Dominance

In a normal form game $G = \{I, \{S_i\}_{i \in I}, \{u_i\}_{i \in I}\}$, let $S_{-i} = \Pi_{j \neq i} S_j$ be the set of of pure strategy profiles for i's opponents (i.e. one strategy for each of player i's opponents. Then,

Definition: A pure strategy $s_i \in S_i$ is **strictly dominant** if

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \text{ for all } s_i' \neq s_i \text{ and } s_{-i} \in S_{-i}, \text{ where} \tag{1}$$

Note:

- A mixed strategy can never be strictly dominant.
- To check if a strategy is strictly dominant, it suffices to check only against pure strategy profiles.
- If the inequality in (1) is made weak, we say s_i is **weakly dominant**.

Let $\Sigma_{-i} = \Pi_{j \neq i} \Delta S_j$ be the set of mixed strategy profiles for i's opponents (i.e. one mixed strategy for each of i's opponents).

Definition: A (possibly mixed) strategy σ_i is **strictly dominated** if there exists $\sigma_i' \in \Sigma_i$ such that

$$u_i(\sigma_i', s_{-i}) > u_i(\sigma_i, s_{-i}) \text{ for all } s_{-i} \in S_{-i} \tag{2}$$

Notes:

- A strategy that is not dominated by any of one's own pure strategies may be dominated by a mixed strategy.
- If a pure strategy s_i is strictly dominated, then so is any mixed strategy σ_i with s_i in its support.
- Even if none of a set of pure strategies is dominated, a mixed strategy that combines them may be.
- If the inequality in (2) is made weak, we say σ_i is **weakly dominated**.