Homework 7

answers

Problem 1 Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$2,000, and if an unqualified worker is assigned to the good job the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^2. The probability that an unqualified worker will have a test score less than t is t. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of \$4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $\$3,000$, for both A-workers and B-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

a. Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?

The fair bet condition simplifies to $4s = \frac{1-\pi}{\pi}$, while the proportion of workers who invest in becoming qualified is $\pi = \frac{4}{3}s(1-s)$. An equilibrium is any (π, s) pair that solves both equations. See figure 1 at the end of this writeup. From inspection, there are equilibria at $(\pi, s) = (\frac{1}{3}, \frac{1}{2})$ and $(\pi, s) = (\frac{1}{4}, \frac{3}{4})$ (feel free to solve for these numerically, but if the answers come out to be such round numbers, you should go back and guess and verify to get a good solution). Therefore, with two groups, there is an equilibrium in which $\frac{1}{4}$ of group B workers become qualified and $\frac{1}{3}$ of group A workers become qualified. In this equilibrium, $\frac{7}{12}$ of group A workers take good jobs, while only $\frac{19}{64}$ of group B workers become qualified.

b. Now suppose that employers are subject to a rule that requires the proportion of A-workers assigned to the good job to be the same as the proportion of B-workers. Otherwise employers maximize expected profits. What is the effect of this rule?

Two equilibria are immediate: workers can be treated identically, with standards of either $\frac{1}{2}$ or $\frac{3}{4}$ (that is, both workers can be treated under the same equilibrium from part a). There may or may not also be a patronizing equilibrium, in which, in order to accomodate more group B workers in the good job, the standard s_B is lowered so far that fewer group B workers become qualified. If there is such an equilibrium, it is a solution to the 4 equations from class (the affirmative action condition, the fair bet condition, and the two incentive constraints). Do not spend time trying to solve such difficult systems unless 1-you see some shortcut, or 2- it is clear that you need the solution, i.e. for a paper.

Problem 2 Consider an economy in which there are equal numbers of men and women, and two kinds of jobs, good and bad. Some workers are qualified for the good job, and some are not. Employers believe that the proportion of men who are qualified is $\frac{2}{3}$ and the proportion of women who are qualified is $\frac{1}{3}$. If a qualified worker is assigned to the good job, the employer gains \$1,000, while if an unqualified worker is assigned to the good job, the employer loses \$1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t. The probability that an unqualified worker will have a test score less than t is $t(2-t)$. Employers are subject to a rule that requires the proportion of men assigned to the good job to be the same as the proportion of women. Otherwise, employers maximize expected profits.

a. Find the profit-maximizing policy for an employer. Note that in this problem we take as given employer attitudes towards men and women; they do not need to be determined endogenously.

As π_A and π_B are given exogenously, employers choose only s_A and s_B subject to the affirmative action constrain ((1) below) and the fair bet conditions ((2) below). Note that the fair bet condition corrects a slight error that made its way into my lecture notes:

$$\pi_A(1-F_q(s_A)) + (1-\pi_A)(1-F_o(s_A)) = \pi_B(1-F_q(s_B)) + (1-\pi_B)(1-F_o(s_B)) \tag{1}$$

$$\frac{x_u}{x_u+x_q} = \frac{\frac{1}{2}}{1+\frac{1-\pi_A}{\pi_A}\frac{f_o(s_A)}{f_q(s_A)}} + \frac{\frac{1}{2}}{1+\frac{1-\pi_B}{\pi_B}\frac{f_o(s_B)}{f_q(s_B)}} \tag{2}$$

Modulo any math errors, these reduce to:

$$2s_B^2 - 5s_B = s_A^2 - 4s_A \tag{3}$$

$$s_B = \frac{3-4s_A}{4-4s_A} \tag{4}$$

Solve this system using your prefered numeric method (or, as substituting the second equation into the first results in a cubic in s_A, solve it analytically), we get $s_A = .54$ and $s_B = .456$ (with rounding errors).

b. Test your policy as follows. If you are told that a worker has just barely passed the test (and you are not told whether the worker is male or female), what is the probability that the worker is qualified? Is it the case that such a worker is a fair bet from the employer's point of view? If not, should the policy be adjusted?

Simply put the values of s_A and s_B you solved for back into the fair bet equation, and verify that each side gives you $\frac{1}{2}$. This should tell you that an employer who knows that a workers has just barely passed the test, but whose gender is unknown, is a fair bet, in that the employer is indifferent between assigning such a worker to a good job or a bad job.

Problem 3 Suppose that business travelers have marginal willingness to pay $40-q$ for a seat of quality $q \in [0,40]$, meaning that their total willingness to pay for a seat of quality $\hat{q} \in [0,40]$ is $\int_0^{\hat{q}}(40-q)dq$ (assume that marginal willingness to pay is 0 for $q > 40$). Tourists have marginal willingness to pay of $30-q$ for $q \in [0,30]$, meaning their total willingness to pay for a seat of quality $\hat{q} \in [0,30]$ is $\int_0^{\hat{q}}(30-q)dq$ (assume tourists have marginal willingness to pay of 0 for $q > 30$). Assume that 80 tourists and 20 business travelers typically fly a given route, and the the plane used on this route is more than big enough to hold all 100 travelers, so the airline never has to worry about a capacity constraint. However, the airline cannot tell which type a given traveler is, and so cannot condition price on group membership.

Suppose the airline is able to put two sections on the plane (i.e. 1st class and coach), each with its own quality level. Assume that the cost of setting quality level q in coach is $K_c * q$ and that the cost of setting quality q in 1st class is $K_{fc} * q$, for $K_{fc} \geq K_c$.

a. For parts a-d, set $K_{fc} = K_c = 0$. Suppose the airline sets $q = 30$ in coach and $q = 40$ in 1st class. Solve for the profit maximizing prices, taking these quality levels as given.

It helps to draw a picture with this problem. They will charge coach customers their full willingness to pay of \$450, and business travelers \$500, leaving them 300 surplus, the same amount they would get from buying a coach ticket (in order to incentivize business travelers to buy a 1st class ticket instead of coach. Note that it is also an option to set the price of business class to be \$800, the price of coach to be \$10M (so no one buys a coach ticket), and sell only to business travelers. Since the business travelers are a small percentage of all the travelers here, it is easy to see that this option is not optimal in this case.

b. You are hired as a consultant to advise the airline on how it can increase profits. Explain why decreasing the quality in coach — and in turn decreasing the price — can increase the airline's profit, even if the number of passengers flying the route remains 100, with 80 tourists and 20 business travelers.

Suppose the airline lowers the quality in coach to 28. Then, the price of a coach ticket is lowered to \$448, while the price of a business class ticket is raised to \$520. Thus, 20% of travelers are now paying \$20 more relative to part a, while the remaining 80% are paying only \$2 less, and so profits are higher. The idea is that the lower the quality in coach, the higher the price in 1st class, as the incentive constraint for business travelers has less bite.

c. Solve for the profit-maximizing price and quality levels in both coach and business class.

Maximizing profits means choosing four variables, p_c, p_{FC}, q_c, q_{FC} to maximize $80 * p_c + 20 * p_{BC}$ subject to an incentive constraint for business travelers and an individual rationality constraint for tourists.

First, clearly $q_{FC} = 40$. Second, individual rationality implies $p_c = 450 - \frac{1}{2}(30 - q_c)^2$. Third, the business travelers' incentive constraint implies $p_{FC} = p_c + \frac{1}{2}(40 - q_c)^2$. Therefore, the airlines profit maximizing coach quality level is given by the following unconstrained maximization problem:

$$\max_{q_c} 80 * (450 - \frac{1}{2}(30 - q_c)^2) + 20 * (p_c + \frac{1}{2}(40 - q_c)^2) \tag{5}$$

(drawing a picture will help quite a bit with determining what prices are implied by the constraints). Evidently, the maximizer of the above is $q_c^* = 27.5$, which implies the optimal coach price is \$446.87 and the optimal first class price is \$525.

d. Now suppose that the composition of travelers changes, so that fraction t of all travelers are business travelers, and fraction $1-t$ are tourists (the plane is still plenty big enough to hold all travelers, so constraints like there needing to be more seats in coach than there are passengers are not binding). Solve for the optimal price and quantity levels in coach and 1st class, as a function of t.

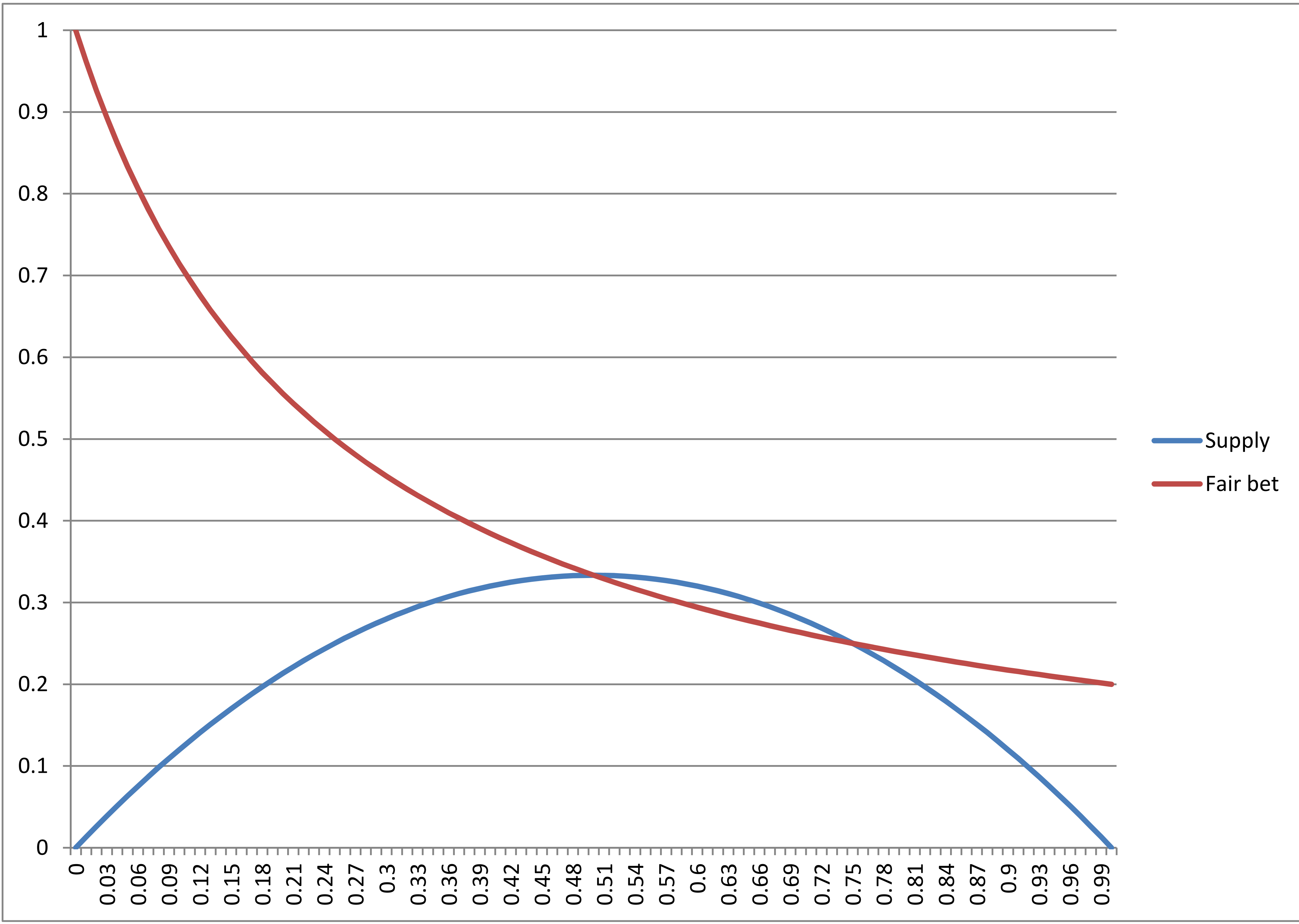
Same setup as in the previous subsection, except we maximize $(1 - t)p_c + tp_{FC}$. The solution is:

$$q_c^* = \max\left\{\frac{30 - 40t}{1 - t}, 0\right\} \tag{6}$$

with prices set accordingly. Note that if the fraction of business travelers is over $\frac{3}{4}$, the airline optimally sells only to business travelers.

e. Finally, suppose that $K_c = 1$ and $K_{fc} = \$K$. Suppose again that there are 80 tourists and 20 business travelers. Solve for the relationship between the price of coach and K, and give an intuitive explanation for why these two variables are related in this way.

A high K would lower the quality set in first class, but this would not change the price or quality set in coach (note that K_c being one will lower the coach quality slightly relative to the case where $K_c = 0$). Again, the easiest way to see this is to draw a picture (see final page). As K_{fc} increases, q_{fc}^* will decrease from 40, but the airline's calculation in determining optimal q_c is unaffected, and so price and quality in coach are unchanged.



Supply
Fair bet
1
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0.6
0.5
0.4
0.3
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0
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0.06
0.09
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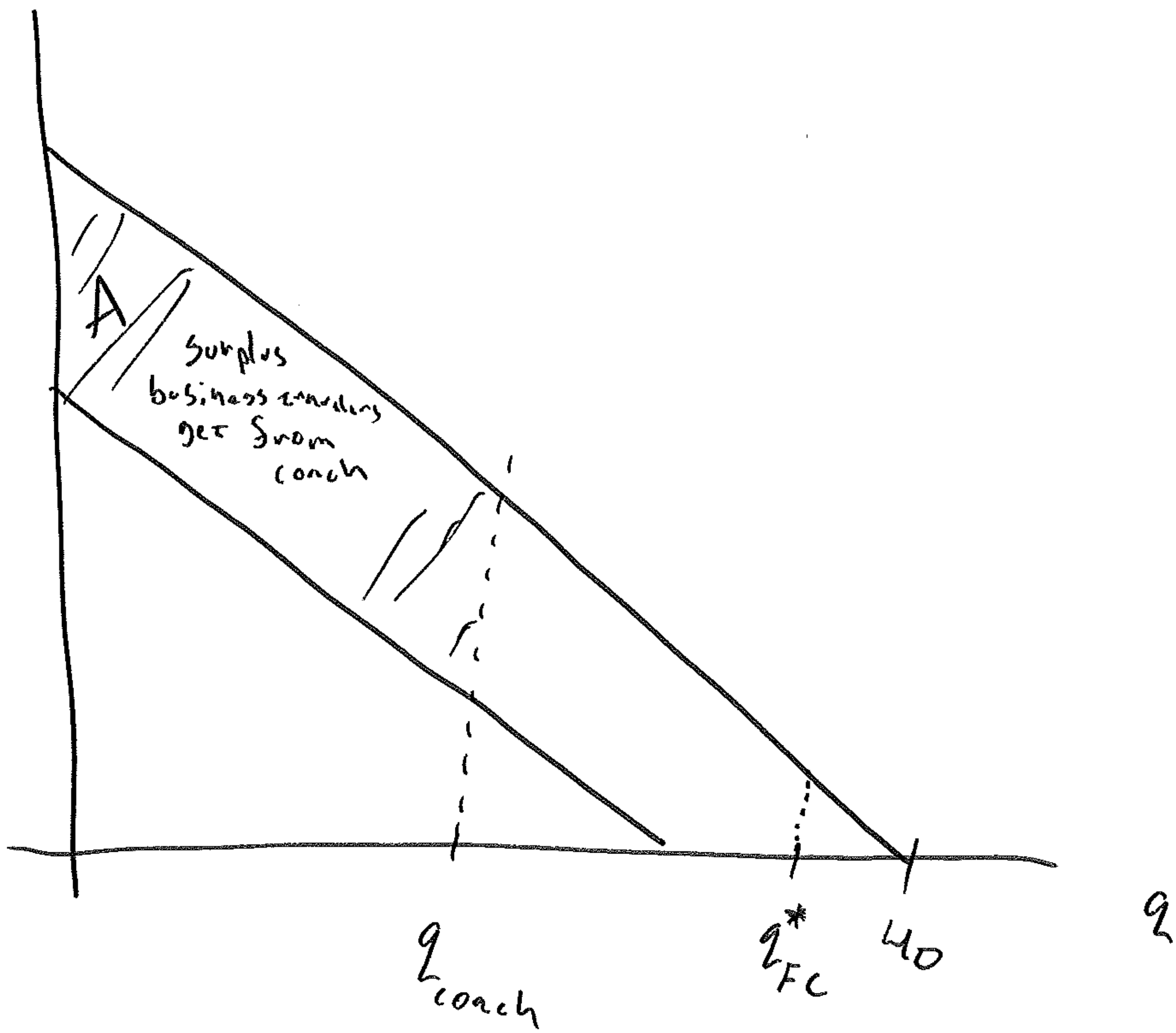

p^*_{FC} equals $\int_0^{q^*_{FC}} 40-q$ — area A

q_{coach} optimally set s.t. MB of lowering q_c (higher price in FC, area A is smaller) equals MC (lower price in coach, $\int_0^{q_c} 30-q$ is smaller). Unaffected by q^*_{FC}