Extensive form game notation

An extensive form game consists of:

- A set of players $I = \{1, 2, ..., n\}$
- A set of nodes $X = D \bigcup T$. T is the set of terminal nodes (nodes at which the game ends and payoffs are realized). D is the set of decision nodes, nodes at which one player makes a decision about which action to play.
- A set of **information sets** \mathcal{H} . \mathcal{H} is a partition of X. Each information set consists of one or more nodes at which one agent makes a decision; there are more than one node in an information set only if the agent is uncertain about what has happened previously. Let $\mathcal{H} = H_0 \bigcup H_1 \bigcup ... \bigcup H_n$, i.e. \mathcal{H} is composed of each individual player's information sets.
- A set of edges defined by $e: X \to X \bigcup \{\phi\}$, where e(x) is the node immediately preceding node $x \in X$. $e(x_0) = \phi$ iff x_0 is the **initial node**.
- A set of actions A_h available at information set h, for all $h \in \mathcal{H}$.
- A list of which actions are required to get to each node, α : X − {x₀} → U_{h∈H} A_h. What this function does is list the action immediately preceeding each node in the game, minus the initial node. If we know this function, it allows us to say, for example "to get to node 27, player 1 must play action 3, then player 5 must play action 1, and then player 2 must play action 3".
- **payoffs** at terminal nodes, $u_i: T \to \mathbb{R}$.

As this is a lot of information to keep track of, in practice, whenever possible, we draw a picture of the game. A good picture will include all of the information above in an easier-to-understand form.