

Midterm exam

3/4/11

Note: Throughout, points will be deducted mercilessly for insufficiently supported answers. When in doubt, err on the side of maximum verbosity.

Problem 1 (20 points (5, 5, 5, 5)) This problem asks you to produce examples of extensive form games with various characteristics. Be sure to give a complete description of the game (a picture is fine), as well as a clear explanation about why it has the required properties.

- a. Give an example of a game that has a Nash equilibrium which is not subgame perfect.
- b. Give an example of a game that has a subgame perfect equilibrium which is not perfect Bayesian.
- c. Give an example of a game that has a perfect Bayesian equilibrium which is not subgame perfect.
- d. Give an example of a game in which every Nash equilibrium is also a sequential equilibrium.

Problem 2 (36 points (14, 14, 8)) Consider the normal form game G in figure 1.

- a. Compute all Nash equilibria of G .
- b. Sketch the set of payoffs which are obtainable in a Nash equilibrium of the infinitely repeated game $G^\infty(\delta)$ if δ is close enough to 1.
- c. Suppose that σ is a strategy profile for $G^\infty(\delta)$ which yields payoffs outside of the closure of the set you sketched in part b. (this means that the payoffs are neither in the set, nor on the boundary of the set). Prove that σ is not a Nash equilibrium of $G^\infty(\delta)$.

		2		
		a	b	c
A	7, 1	4, 7	2, 4	
1 B	5, -2	5, 4	1, 0	
C	2, 5	8, 3	2, 4	

Figure 1: Normal form game G

Problem 3 (15 points) For this question, let G denote a 2-player zero-sum game (hint: while G is an arbitrary zero-sum game, it may be helpful to think of a particular, simple zero-sum game in answering this question, such as matching pennies).

- a. Suppose that $\sigma_1 \in \Delta S_1$ is a rationalizable strategy for player 1. Must σ_1 be a maxmin strategy for player 1? Provide a proof or counterexample.
- b. Suppose that $\sigma_1 \in \Delta S_1$ is a maxmin strategy for player 1. Must σ_1 be a rationalizable strategy for player 1? Provide a proof or counterexample.

Problem 4 (14 points) Suppose that players 1 and 2 play the game below, and that it is common knowledge between them that both of them are rational. If we make no other assumptions about the players' knowledge, what is our best prediction about how they will play the game?

		2				
		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
1	<i>A</i>	1, 4	0, 6	5, 2	2, 3	2, 5
	<i>B</i>	2, 2	3, 1	3, 1	4, 3	3, 3
	<i>C</i>	7, 3	3, 5	0, 0	3, 2	0, 0

Problem 5 (15 points (10, 5)) Consider the 3-player game between two entrants ($E1$ and $E2$) and an incumbent (I) in figure 2. Entrant $E1$ will either enter on his own or as part of a joint venture with entrant $E2$. If entry occurs, firm I chooses between fighting entry with a price war and acquiescing to normal competition. Firm I cannot observe whether or not entry was the result of a joint venture.

- Find all perfect Bayesian equilibria of this game.
- Explain why the perfect Bayesian refinement is needed in this game. Why is NE or SPE unsatisfactory?

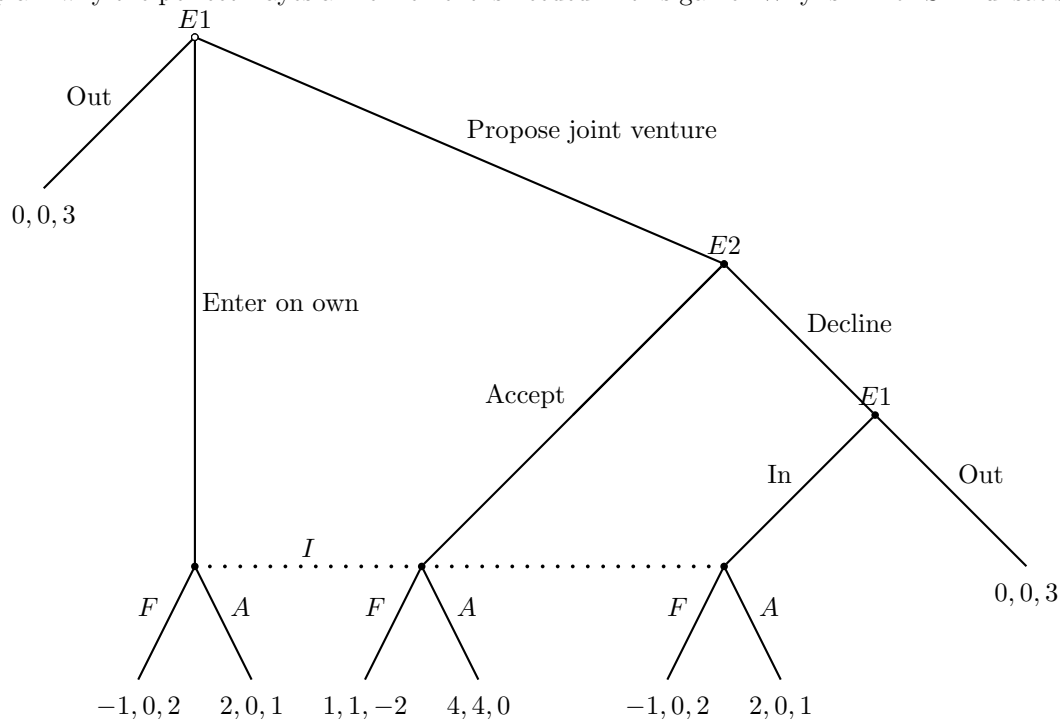


Figure 2: modified chain store game