Repeated games notation

In the stage game $G = \{I, \{A_i\}_{i\in I}, \{u_i\}_{i\in I}\}$, recall that:

- $a_i \in A_i$ is a pure strategy
- $\alpha_i \in \Delta A_i$ is a mixed strategy
- $\alpha \in_i \Delta A_i$ is mixed strategy profile

In the *finitely repeated game* G^T,

- the stage game G is repeated $T + 1$ times, beginning in period 0 and ending in period T.
- Before playing period t, the results of all previous periods are observed.
- Payoffs are the discounted sum of stage game payoffs; all players share the same discount factor.
- Notation:
  - $h^t \in H^t = \left\{(a^0, a^1, ..., a^{t-1}) : a^\tau \in A\right\}$ gives histories at the beginning of period $t \geq 1$
  - $H^0 = \{h^0\}$ is the null history
  - $H = \bigcup_{t=0}^{T} H^t$ is the set of all possible histories
  - $s_i : H \to A_i$ is a pure strategy
  - $\sigma_i : H \to \Delta A_i$ is a behavior strategy
  - H^{T+1} is the set of terminal histories (all possible ways the game can be played out over periods $0, 1, ..., T$)
  - $\delta \in (0, 1)$ is the common discount rate.
  - $\pi : H^{T+1} \to \mathbb{R}$, $\pi_i(h^{T+1}) = \sum_{t=0}^{T} \delta^t u_i(a^t)$ is player i's payoff function

In the *infinitely repeated game* G^∞,

- Same rules as G^T except game either never ends or has an uncertain ending.
- Notation:
  - $H = \bigcup_{t=0}^{\infty} H^t$ is the set of histories
  - $H^\infty = \left\{(a^0, a^1, ...) : a^\tau \in A\right\}$
  - $s_i : H \to A_i$ is a pure strategy
  - $\sigma_i : H \to \Delta A_i$ is a behavior strategy
  - $\delta \in (0, 1)$ is the common discount rate
  - $\pi_i : H^\infty \to \mathbb{R}$, $\pi_i(h^\infty) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_i(a^t)$ is i's payoff function