand curve for this market is $P(Q) = A - BQ$, and each entrant has cost function $c(q) = K$ (constant marginal cost of zero).

a. Solve for the subgame perfect equilibrium number of firms, as a function of K.

If there are J firms active, each firm will produce $\frac{A}{B(J+1)}$, and so the price will be $\frac{A}{J+1}$. Each firm will earn profit equal to $\frac{A^2}{B(J+1)^2)}$, meaning there will be $\lfloor \frac{A}{\sqrt{KB}} - 1 \rfloor$ firms in the market (the $\lfloor \cdot \rfloor$ function returns the highest integer less than or equal to its argument).

b. As entry cost K approaches zero, the number of firms entering the market in the SPE of the 2-stage game goes approaches ∞. What happens to the total entry cost paid by all J firms, $J * K$? Does it approach 0? ∞? Something in between?

If the entry cost K is paid J times, then the total entry cost paid is no more than $\frac{A\sqrt{K}}{\sqrt{B}} - K$, which converges to 0 as $K \to 0$.

Problem 2 A seller has one unit of a good which she may sell to a buyer. The seller has private information about her valuation of the good, v, which is drawn from $[0, 1]$ according to the uniform distribution. When the seller's valuation of the good is v, the buyer's valuation is kv, where $k > 1$. The buyer does not observe his valuation, however, but does have accurate knowledge of the distribution of the seller's valuation. Both players are risk-neutral.

a. Suppose that the buyer makes a take-it-or-leave-it offer to the seller. That is, the buyer offers a price at which he is willing to buy, the seller either accepts or rejects, and rejection results in no sale. Describe all subgame perfect equilibria in pure strategies. How does your analysis depend on the value of k?

If the buyer makes an offer of p, the seller will accept it iff $v \leq p$. The buyer's expected utility from an offer of p is then $E[kv|v \leq p] = \frac{kp}{2}$. If $k < 2$, this is less than p for all p, and so the buyer's optimal offer is $p = 0$ (in other words, the buyer is never willing to buy). If $k \geq 2$, the buyer's expected utility as a function of his offer p is $\frac{kp}{2} - p$ for all $p \leq 1$, which is a strictly increasing function of p, and so the buyer's optimal offer is $p = 1$.

b. Suppose now that the seller makes a take-it-or-leave-it offer. That is, the seller charges a price, the buyer either accepts or rejects, and rejection results in no sale. Describe all perfect Bayesian equilibria in pure strategies. How does your analysis depend on the value of k?

A strategy for the seller is a function $p(v)$ mapping her observed value of v into a price p. Focus first on equilibria in which offer $p(v)$ is accepted for all v. Clearly, $p(v)$ cannot be increasing, as were $p(v') < p(v'')$ for some $v' < v''$ the seller with $v = v'$ would prefer to deviate from price $p(v')$ to $p(v'')$. A similar argument says that $p(v)$ cannot be decreasing. Therefore, in any such equilibrium, $p(v) = p$ for all v. Now, under Bayesian beliefs, a buyer will accept such an offer iff $p \leq \frac{k}{2}$, meaning the optimal price for the seller is $p = \frac{k}{2}$ so long as $k \geq 2$. If $k < 2$, there is no equilibrium in which the seller's offer is always accepted.

Now, there are always equilibria in which the seller's offer $p(v)$ is rejected for all v (eg. seller sets $p(v) = \$1M$ for all v, buyer always rejects, and believes that the quality behind any price other than \$1M is 0). If $k \geq 2$, there are also equilibria in which the seller's offer is accepted for v in some nonempty subset of $[0, 1]$, though the above logic implies that this subset must be $[0, \overline{v}]$ for $\overline{v} < 1$ (example: $p(v) = \frac{k}{4}$ for $v \in [0, \frac{1}{2}]$, $p(v) = \$1M$ for $v > \frac{1}{2}$, with appropriate beliefs). This last class of equilibria does not seem very compelling. So basically there are equilibria in which the item is always sold for price $\frac{k}{2}$ ($k \geq 2$ only) and equilibria in which the item is never sold (all k).

Problem 3 Consider a labor market in which two firms are hiring workers of unknown productivity θ. In this market, both firms simultaneously post wage offers, which workers than decide to accept or not. Firms also have the option of shutting down and making zero profit. If a worker of productivity θ is hired by a firm, he is able to produce θ units; if he is hired by neither firm, he has reservation utility $r(\theta) = b\theta - a$. Interpret $b\theta$ as the amount a worker can produce at home, and a as the stigma from being unemployed. The productivity θ is distributed uniformly on $[0, 1]$. An equilibrium wage in this market is defined as a SPE of the game described above. Assume that a worker who is indifferent between working and staying at home will choose to work.

a. Give necessary conditions for a and b such that the equilibrium wage is an interior solution, i.e. one in which both employed and unemployed workers are of positive measure.

Any interior equilibrium wage w satisfies $E[\theta | r(\theta) \leq w] = w$, or $w = \frac{a}{2b-1}$. For there to be at least some unemployed workers at this wage, we need $r(1) > \frac{a}{2b-1}$, or $a < b - \frac{1}{2}$. For there to be at least some employed workers, we need $a > 0$.

b. Assuming the conditions you have derived in a., what is the equilibrium wage in this market? Is the wage increasing or decreasing in the stigma of being unemployed? Give an intuitive explanation.

The equilibrium wage $\frac{a}{2b-1}$ is increasing in the stigma to being unemployed a. Evidently, this is because as a increases, the average quality of workers who will take any given wage offer increases, and so the $E[\theta | r(\theta) \leq w]$ curve shifts up. What this means is that, beginning from equilibrium, if a were to increase, labor demand would exceed labor supply, and so wage w must increase in order to equilibriate the market.

Now suppose that the game described above is repeated twice, and that furthermore, workers are not anonymous. Specifically, at the end of the first period, each firm observes the wage posted by the other firm and the identity of all workers who have accepted a wage offer by either firm. To rule out any multiplicities, assume that firms do not post any offers that they expect no one to accept, and that in equilibrium either both firms shut down or none do.

c. Assume again the conditions you derived in a. Call a wage schedule *separating* if it never offers the same second-period wage to both the previously unemployed and the previously unemployed. Show that in any SPE of this game, if firms do not offer a separating wage schedule, they must post the same wage to all workers in both periods. Go on to argue that such a constant wage schedule can never be an equilibrium.

d. Now assume that $b = \frac{5}{4}$ and $a = \frac{1}{4}$. Derive the equilibrium wage schedules (you can assume that all wages are interior).

Let w_e be the equilibrium wage paid to workers who were employed in period 1, and w_u the wage paid to workers unemployed in period 1. For any first period wage w_1, a worker will accept if $w_1 + w_e \geq r(\theta) + w_u$, or if $\theta \leq \frac{4}{5}w_1 + \frac{4}{5}w_e - \frac{4}{5}w_u - \frac{1}{5}$. Therefore, in a competitive labor market, $w_1 = \frac{1}{6} - \frac{2}{3}[w_u - w_e]$, and w_1 is accepted by any worker with $\theta \leq \frac{1}{3} - \frac{4}{3}[w_u - w_e]$.

Now, second period offer w_u is accepted by a previously unemployed worker if $\theta \leq \frac{4}{5}w_u + \frac{1}{5}$. Therefore, the set of workers accepting offer w_u is $[\frac{1}{3} - \frac{4}{3}[w_u - w_e], \frac{4}{5}w_u + \frac{1}{5}]$. w_u must equal the expectation of θ over this set, or $w_u = \frac{4}{19} + \frac{10}{19} w_e$. Finally, in an interior solution, we must have $w_e = w_1$. Solving this system of three equations in three unknowns yields

$$w^1 = \frac{1}{26}, w_e = \frac{1}{26}, w_u = \frac{3}{13}$$