Homework 2

due 2/8/2012

Problem 1 A newspaper runs the following contest: Each participant mails in a postcard on which he writes an integer between 0 and 1000 (inclusive). Given the entries, the *target integer* is defined to be $\frac{9}{10}$ times the highest entry, rounding downward if the result is not an integer. All participants who chose the target integer split a $10,000 prize.

a. Suppose this contest is modeled as a simultaneous move game among 100 players. Using only common knowledge of rationality, determine a unique prediction of play.

b. If you entered such a contest, what number would you personally play, and why?

Problem 2 Consider a 3-player, simultaneous move game with $S_1 = \{L, M, R\}$, $S_2 = \{U, D\}$, and $S_3 = \{l, r\}$. Figure 1 gives **player 1's payoffs** from each of his three pure strategies conditional on the strategy choices of players 2 and 3. So, for example, if 2 plays U and 3 plays l, $u_1(L) = \pi + 4\epsilon$, $u_1(M) = \pi - \eta$, and $u_1(R) = \pi - 4\epsilon$. Assume that pi, ϵ, and η are strictly greater than 0, and that $\eta < 4\epsilon$.

		Player 3's strategy	
		l	r
Player 2's strategy	U	$\pi + 4\epsilon, \pi - \eta, \pi - 4\epsilon$	$\pi - 4\epsilon, \pi + \frac{\eta}{2}, \pi + 4\epsilon$
	D	$\pi + 4\epsilon, \pi + \frac{\eta}{2}, \pi - 4\epsilon$	$\pi - 4\epsilon, \pi - \eta, \pi + 4\epsilon$

Figure 1: Player 1's payoffs $(u_1(L), u_1(M), (u_1(R))$ are depicted.

a. Argue that pure strategy M is never a best response for player 1 to any mixed strategy combinations for players 2 and 3.[1]

b. Show that pure strategy M is not strictly dominated for player 1.

c. Which (generically) eliminates more strategies? Iterated removal of strictly dominated strategies or iterated removal of non-rationalizable strategies?

Problem 3 Find all Nash equilibria of the normal form game in figure 2:

		2		
		a	b	c
	A	2, 4	10, 2	2, 0
1	B	4, 2	8, 8	0, 0
	C	0, 2	0, 0	0, 0

Figure 2: game for problem 3

[1]Hint: this problem is from MWG, and I'm not sure why three variables are needed to write player 1's utilities. I would start by simplifying player 1's utility. To do so, remember that VNM utility functions are only unique to affine transformations.

Problem 4 Consider the simultaneous-move game in figure 3.

		2: a	b	c
	x	$6,0$	$0,1$	$0,1$
1	y	$0,1$	$6,0$	$0,1$
	z	$5,1$	$5,1$	$5,0$

Figure 3: game for problem 4

a. Draw the best response correspondences for player 1 and player 2.

b. Describe the set of rationalizable strategies for player 1 and player 2.

c. Find all Nash equilibria of this game.

Problem 5 Compute all Nash equilibria of the reduced normal form of the game in figure 1 of HW1.

Problem 6 Compute all Nash equilibria of the symmetric normal form game in figure 4:

		2: L	C	R
	T	$0,0$	$6,-3$	$-4,-1$
1	M	$-3,6$	$0,0$	$5,3$
	B	$-1,-4$	$3,5$	$0,0$

Figure 4: game for problem 6

Problem 7 Consider the following payoffs for player 1 (player 2's payoffs are irrelevant to this question):

		2: l	r
	T	$3,\cdot$	$0,\cdot$
1	M	$0,\cdot$	$3,\cdot$
	B	$2,\cdot$	$2,\cdot$

Figure 5: game for problem 7

We saw in class that all mixtures of T and M are strictly dominated by some other strategy. Identify, for each mixture of T and M, a strategy that strictly dominates that mixture. That is, for the mixture $\alpha T + (1-\alpha)M$, there is some strategy $\sigma(\alpha) \in \Sigma_1$ which strictly dominates $\alpha T + (1-\alpha)M$, and which does not put positive probability on both T and M. Find $\sigma(\alpha)$ for all $\alpha \in (0,1)$.